WESTERN REGION

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

1997

MATHEMATICS

3 Unit (Additional)
and
3/4 Unit (Common)

Time allowed - TWO hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- ALL necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on the last page. These may be removed for your convenience.
- Board-approved calculators may be used.
- Each question should be started on a new page.

Question 1 *Start a new page* **Marks**

(a) Evaluate $\displaystyle\int_0^1 \frac{dx}{x^2+3}$ **2**

(b) Solve the inequality $\dfrac{4x+3}{x-4} > 1$ **2**

(c) Find the acute angle between the lines $x + 2y = 5$ and $x = 3y - 3$ **2**

(d) Find the second derivative of $\dfrac{1}{\sqrt{x}}$ **2**

(e) Given that $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$, find the exact value of $\sin 75°$ **2**

(f) If α and β are the roots of the quadratic equation $x^2 + px + q = 0$, write down the value of $\alpha^2 + \beta^2$ **2**

Question 2 *Start a new page* **Marks**

(a) It is known that the equation $e^{-x^2} - 5x^2 - 0\cdot 99 = 0$ has a positive root close to the origin. Attempt to find the root using one application of Newton's method, starting with $x = 0$ as the first approximation and hence explain why Newton's method fails with $x = 0$ as the first approximation. **3**

(b) The motion of a particle is described by the equation $x = 3\sin\left(\frac{t}{3}\right)$ **3**

Find the acceleration of the particle when $t = \frac{\pi}{4}$

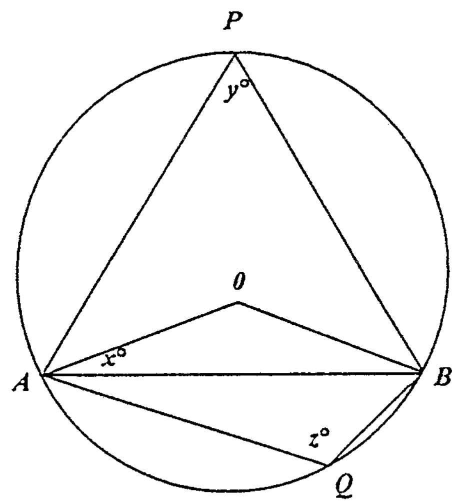
(c) Show that the derivative of $x\tan x - \ln(\sec x)$ is $x\sec^2 x$ **3**

Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{4}} x\sec^2 x\,dx$

(d) Draw a neat sketch of the graph $y = \sin^{-1} x$ and state its domain and range. **3**

Question 3 *Start a new page* **Marks**

(a) O is the centre of the circle 4



(i) Prove that $x + y = 90°$

(ii) Prove that $z - y = 2x$

(b) The area under the curve $y = \sin x$ for $0 \leq x \leq \pi$ is rotated about the x axis. 3
Calculate the volume of the solid generated.

(c) A particle P is moving in simple harmonic motion. The displacement of P is 5
related to the time t by the equation $x = 3\cos 2\pi\, t$

(i) What is the initial position of the particle?

(ii) When will P first pass through the centre of motion?

(iii) What is the maximum velocity of the particle?

(iv) When does the maximum acceleration <u>first</u> occur?

Question 4 *Start a new page* **Marks**

(a) P is the point $(2at, at^2)$ on the parabola $x^2 = 4ay$ and ℓ is the tangent at P. S is the focus. 5

(i) Prove that the equation of ℓ is $y = tx - at^2$

(ii) If ℓ cuts the y axis at A, show that A is the point $(0, -at^2)$

(iii) Show that ℓ makes equal angles with the y axis and with the interval PS.

(b) Six students are to be selected at random from a group of 10 to form a committee. The group of 10 consists of 6 males and 4 females. 3

(i) How many such unordered selections are possible if there are no restrictions?

(ii) In how many ways can the committee be chosen if it must contain at least 1 male and 1 female?

(iii) The committee contains 2 males and 4 females. In how many ways can they sit around a circular table if those of the same sex sit together?

(c) The volume of a sphere is increasing at the rate of $5cm^3$ per second. At what rate is the surface area increasing when the radius is $20cm$? 4

Question 5 *Start a new page* **Marks**

(a) $x^2 - x - 2$ is a factor of $x^4 + 3x^3 + ax^2 - 2x - b$ **2**

find the values of a and b.

(b) (i) Given that $\cos(a+b) = \cos a \cos b - \sin a \sin b$ **10**

prove that $\cos 2\omega = 1 - 2\sin^2 \omega$

(ii) Prove the identity $$\frac{\cos y - \cos(y + 2\omega)}{2\sin\omega} = \sin(y + \omega)$$

(iii) Use mathematical induction and the result of part (ii) to prove the identity

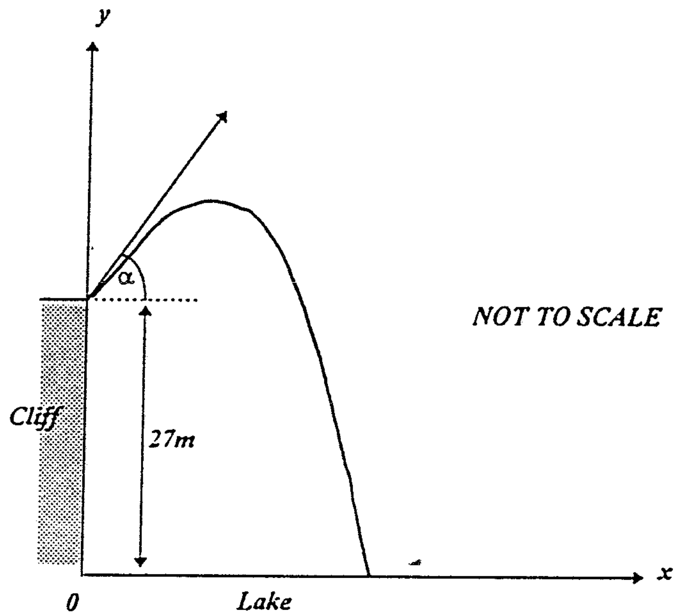
$$\sin\omega + \sin 3\omega + \sin 5\omega + \ldots\ldots\ldots + \sin(2N-1)\omega = \frac{1 - \cos 2N\omega}{2\sin\omega}$$

Question 6 *Start a new page* **Marks**

(a) Use the substitution $u = x + 1$ to evaluate $\int_0^1 x(x+1)^4 \, dx$ 2

(b) Solve the equation $\sin\alpha + \sin 2\alpha = 0$ for $0 \le \alpha < 2\pi$ 3

(c) A stone is projected with a velocity of 10 metres per second at an angle of elevation $\theta = \tan^{-1}\left(\frac{3}{4}\right)$ from the top of a vertical cliff 27 metres high, overlooking a lake. 7



The equations of motion of the stone with air resistance being neglected are

$\ddot{x} = 0$ and $\ddot{y} = -g$

(i) By taking the origin, O, to be the base of the cliff show that the horizontal and vertical components of the stone's displacement from the origin after t seconds is given by: $x = 8t$ and $y = -\frac{1}{2}gt^2 + 6t + 27$

(ii) Hence, or otherwise, calculate the time which elapses before the stone hits the lake and find the horizontal distance of the point of contact from the base of the cliff. (Assume the approximate value of $10 m/s^2$ for g)

(iii) What is the maximum height reached by the stone?

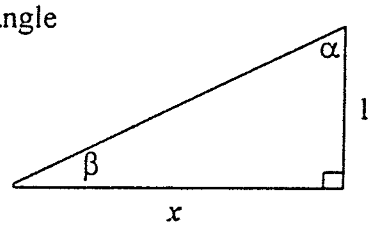
Question 7 *Start a new page* **Marks**

(a) 90% of students will pass an examination. In a sample of 10 students, find the probability that exactly 3 students fail the examination. **2**

(b) (i) Differentiate $f(x) = \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ for $x \neq 0$ and show that $f'(x) = 0$ **5**

(ii) What does the result in (i) imply about $f(x)$

(iii) By considering the given right-angled triangle find the value of $f(x)$



(c) Assume that, for all real numbers x and all positive integers n, **5**

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$$

(i) Show that $0 = \sum_{r=0}^{n} (-1)^r \binom{n}{r}$

(ii) Find a simple expression for $\sum_{r=0}^{n} 2^r \binom{n}{r}$

(iii) Find a simple expression for $\sum_{r=0}^{n} r \binom{n}{r}$

(1)(a) $\frac{\sqrt{3}\,\pi}{18}$

(b) $x < -\frac{7}{3}, x > 4$

(c) 45^0

(d) $\frac{3}{4\sqrt{x^5}}$

(e) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(f) $p^2 - 2q$

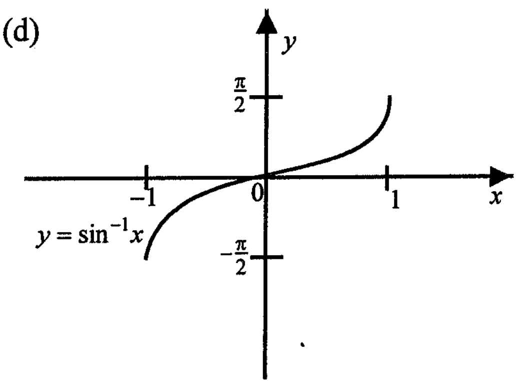
(2) (a) Newton's method fails because

$f'(0) = 0$, so x_2 cannot be found.

(b) $-\frac{1}{3}\sin\frac{\pi}{12}$

(c) Proof; then $\frac{\pi}{4} - \ln\sqrt{2}$

(d)



$D: -1 \le x \le 1; R: -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(3)(a)(i) Proof (ii) Proof

(b) $\frac{\pi^2}{2}$ units^3

(c) (i) $x = 3$ (ii) $t = \frac{1}{4}$

(iii) $v_{\max} = 6\pi$ (iv) $t = 0$

(4) (a) (i) Proof (ii) Proof (iii) Proof

(b) (i) 210 (ii) 209 (iii) 48

(c) $\frac{1}{2}$ cm^2/s

(5) (a) $a = b = -12$

(b) (i) Proof (ii) Proof (iii) Proof

(6)(a) (i) $4\frac{3}{10}$

(b) $\alpha = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$

(c) (i) Proof (ii) 24 m (iii) $28\frac{4}{5}$ m

(7) (a) $\frac{4(3^{15})}{10^9}$

(b) (i) Proof

(ii) Graph is parallel to the x–axis

(iii) $f(x) = \frac{\pi}{2}$

(c) (i) Proof (ii) 3^n (iii) $n.2^{n-1}$

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MATHEMATICS

3/4 Unit Common

Solutions and Marking Scheme

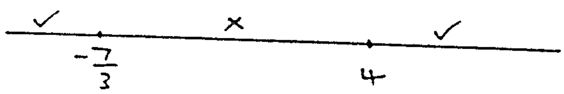
PLEASE NOTE:

- These are suggested solutions. They are not intended to specify the amount of working required or the method to be applied.

  Teachers should accept any valid method of solution providing adequate working is shown.

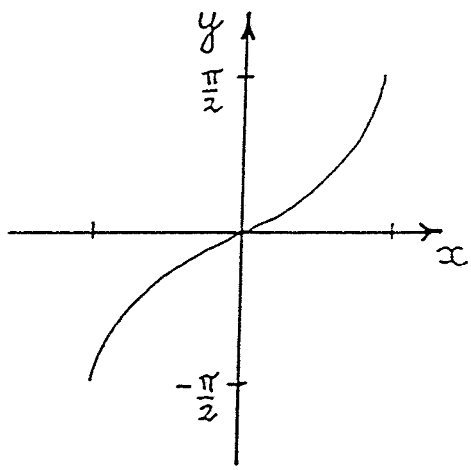
1997 Western Region Trial H.S.C. Marking Scheme

Course: 3 Unit Mathematics

Solutions	Marks	Comments
Question 1		
(a) $\int_0^1 \frac{dx}{x^2+3} = \left[\frac{1}{\sqrt{3}} \cdot \tan^{-1}\frac{x}{\sqrt{3}}\right]_0^1$	1	
$= \frac{1}{\sqrt{3}} \cdot \frac{\pi}{6} - \frac{1}{\sqrt{3}} \cdot 0$		
$= \frac{\pi}{6\sqrt{3}}$		
$= \frac{\sqrt{3}\,\pi}{18}$	1	
(b) $\frac{4x+3}{x-4} > 1$		
Consider $\frac{4x+3}{x-4} = 1$		
$x \neq 4$ and $4x+3 = x-4$		
$3x = -7$		
$x = \frac{-7}{3}$		
Testing regions:		
		
$\therefore$ Solutions are $x < \frac{-7}{3}$, $x > 4$	1 each	
(c) If $x+2y=5$ $2y = -x+5$ $y = -\frac{1}{2}x + \frac{5}{2}$ $\therefore m_1 = -\frac{1}{2}$ If $x = 3y-3$ $3y = x+3$ $y = \frac{1}{3}x + 1$ $\therefore m_2 = \frac{1}{3}$	1	
So $\tan\theta = \left\lvert \frac{m_1 - m_2}{1 + m_1 m_2} \right\rvert$		
$= \left\lvert \frac{-\frac{1}{2} - \frac{1}{3}}{1 + -\frac{1}{2} \times \frac{1}{3}} \right\rvert$		
$= \left\lvert \frac{-\frac{5}{6}}{\frac{5}{6}} \right\rvert$		
$= 1$		
$\therefore \theta = \frac{\pi}{4}$ or 45°	1	

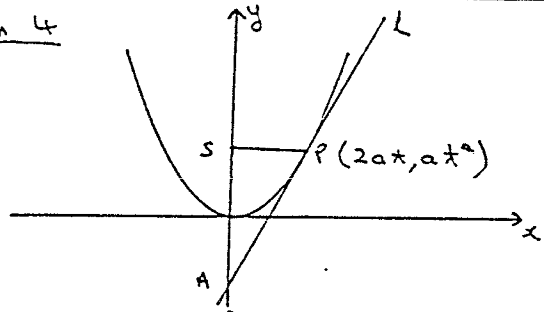
Solutions	Marks	Comments
Question 1 continued		
(d) $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$		
$f'(x) = -\frac{1}{2}x^{-3/2}$	1	
$f''(x) = \frac{3}{4}x^{-5/2}$	1	
$= \frac{3}{4\sqrt{x^5}}$		
(e) $\sin 75° = \sin(30+45)$		
$= \sin 30°.\cos 45° + \cos 30°.\sin 45°$	1	
$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$		
$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$		
$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$		
$= \frac{\sqrt{6}+\sqrt{2}}{4}$	1	
(f) $x^2 + px + q = 0$		
So $\alpha + \beta = \frac{-b}{a} = -p$		
and $\alpha\beta = \frac{c}{a} = q$	1	
$\therefore \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$		
$= (-p)^2 - 2 \times q$		
$= p^2 - 2q$	1	

Solutions	Marks	Comments
Question 2		
(a) $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$		
Now $f(x) = e^{-x^2} - 5x^2 - 0.99$		
so $f'(x) = e^{-x^2} \times (-2x) - 10x$		
$= -2x\,e^{-x^2} - 10x$	1	
And $f(0) = 1 - 0 - 0.99 = 0.01$		
$f'(0) = 0 - 0 = 0$		
$\therefore\ x_2 = 0 - \dfrac{0.01}{0}$??	1	
1. This method fails because $f'(0) = 0$ and division by zero has no meaning. 2. The value $x = 0$ chosen as the first approx. is a stationary point and the tangent drawn at this point does not cut the x axis – which is the basis for Newton's method.	1	
(b) $x = 3\sin\dfrac{t}{3}$		
$\dot{x} = 3\cos\dfrac{t}{3} \cdot \dfrac{1}{3}$		
$= \cos\dfrac{t}{3}$	1	
$\ddot{x} = -\sin\dfrac{t}{3} \times \dfrac{1}{3}$		
$= -\dfrac{1}{3}\sin\dfrac{t}{3}$	1	
when $t = \dfrac{\pi}{4}$, $\ddot{x} = -\dfrac{1}{3}\sin\dfrac{\pi}{12}$	1	
(c) Firstly, if $w(x) = \sec x = \dfrac{1}{\cos x}$		
then $w'(x) = -1 \times (\cos x)^{-2} \times -\sin x$		
$= \dfrac{\sin x}{\cos^2 x}$		

Solutions	Marks	Comments
Question 2 continued		
So if $f(x) = x.\tan x - \ln(\sec x)$		
$f'(x) = x.\sec^2 x + \tan x - \dfrac{\frac{\sin x}{\cos^2 x}}{\sec x}$		
$= x\sec^2 x + \tan x - \dfrac{\sin x}{\cos^2 x} \times \dfrac{\cos x}{1}$		
$= x\sec^2 x + \tan x - \dfrac{\sin x}{\cos x}$		
$= x\sec^2 x$	2	
$\therefore \int_0^{\pi/4} x\sec^2 x\,dx = \left[x\tan x - \ln(\sec x)\right]_0^{\pi/4}$		
$= \left(\frac{\pi}{4}\tan\frac{\pi}{4} - \ln\sec\frac{\pi}{4}\right) - (0 - \ln\sec 0)$		
$= \frac{\pi}{4} - \ln\sqrt{2} + \ln(1)$		
$= \frac{\pi}{4} - \ln\sqrt{2}$	1	
(d) 	3	1 for curve 1 for DOMAIN 1 for RANGE
DOMAIN: $-1 \leq x \leq 1$		
RANGE: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$		

Solutions	Marks	Comments
Question 3		
(a)(i) $2\angle APB = \angle AOB$ (angle at the centre is twice the angle at the circumference)	1	
i.e. $2y = \angle AOB$		
Now $\angle OAB = \angle OBA = x^\circ$ (isosceles Δ)		
so. $\angle OAB + \angle OBA + \angle AOB = 180^\circ$ (angle sum of a Δ)		
i.e. $x + x + 2y = 180^\circ$		
$2(x+y) = 180^\circ$		
$x + y = 90^\circ$	1	
(ii) $z + y = 180^\circ$ (opposite angles of a cyclic quadrilateral are supplementary)	1	
and $x + y = 90^\circ$ (from (i) above)		
so $z + y = 2(x+y)$		
$z + y = 2x + 2y$		
$z + y - 2y = 2x$		
$z - y = 2x$	1	
(b) $V = \pi \int_a^b y^2\,dx$		
$= \pi \int_0^\pi \sin^2 x\,dx$	1	
$= \pi \int_0^\pi \frac{1}{2}(1 - \cos 2x)\,dx$	1	
$= \frac{\pi}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^\pi$		
$= \frac{\pi}{2}\left[\left(\pi - \frac{1}{2}\sin 2\pi\right) - \left(0 - \frac{1}{2}\sin 0\right)\right]$		
$= \frac{\pi}{2}\left[\pi - 0\right]$		
$= \frac{\pi^2}{2}$ units^3	1	

Solutions	Marks	Comments
Question 3 continued		
(c) $x = 3\cos 2\pi t$		
(i) Initial position is when $t=0$.		
i.e. $x = 3\cos 0$		
$= 3$		
i.e. 3 units from the centre of motion	1	
(ii) passes through centre of motion when $x=0$		
$\therefore 3\cos 2\pi t = 0$		
$\cos 2\pi t = 0$		
Now if $\cos\theta = 0$, then $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$		
So $2\pi t = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$		
$t = \frac{1}{4}, \frac{3}{4}, \ldots$		
$\therefore$ first passes through centre of motion when $t = \frac{1}{4}$.	2	
(iii) $x = 3\cos 2\pi t$		
$\therefore v = -3\sin 2\pi t \times 2\pi$		
$= -6\pi \sin 2\pi t$		
This will be a max. when $\sin 2\pi t = \pm 1$		
$\therefore$ Max velocity $= -6\pi \times \pm 1$		
$= \pm 6\pi$	1	
(iv) acc $= -6\pi\cos 2\pi t \times 2\pi$		
$= -12\pi^2 \cos 2\pi t$		
This is first a max when $t = 0$.	1	

Solutions	Marks	Comments
Question 4		
		
(i) If $4ay = x^2$		
$y = \frac{1}{4a} x^2$		
and $\frac{dy}{dx} = \frac{1}{4a} . 2x = \frac{x}{2a}$		
At $(2at, at^2)$ $\frac{dy}{dx} = \frac{2at}{2a} = t$		
$\therefore$ Equation of tangent is		
$y - y_1 = m(x - x_1)$		
$y - at^2 = t(x - 2at)$		
$y - at^2 = tx - 2at^2$		
$y = tx - at^2$	2	
(ii). When l cuts the y axis, $x = 0$		
i.e. $y = t \times 0 - at^2 = -at^2$		
$\therefore$ A is $(0, -at^2)$	1	
(iii) $SA = a + at^2$		
and $SP = \sqrt{(2at - 0)^2 + (at^2 - a)^2}$		
$= \sqrt{4a^2t^2 + a^2t^4 - 2a^2t^2 + a^2}$		
$= \sqrt{a^2t^4 + 2a^2t^2 + a^2}$		
$= \sqrt{a^2(t^4 + 2t^2 + 1)}$		
$= \sqrt{a^2(t^2+1)^2}$		
$= a(t^2 + 1)$		
$\therefore SA = SP$		
$\therefore \triangle SAP$ is isosceles	2	
$\therefore \angle SPA = \angle SAP$		

Solutions	Marks	Comments
Question 4 continued		
(b) (i) Number of ways $= {}^{10}C_4$		
$= 210$	1	
(ii) The restriction is obeyed unless the committee consists of 6 males		Longer alternative
$\therefore$ Number of ways $= {}^{10}C_6 - {}^6C_6$		$(2m, 4f) = \binom{6}{2}\binom{4}{4} = 15$
		$(3m, 3f) = \binom{6}{3}\binom{4}{3} = 80$
		$(4m, 2f) = \binom{6}{4}\binom{4}{2} = 90$
$= 210 - 1$		$(5m, 1f) = \binom{6}{5}\binom{4}{1} = 24$
$= 209$	1	total 209
(iii) Number of ways $= 4! \times 2!$		
$= 48$	1	
(c) $\frac{dV}{dt} = 5\text{ cm}^3/\text{s}$. Find $\frac{dA}{dt}$ when $r = 20$.		
Now $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ and $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$		
Since $V = \frac{4}{3}\pi r^3$		
$\frac{dV}{dr} = 4\pi r^2$		
and $\frac{dr}{dV} = \frac{1}{4\pi r^2}$	1	
Also if $A = 4\pi r^2$		
$\frac{dA}{dr} = 8\pi r$	1	
Now $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$ (from above)	1	
So $\frac{dA}{dt} = \frac{8\pi r}{1} \times \frac{1}{4\pi r^2} \times 5$		
$= \frac{10}{r}$		
$\therefore$ when $r = 20$, $\frac{dA}{dt} = \frac{10}{20}$		
$= \frac{1}{2}\text{ cm}^2/\text{sec}$	1	

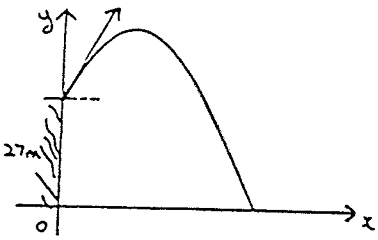
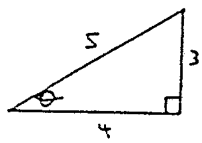
Solutions	Marks	Comments
Question 5		
(a) If $x^2 - x - 2$ is a factor then $(x-2)(x+1)$ is a factor i.e. $(x-2)$ is a factor as is $(x+1)$.		
If $(x-2)$ is a factor then $P(2) = 0$		
$\therefore 2^4 + 3\times 2^3 + a\times 2^2 - 2\times 2 - b = 0$		
$16 + 24 + 4a - 4 - b = 0$	1	for the general idea
$4a - b + 36 = 0 \quad \ldots (1)$		
If $(x+1)$ is a factor then $P(-1) = 0$		
$\therefore (-1)^4 + 3\times(-1)^3 + a\times(-1)^2 - 2\times(-1) - b = 0$		
$1 - 3 + a + 2 - b = 0$		
$a - b = 0$		
$\therefore a = b$		
Substitute $a = b$ into equation (1)		
$4b - b + 36 = 0$		
$3b = -36$		
$b = -12$		
and $a = -12$	1	for correct solution
(b)(i) $\cos(a+b) = \cos a.\cos b - \sin a.\sin b$		
$\therefore \cos(w+w) = \cos w.\cos w - \sin w.\sin w$	1	
$\therefore \cos 2w = \cos^2 w - \sin^2 w$		
$= (1 - \sin^2 w) - \sin^2 w$		
$= 1 - 2\sin^2 w$	1	
(ii) Prove $\dfrac{\cos y - \cos(y+2w)}{2\sin w} = \sin(y+w)$		
L.H.S. $= \dfrac{\cos y - [\cos y.\cos 2w - \sin y \sin 2w]}{2\sin w}$	1	
$= \dfrac{\cos y - \cos y(1 - 2\sin^2 w) + \sin y.\, 2\sin w.\cos w}{2\sin w}$	1	

Solutions	Marks	Comments
Question 5 continued		
$LHS = \dfrac{\cos y - \cos y + 2\cos y \sin^2 w + 2\sin y . \sin w . \cos w}{2\sin w}$		
$= \dfrac{2\sin w\,(\cos y \sin w + \sin y . \cos w)}{2\sin w}$	1	
$= \sin(w+y)$		
$= \sin(y+w)$		
$= R.H.S.$		
(iii) R.T.P. $\sin w + \sin 3w + \sin 5w + \dots + \sin(2N-1)w = \dfrac{1-\cos 2Nw}{2\sin w}$		
Step1 when $N=1$, $LHS = \sin w$		
$RHS = \dfrac{1-\cos 2w}{2\sin w}$		
$= \dfrac{1-(1-2\sin^2 w)}{2\sin w}$		
$= \dfrac{2\sin^2 w}{2\sin w}$		
$= \sin w$	1	
$= LHS$		
$\therefore$ True for $N=1$		
Step2. Assume true for $n=k$		
i.e. $\sin w + \sin 3w + \dots + \sin(2k-1)w = \dfrac{1-\cos 2kw}{2\sin w}$		
Now if $n=k+1$, we need to show that		
$\sin w + \sin 3w + \dots + \sin(2k-1)w + \sin(2(k+1)-1)w = \dfrac{1-\cos 2(k+1)w}{2\sin w}$		
i.e. that		
$\sin w + \sin 3w + \dots + \sin(2k-1)w + \sin(2k+1)w = \dfrac{1-\cos 2(k+1)w}{2\sin w}$	1	

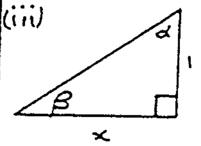
Solutions	Marks	Comments
Question 5 continued again!		
So L.H.S. $= \dfrac{1-\cos 2kw}{2\sin w} + \sin(2k+1)w$ (from assumption)		
$= \dfrac{1-\cos 2kw}{2\sin w} + \sin(2kw+w)$		
$= \dfrac{1-\cos 2kw}{2\sin w} + \dfrac{\cos 2kw - \cos(2kw+2w)}{2\sin w}$ (from part ii)	1	
$= \dfrac{1-\cos 2kw + \cos 2kw - \cos(2kw+2w)}{2\sin w}$		
$= \dfrac{1-\cos(2kw+2w)}{2\sin w}$		
$= \dfrac{1-\cos 2w(k+1)}{2\sin w}$		
$= \dfrac{1-\cos 2(k+1)w}{2\sin w}$	1	
$\therefore$ If true for $n=k$, it is also true for $n=k+1$.		
Since it is true for $n=1$	1	
then it is also true for $n=1+1=2$		
$n=2+1=3$ etc.		
i.e. it is true for all n.		
Question 6		
(a) $\int_0^1 x(x+1)^4\,dx$ if $u=x+1$ then $x=u-1$ $\frac{du}{dx}=1 \therefore du=dx$ when $x=0, u=1$ $x=1, u=2$	1	
$= \int_1^2 (u-1).u^4\,du$		
$= \int_1^2 u^5 - u^4\,du$		
$= \left[\frac{u^6}{6} - \frac{u^5}{5}\right]_1^2$		
$= \left[\left(\frac{64}{6} - \frac{32}{5}\right) - \left(\frac{1}{6} - \frac{1}{5}\right)\right]$		
$= 4\frac{3}{10}$	1	

1997 Western Region Trial H.S.C. Marking Scheme

Course: 3 Unit Mathematics　　　　　Page No : 12

Solutions	Marks	Comments
Question 6 continued		
(b) $\sin\alpha + \sin 2\alpha = 0$		
$\sin\alpha + 2\sin\alpha\cos\alpha = 0$	1	
$\sin\alpha(1 + 2\cos\alpha) = 0$		
$\therefore \sin\alpha = 0$ or $\cos\alpha = -\frac{1}{2}$		
$\therefore \alpha = 0, \pi, 2\pi, \ldots$ or $\alpha = \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$	2	
$\therefore \alpha = 0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$		
(c)  		
$\dot{x} = V\cos\theta = 10 \cdot \frac{4}{5} = 8\text{ m/s}$		
(i) $\dot{y} = V\sin\theta = 10 \cdot \frac{3}{5} = 6\text{ m/s}$	1	
Horizontal: $\ddot{x} = 0$; $\therefore \dot{x} = c = 8\text{ m/s}$; $x = 8t + k$; when $t = 0$, $x = 0$ so $k = 0$; $\therefore x = 8t$	1	for horizontal eqn
Vertical: $\ddot{y} = -g$; $\therefore \dot{y} = -gt + c$; when $t = 0$, $\dot{y} = 6\text{ m/s}$ $\therefore c = 6$ and $\dot{y} = -gt + 6$; so $y = -g\frac{t^2}{2} + 6t + k$; when $t = 0$, $y = 27$ $\therefore k = 27$; $\therefore y = -g\frac{t^2}{2} + 6t + 27$	1	for vertical eqn
(ii) When stone hits the lake $y = 0$		
i.e. $-10 \times \frac{1}{2}t^2 + 6t + 27 = 0$		
$-5t^2 + 6t + 27 = 0$		
$5t^2 - 6t - 27 = 0$		
$(5t + 9)(t - 3) = 0$		
$\therefore t = \frac{-9}{5}$ or 3		
i.e. hits lake after 3 seconds.	1	
When $t = 3$, $x = 8 \times 3 = 24$ $\therefore$ hits <u>24m</u> from base of cliff	1	

Solutions	Marks	Comments
Question 6 concludes		
(iii) Reaches maximum height when $\dot{y}=0$		
i.e. $-gt+6=0$		
i.e. $-10t+6=0$		
$t=\frac{6}{10}$		
$t=\frac{3}{5}$	1	
So when $t=\frac{3}{5}$, $y=-\frac{1}{2}\times 10\times\left(\frac{3}{5}\right)^2+6\times\frac{3}{5}+27$		
$=28\frac{4}{5}$		
i.e. Max height is $28\frac{4}{5}$ metres.	1	
Question 7		
(a) $P(\text{pass})=0.9$ $\quad$ $P(\text{fail})=0.1$		
$P(3\text{ fail})={}^{10}C_3\,(0.1)^3\,(0.9)^7$	1	
$=\frac{10!}{3!\,7!}\times\frac{1}{10^3}\times\frac{9^7}{10^7}$		
$=\frac{10.9.8.}{3.2.1}\times\frac{9^7}{10^{10}}$		
$=\frac{3.4.(3^2)^7}{10^9}$		
$=\frac{4.3^{15}}{10^9}$	1	There will be many variations on this solution. eg. $4=2^2$
(b) (i) Firstly, find $\frac{d}{dx}\left(\tan^{-1}\frac{1}{x}\right)$.		
If $y=\tan^{-1}\frac{1}{x}$		
then $y=\tan^{-1}u$ where $u=\frac{1}{x}$		
Now $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$ and $\frac{du}{dx}=-x^{-2}$		
$=\frac{1}{1+u^2}\times\frac{-1}{x^2}$		
$=\frac{1}{1+\left(\frac{1}{x}\right)^2}\times\frac{-1}{x^2}$		
$=\frac{-1}{x^2+1}$		
$=\frac{-1}{1+x^2}$	2	

Solutions	Marks	Comments
Question 7 continued		
So if $f(x) = \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$		
then $f'(x) = \frac{1}{1+x^2} + \frac{-1}{1+x^2}$		
$= 0.$		
(ii) This implies that the graph of $f(x)$ is parallel to the x axis. (i.e. horizontal) $\therefore$ zero gradient.	1	
(iii) 		
$\tan\alpha = \frac{1}{x}$		
$\therefore \alpha = \tan^{-1}\frac{1}{x}$		
$\tan\beta = x$		
$\therefore \beta = \tan^{-1} x$		
$\therefore \tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \alpha + \beta$		
i.e. $f(x) = \frac{\pi}{2}$	2	
(c) $(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$		
(i) let $x = -1$, $(1+-1)^n = \sum_{r=0}^{n} \binom{n}{r}(-1)^r$		
i.e. $0 = \sum_{r=0}^{n} (-1)^r \binom{n}{r}$	1	
(ii). Let $x = 2$, $(1+2)^n = \sum_{r=0}^{n} \binom{n}{r} 2^r$		
i.e. $3^n = \sum_{r=0}^{n} 2^r \binom{n}{r}$	1	
(iii) Since $(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$		
Differentiate both sides with respect to x	1	
$n(1+x)^{n-1} = \sum_{r=0}^{n} \binom{n}{r} r x^{r-1}$	1	
Let $x=1$, $n\,2^{n-1} = \sum_{r=0}^{n} \binom{n}{r} r$	1	
i.e. $\sum_{r=0}^{n} r\binom{n}{r} = n\,2^{n-1}$		
THE END!! ☺		