WESTERN REGION

2005
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**84**)

- Attempt Questions 1-7
- All questions are of equal value

Total Marks – 84
Attempt Questions 1-7
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

QUESTION 1 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Calculate the acute angle (to the nearest minute) between the lines : **2**

$$2x + y = 4$$
and $$x - 3y = 6$$

b) Use the table of standard integrals to show that $\int_6^{15} \frac{dx}{\sqrt{x^2+64}} = \ln(2)$ **2**

c) Solve $\frac{2x-3}{x-1} \le 4$ **3**

d) Evaluate $\sum_{n=2}^{8} (n^2 - n)$ **1**

e) Show that $2x - 1$ is a factor of $2x^3 + 5x^2 + x - 2$ **2**

f) Find $\int \sin x \cos x \, dx$ using the substitution $u = \sin x$ **2**

QUESTION 2 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Find the Cartesian equation of the curve represented by the parametric equations below: 3

$x = 2t - 1$

$y = t^2 + 2t$

b) Find the volume of the solid of revolution formed when the section of the curve $y = \cos x$ between $x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$ is rotated about the x axis. 3

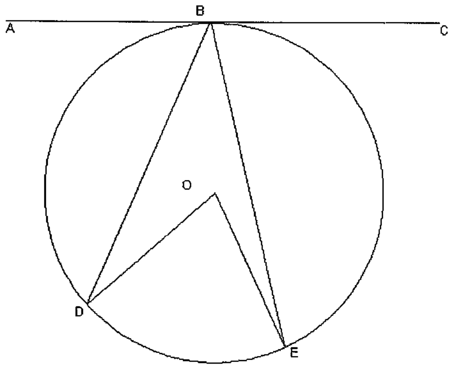
c) Considering the letters which form the word **DESCARTES**.

(i) How many distinct arrangements of the letters are possible? 1

(ii) How many distinct arrangements are possible if the two letter S's must be placed so that one is at the beginning and one at the end? 1

(iii) How many distinct arrangements are possible if the two letter S's must be placed together? 1

d) In the diagram below, O is the centre of the circle, AC is a tangent at B and D and E are points on the circumference. If $\angle ABD = 80^\circ$ and $\angle DBE = 40^\circ$, find the size of $\angle BEO$, giving reasons. 3



QUESTION 3 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) The graph $y = \sin(e^x)$ has a zero close to $x = 1$. Use one application of Newton's method to find a second approximation for this zero, giving your result correct to 3 significant figures. 3

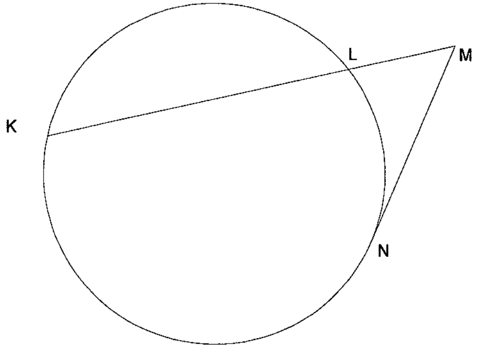
b) (i) Show that $\sin x - \cos 2x = 2\sin^2 x + \sin x - 1$ 2

(ii) Hence or otherwise solve $\sin x - \cos 2x = 0$ for $0 \le x \le 2\pi$ 3

c) In the diagram below the chord KL is produced to M so that LM = $\frac{1}{3}$KL 4

The tangent MN is then drawn.

Show that MN = $\frac{2}{3}$KL



QUESTION 4 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Find the coefficient of x^2 in the expansion of $(2x+4)^5+(x^2-3)^6$ **3**

b) For the cubic equation $2x^3-3x^2+5x-2=0$ with roots, $x=\alpha$, $x=\beta$ and $x=\gamma$, find the value of :

(i) $\alpha^2+\beta^2+\gamma^2$ **2**

(ii) $\dfrac{1}{\alpha^2\beta^2}+\dfrac{1}{\beta^2\gamma^2}+\dfrac{1}{\alpha^2\gamma^2}$ **2**

c) When Lleyton and Bec play tennis, Bec has a 0.1 chance of winning any particular game. On the weekend they intend to play 6 games. Calculate the probability (to 2 significant figures) that :

(i) Bec wins no games. **1**

(ii) Bec wins at least one game. **1**

(iii) Bec wins 4 or more games. **3**

QUESTION 5 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) For the function $f(x)=x^2-6x$:

(i) Give the domain of $f(x)$ for which there exists an inverse function, $f^{-1}(x)$. **1**

(ii) Find the equation of the inverse function $f^{-1}(x)$ and give it's domain. **3**

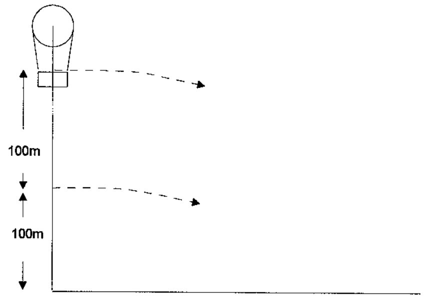
b) Considering the expansion of $(a+b)^n$:

(i) By letting $a=b=1$, show that $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+....+\binom{n}{n}=2^n$ **1**

(ii) By letting $a=1$, and $b=-1$, show that $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}+....+(-1)^n\binom{n}{n}=0$ **1**

(iii) Hence show that $\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+....=2^{n-1}$ **2**

c) A balloon rises vertically from level ground. Two projectiles are fired horizontally in the same direction from the balloon at a velocity of 80ms^{-1}. The first is fired at a point 100 m from the ground and the second when it has risen a further 100 m from the ground. How far apart will the projectiles hit the ground? (Use $g=10ms^{-2}$) **4**



QUESTION 6 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) The chord joining P $(2ap, ap^2)$ and Q $(2aq, aq^2)$ on the parabola $x^2 = 4ay$ subtends a right angle at the vertex of the parabola.

(i) Show that $pq = -4$ **1**

(ii) Show that the locus of the point M, the midpoint of PQ, is also a parabola and give its vertex. **3**

b) (i) By considering the cases where a positive integer k is even $(k = 2x)$ and odd ($k = 2x + 1$), show that $k^2 + k$ is always even. i.e. $k^2 + k = 2m$, where m is also an integer. **2**

(ii) Prove, by Mathematical induction, that for all positive integral values of n, $n^3 + 5n$ is divisible by 6. **3**

c) (i) Given the series expansion for $e^h = 1 + \frac{h}{1!} + \frac{h^2}{2!} + \frac{h^3}{3!} + \ldots$ show that **1**

$$\frac{e^h - 1}{h} = 1 + \frac{h}{2!} + \frac{h^2}{3!} + \ldots$$

(ii) Hence, use the definition $f'(x) = \lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $f(x) = e^x$. **2**

QUESTION 7 (12 MARKS) Begin a NEW sheet of writing paper. **Marks**

a) Using the fact that the inverse trigonometric function

$y = sin^{-1} x \quad \{-1 \le x \le 1\}$ is equivalent to the function

$x = sin\, y \quad \left\{-\frac{\pi}{2} \le y \le \frac{\pi}{2}\right\}$

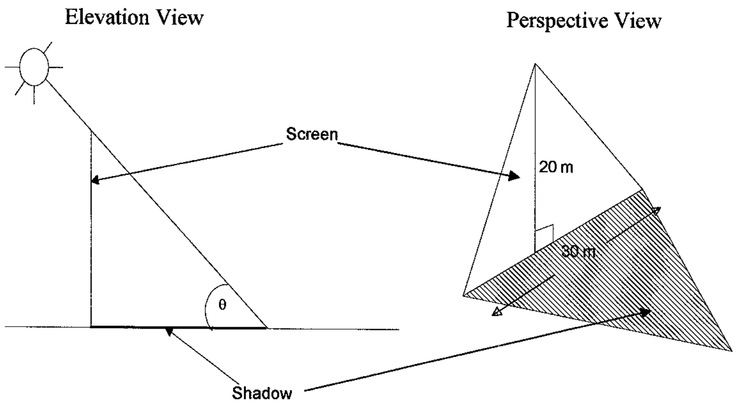
(i) Show that $\frac{d}{dx}\left(sin^{-1} x\right) = \frac{1}{\sqrt{1 - x^2}}$ **3**

(ii) Hence find the value of the derivative of $sin^{-1}\left(\sqrt{x}\right)$ when $x = \frac{1}{2}$. **2**

QUESTION 7 CONTINUES ON PAGE 9

QUESTION 7 CONTINUED **Marks**

b) A thin, 20 m high, triangular screen which has a 30m long base running north to south, stands on level ground. At noon it casts no shadow, and as the afternoon progresses its shadow increases in length as the angle of elevation of the sun (θ) decreases.



(i) Show that the area of the shadow is given by $A = \frac{300}{tan\theta}$ **1**

(ii) Given that the angle of elevation of the sun decreases at a rate of $\frac{\pi}{12}^c$ /hour. (i.e. $\frac{d\theta}{dt} = -\frac{\pi}{12}$) find the rate (in m^2/hour) at which the area of the shadow is increasing at 3 p.m. **3**

(iii) Sketch $A = \frac{300}{\tan\theta}$ for $0 \le \theta \le \frac{\pi}{2}$. Use this graph to help describe what happens to the area of the shadow as the afternoon progresses, and also to the rate of change in the area. In particular look at what happens just after noon and as the time approaches 6 p.m. **3**

STANDARD INTEGRALS

$\int x^n dx$	$= \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0$ if $n < 0$
$\int \frac{1}{x} dx$	$= \ln x,\ x > 0$
$\int e^{ax} dx$	$= \frac{1}{a}e^{ax},\ a \neq 0$
$\int \cos ax dx$	$= \frac{1}{a}\sin ax, a \neq 0$
$\int \sin ax dx$	$= -\frac{1}{a}\cos ax, a \neq 0$
$\int \sec^2 ax dx$	$= \frac{1}{a}\tan ax, a \neq 0$
$\int \sec ax \tan ax dx$	$= -\frac{1}{a}\sec ax, a \neq 0$
$\int \frac{1}{a\ +x^2} dx$	$= \frac{1}{a}\tan^{-1}\frac{x}{a}, a \neq 0$
$\int \frac{1}{\sqrt{a^2-x^2}} dx$	$= \sin^{-1}\frac{x}{a}, a > 0, -a < x < a$
$\int \frac{1}{\sqrt{x^2-a^2}} dx$	$= \ln(x+\sqrt{x^2-a^2}), x > a > 0$
$\int \frac{1}{\sqrt{x^2+a^2}} dx$	$= \ln(x+\sqrt{x^2+a^2})$

Note: $\ln x = \log_e x, x > 0$

Western Region

2005

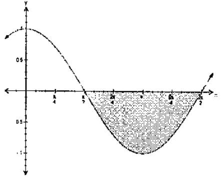
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

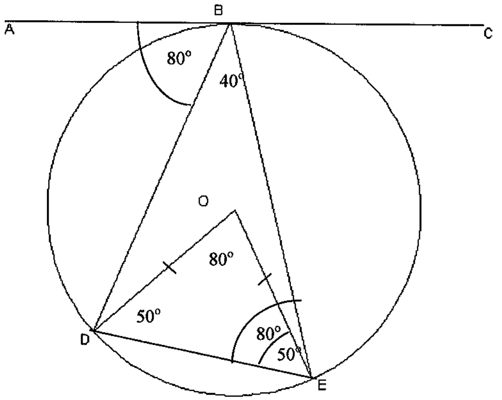
Mathematics Extension 1

Solutions

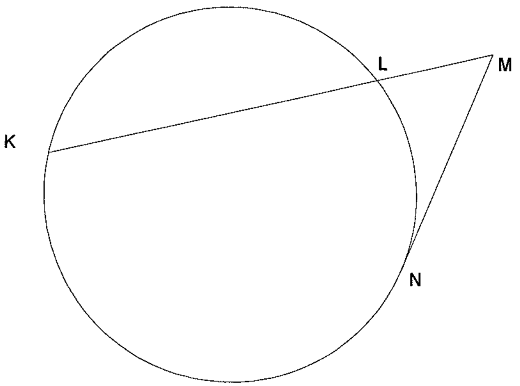
Solutions Question 1 2005	Marks/Comments
1. a) For $2x+y=4$ $m_1=-2$ and for $x-3y=6$ $m_2=\frac{1}{3}$ so: $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $=\left\|\frac{-2-\frac{1}{3}}{1+-2\times\frac{1}{3}}\right\|$ $=\left\|\frac{-2\frac{1}{3}}{\frac{1}{3}}\right\|$ $\tan\theta=\|-7\|$ $\theta=81°52'$	2 marks for correct formula, substitution and evaluation. 1 mark if a single error is made in one step.
1 b) From Standard Integrals $\int_6^{15}\frac{dx}{\sqrt{x^2+64}}=[\ln(x+\sqrt{x^2+64})]^{15}_{6}$ $=\left(\ln\left(15+\sqrt{15^2+64}\right)\right)-\left(\ln\left(6+\sqrt{6^2+64}\right)\right)$ $=\left(\ln\left(15+\sqrt{289}\right)\right)-\left(\ln\left(6+\sqrt{100}\right)\right)$ $=(\ln(32))-(\ln(16))$ $=\ln(2)$	2 marks for correct substitution, and simplification. 1 mark if a single error is made in the substitution or the simplification.
1 c) $\frac{2x-3}{x-1}\leq 4$ $x\neq 1$ $2x-3=4x-4$ $x=\frac{1}{2}$ try $x=0$ $3\leq 4$✓ try $x=0.75$ $6\leq 4$ ✗ try $x=2$ $1\leq 4$✓ $x\leq\frac{1}{2},x>1$ $\frac{2x-3}{x-1}\leq 4$ $(x-1)^2\frac{2x-3}{x-1}\leq 4(x-1)^2$ $(x-1)(2x-3)\leq 4(x-1)^2$ $2x^2-5x+3\leq 4x^2-8x+4$ $0\leq 2x^2-3x+1$ $(2x-1)(x-1)\geq 0$ $x\leq\frac{1}{2}'x>1$	1 for values 1 for test 1 for statement. No 3rd mark if $x\geq 1$ Similar break up of marks for alternative methods.

1. d) $\sum_{n=2}^{8}(n^2-n)=(4-2)+(9-3)+(16-4)+(25-5)$ $+(36-6)+(49-7)+(64-8)$ $=2+6+12+20+30+42+56$ $=168$	1 mark
1. e) $2x-1$ is a factor of $P(x)=2x^3+5x^2+x-2$ if $P\left(\frac{1}{2}\right)=0$ by factor theorem. $P\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+5\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)-2$ $=\frac{1}{4}+\frac{5}{4}+\frac{1}{2}-2$ $=0$ $\therefore 2x\text{-}1$ is a factor	2 marks for substitution and explanation, or other valid method such as dividing. 1 mark if error in substitution, but right reasoning to substitute $x=\frac{1}{2}$
1. f) If $u=\sin x$, then $\frac{du}{dx}=\cos x$ and $du=\cos x dx$ $\int \sin x \ \cos x \ dx=\int u du$ $=\frac{u^2}{2}+c$ $=\frac{\sin^2 x}{2}+c$	2 marks for correct substitution and integration. 1 mark if error made in substitution or integration.

Solutions Question 2 2005	Marks/Comments
2 a) $x=2t-1\Rightarrow t=\frac{x+1}{2}$ $y=t^2+2t$ $y=\left(\frac{x+1}{2}\right)^2+2\left(\frac{x+1}{2}\right)$ OR $4y=x^2+2x+1+4x+4$ $4y=x^2+6x+5$ $\quad 4y+4=x^2+6x+9$ $y=\frac{x^2+6x+5}{4}$ $\quad 4(y+1)=(x+3)^2$	1 mark for making t the subject of either equation. 1 for substitution into the other equation 1 for simplifying to either form.
2 b) $y=\cos x$ $V=\pi\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} y^2 dx$ $=\pi\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(\cos x)^2 dx$ $=\pi\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\frac{(1+\cos 2x)}{2}dx$ $=\frac{\pi}{2}\left[x+\frac{1}{2}\sin 2x\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$ $=\frac{\pi}{2}\left[\left(\frac{3\pi}{2}+\frac{1}{2}\sin 2\left(\frac{3\pi}{2}\right)\right)-\left(\frac{\pi}{2}+\frac{1}{2}\sin 2\left(\frac{\pi}{2}\right)\right)\right]$ $=\frac{\pi}{2}\left[\left(\frac{3\pi}{2}+\frac{1}{2}\sin 3\pi\right)-\left(\frac{\pi}{2}+\frac{1}{2}\sin\pi\right)\right]$ $=\frac{\pi}{2}\left[\left(\frac{3\pi}{2}\right)-\left(\frac{\pi}{2}\right)\right]$ $=\frac{\pi}{2}[\pi]$ $=\frac{\pi^2}{2} units^2$	 1 for substituting into Volume formula. 1 for finding integral of $\cos^2 x$ 1 for evaluating resulting definite integral.

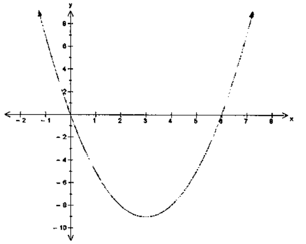
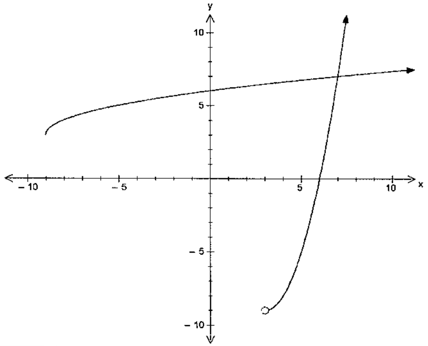
2 c) i) There are $\frac{9!}{2!2!} = 90720$ arrangements with the S's and E's repeated.	1 mark.
ii) When the two S's are placed there are $\frac{7!}{2!} = 2520$ arrangements of the remaining 7 letters with E's repeated.	1 mark
iii) Consider the two S's as a single unit, so eight to arrange in $\frac{8!}{2!} = 20160$ ways with E's repeated.	1 mark
2 d)  $\angle BED = \angle ABD = 80°$ Angle between tangent and chord is equal to the angle in alternate segment. $\angle DOE = 2 \times \angle DBE = 80°$ Angle at the centre is twice angle at circumference on same arc. DO = OE Equal radii $\angle OED = \angle ODE = 50°$ Equal angles in isosceles ΔODE $\angle BEO = \angle BED - \angle OED = 80° - 50° = 30°$	1 mark for ∠BED with reasons. 1 mark for ∠DOE with reasons. 1 mark for ∠BEO with reasons.

Solutions Question 3 2005	Marks/Comments
3(a) By Newtons Method $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ Here $x_1 = 1, \quad f(x) = \sin(e^x)$ $f'(x) = \frac{d}{dx}[\sin(e^x)]$ $= e^x \cos(e^x)$ So $x_2 = 1 - \frac{\sin(e^1)}{e^1 \cos(e^1)}$ $= 1 - (-0{\cdot}1657)$ $= 1{\cdot}1657 = 1{\cdot}17$ (3 Significant figures)	1 for derivative 1 for sub in formula 1 for correct evaluation.
3(b) (i) $\sin x - \cos 2x = \sin x - (1 - 2\sin^2)$ $= \sin x - 1 + 2\sin^2 x$ $= 2\sin^2 x + \sin x - 1$	1 mark for expression for cos2x 1 for simplifying
3(b) (ii) $\sin x - \cos 2x = 0$ $2\sin^2 x + \sin x - 1 = 0$ $2\sin^2 x + 2\sin x - \sin x - 1 = 0$ $2\sin x(\sin x + 1) - (\sin x + 1) = 0$ $(2\sin x - 1)(\sin x + 1) = 0$ So $\sin x = \frac{1}{2}$ or $\sin x = -1$ $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$	1 for factorising quadratic equation 1 for values of sin$x$ 1 for the solutions for x

3(c)  $KM.LM = MN^2$ Product of intercepts on secant = square on tangent. $LM = \frac{1}{3}KL$ Given $KM.\frac{1}{3}KL = MN^2$ $(KL + LM).\frac{1}{3}KL = MN^2$ $\left(KL + \frac{1}{3}KL\right).\frac{1}{3}KL = MN^2$ Substituting and simplifying. $\left(\frac{4}{3}KL\right).\frac{1}{3}KL = MN^2$ $\frac{4}{9}KL^2 = MN^2$ $\frac{2}{3}KL = MN$ (As required)	**Other solutions are possible with commensurate marks.** 1 for product = square 1 for sub given relation 1 for replacing KM by $\frac{4}{3}KL$ 1 for square root to get required answer.

Solutions Question 4 2005	Marks/Comments
4(a) In $(2x+4)^5$ the term in x^2 comes from $\binom{5}{3}(2x)^2 4^3$ In $(x^2-3)^6$ the term in x^2 comes from $\binom{6}{5}(x^2)^1(-3)^5$ In $(2x+4)^5 + (x^2-3)^6$ the term in x^2 comes from $\binom{5}{3}(2x)^2 4^3 + \binom{6}{5}(x^2)^1(-3)^5 = 10.(4x^2).64 + 6.(x^2)(-243)$ $= 2560x^2 - 1458\ x^2$ $= 1102x^2$ Coefficient of x^2 is 1102	1 mark for the term in x^2 in each expansion (2 marks) 1 mark for adding the terms in x^2
4(b) $2x^3 - 3x^2 + 5x - 2 = 0$ $\alpha+\beta+\gamma = \frac{-b}{a} = \frac{3}{2}$ $\alpha\beta+\beta\gamma+\alpha\gamma = \frac{c}{a} = \frac{5}{2}$ $\alpha\beta\gamma = \frac{-d}{a} = \frac{2}{2} = 1$ (i) $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\alpha\gamma)$ $= \left(\frac{3}{2}\right)^2 - 2\left(\frac{5}{2}\right)$ $= -\frac{11}{4} = -2\frac{3}{4}$	1 mark for correctly re arranging the required expression in terms of sums and products. 1 mark for correctly substituting and simplifying.
(ii) $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\alpha^2\gamma^2} = \frac{\alpha^2+\beta^2+\gamma^2}{\alpha^2\beta^2\gamma^2}$ $= \frac{\alpha^2+\beta^2+\gamma^2}{(\alpha\beta\gamma)^2}$ $= \frac{-11}{4} \Big/ (1)^2$ $= \frac{-11}{4} = -2\frac{3}{4}$	1 mark for correctly re arranging the required expression in terms of sums and products. 1 mark for correctly substituting and simplifying.

4 (c) Let N denote the number of games Bec wins. For a given game $P(\text{Bec wins}) = b = 0.1$. and $P(\text{Lleyton wins}) = l = 0.9$	
(i) Bec wins no games. $P(N=0) = \binom{6}{0}(0.1)^0.(0.9)^6$ $= (1).(1).(0.531441)$ $= 0.53$ (2 sig figs)	1 mark
(ii) Bec wins at least one game. $P(N>0) = 1 - P(N=0)$ $= 1 - \binom{6}{0}(0.1)^0.(0.9)^6$ $= 1 - (0.531441)$ $= 0.47$ (2 sig figs)	1 mark
(iii)Bec wins 4 or more games. $P(N \geq 4) = P(N=4) + P(N=5) + P(N=6)$ $= \binom{6}{4}(0.1)^4.(0.9)^2 + \binom{6}{5}(0.1)^5.(0.9)^1 + \binom{6}{6}(0.1)^6.(0.9)^0$ $= (15).(0.0001).(0.81) + (6).(0.00001).(0.9) + (1).(0.000001).(1)$ $= 0.001215 + 0.000054 + 0.000001$ $= 0.00127$ $= 0.0013$ (2 sig figs)	1 mark for recognising required terms to add. 1 mark for binomial terms. 1 mark for calculation.

Solutions Question 5 2005	Marks/Comments
5(a) i) Since the sketch of $f(x) = x^2 - 6x$ is symmetrical at $x = 3$, it will not have an inverse function unless the domain is restricted to $\{x : x \geq 3\}$ or $\{x : x \leq 3\}$ 	1 mark for either of the domains given.
(a) ii) If function is $y = x^2 - 6x$ $\{x : x \geq 3\}$, then inverse is $x = y^2 - 6y$ $x + 9 = y^2 - 6y + 9$ $x + 9 = (y-3)^2$ $y - 3 = \sqrt{x+9}$ $y = \sqrt{x+9} + 3$ $f^{-1}(x) = \sqrt{x+9} + 3$ The domain is $\{x : x \geq -9\}$ 	Answers refer to domain $\{x : x \geq 3\}$, but would be similar for $\{x : x \leq 3\}$ 1 marks for interchanging x and y and attempting to find inverse. 1 mark for successfully completing this to get $f^{-1}(x)$ 1 mark for domain. Graph not required.

5. (b) i) $(a+b)^n = \binom{n}{0}a^nb^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{n}a^0b^n$ When $a=1$ and $b=1$, this becomes $(1+1)^n = \binom{n}{0}1^n1^0 + \binom{n}{1}1^{n-1}1^1 + \binom{n}{2}1^{n-2}1^2 + ... + \binom{n}{n}1^01^n$ $2^n = \binom{n}{0}+\binom{n}{1}+\binom{n}{2}+....+\binom{n}{n}$	1 mark
b) ii) $(a+b)^n = \binom{n}{0}a^nb^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{n}a^0b^n$ When $a=1$ and $b=-1$, this becomes $(1+(-1))^n = \binom{n}{0}1^n(-1)^0 + \binom{n}{1}1^{n-1}(-1)^1 + \binom{n}{2}1^{n-2}(-1)^2 + ... + \binom{n}{n}1^0(-1)^n$ $(0)^n = \binom{n}{0}1+\binom{n}{1}(-1)+\binom{n}{2}1+....+\binom{n}{n}(-1)^n$ $0=\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}....+(-1)^n\binom{n}{n}$	1 mark
c) iii) From ii) above $\binom{n}{0}-\binom{n}{1}+\binom{n}{2}-\binom{n}{3}+....+(-1)^n\binom{n}{n}=0$ $\therefore \binom{n}{0}+\binom{n}{2}+...=\binom{n}{1}+\binom{n}{3}+....$ Now since $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+....+\binom{n}{n}=2^n$ $\therefore \binom{n}{0}+\binom{n}{2}+...=\binom{n}{1}+\binom{n}{3}+....=\frac{1}{2}\left[\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+....+\binom{n}{n}\right]$ $\binom{n}{0}+\binom{n}{2}+...=\binom{n}{1}+\binom{n}{3}+....=\frac{1}{2}(2^n)$ $\binom{n}{0}+\binom{n}{2}+...=\binom{n}{1}+\binom{n}{3}+....=2^{n-1}$	1 mark for rearranging answer to ii) 1 mark for simplifying to get required result.

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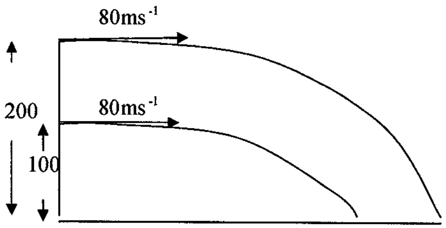
5. c)

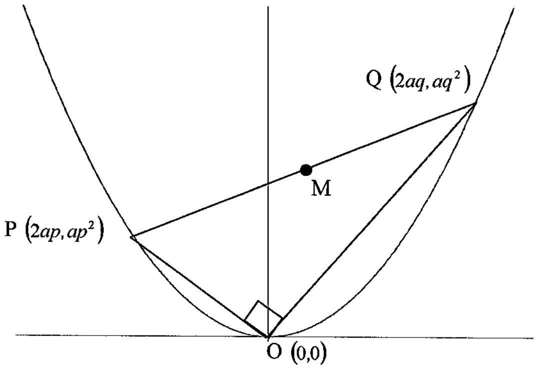


Horizontally	Vertically	
For both projectiles	For both projectiles	
V=80ms^{-1}	V=0ms^{-1}	
$\ddot{x}=0$	$\ddot{y}=-10$	
$\dot{x}=C_1=80,$	$\dot{y}=-10t+C_3=-10t,$	
since $\dot{x}=80$, when $t=0$	since $\dot{y}=0$, when $t=0$	
$x=80t+C_2$	$y=-5t^2+C_4$	
$x=80t$	For Projectile 1	1 mark for equation for x
since $x=0$, when $t=0$	$y=-5t^2+100$	
	since $y=100$, when $t=0$	
	For Projectile 2	1 mark for equations for y
	$y=-5t^2+200$	
	since $y=200$, when $t=0$	

Now each projectile hits the ground when $y=0$

For projectile 1	For projectile 2	1 mark for values of t.
$-5t^2+100=0$	$-5t^2+200=0$	
$t^2-20=0$	$t^2-40=0$	
$(t+\sqrt{20})(t-\sqrt{20})=0$	$(t+\sqrt{40})(t-\sqrt{40})=0$	1 mark for distance apart.
$t=\sqrt{20}$	$t=\sqrt{40}$	
$x=80t=80\sqrt{20}=358\text{m}$	$x=80t=80\sqrt{40}=506\text{m}$	

The distance apart is 148 m.

Solutions Question 6 2005	Marks/Comments
6(a) i) 	
i) Gradient of OP = $m_1 = \frac{ap^2}{2ap} = \frac{p}{2}$ Gradient of OQ = $m_2 = \frac{aq^2}{2aq} = \frac{q}{2}$ Now $m_1 m_2 = -1$ since <POQ is a right angle. $\frac{p}{2}\frac{q}{2} = -1$ $pq = -4$	1 mark
ii) M has coordinates $\left(\frac{2ap+2aq}{2}, \frac{ap^2+aq^2}{2}\right) = \left(a(p+q), \frac{a(p^2+q^2)}{2}\right)$	1 co-ords of M
$x = a(p+q) \Rightarrow p+q = \frac{x}{a}$ $y = \frac{a(p^2+q^2)}{2}$ $\frac{2y}{a} = p^2 + q^2$	1 for comp square
$\frac{2y}{a} + 2pq = p^2 + 2pq + q^2$ $\frac{2y}{a} + 2(-4) = (p+q)^2$ $\frac{2y}{a} - 8 = \left(\frac{x}{a}\right)^2$	1 for equation
$2ay - 8a^2 = x^2$ $x^2 = 2a(y-4a)$ This is a parabola with vertex $(0,4a)$	Other methods possible.

6(b) i) If k is even, i.e $k = 2x$, then $k^2 + k = (2x)^2 + 2x$ $= 4x^2 + 2x$ $= 2(2x^2 + x)$ $= 2m$	If k is odd, i.e $k = 2x+1$, then $k^2 + k = (2x+1)^2 + 2x + 1$ $= 4x^2 + 4x + 1 + 2x + 1$ $= 4x^2 + 6x + 2$ $= 2(2x^2 + 3x + 1)$ $= 2m$	1 mark for each case.
$\therefore k^2 + k$ is divisible by 2, no matter whether k is odd or even.		
ii) When $n = 1$, $n^3 + 5n = 1^3 + 5(1) = 6$ $\therefore n^3 + 5n$ is divisible by 6 when $n = 1$		
Assume that $n^3 + 5n$ is divisible by 6 for $n = k$ i.e $k^3 + 5k = 6p$ where p is an integer. Now when $n = k+1$ $(k+1)^3 + 5(k+1) = k^3 + 3k^2 + 3k + 1 + 5k + 5$ $= k^3 + 5k + 3k^2 + 3k + 6$ $= 6p + 3k^2 + 3k + 6$ $= 6p + 6 + 3(k^2 + k)$ * from i) above $= 6p + 6 + 3(2m)*$ $= 6p + 6 + 6m$ $= 6(p + m + 1)$ $(k+1)^3 + 5(k+1)$ is divisible by 6 $\therefore$ if true for $n = k$, then also true for $n = k+1$, but since true for $n = 1$, by induction is true for all integral values, $n \geq 1$		3 marks for full proof including all steps. Delete one mark if case n=1 or conclusion not correct or omitted. Delete one mark for simple error in case for k+1

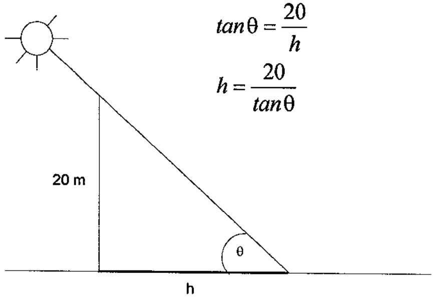
6 (c) (i) $e^h = 1+\frac{h}{1!}+\frac{h^2}{2!}+\frac{h^3}{3!}+...$ $e^h - 1 = \frac{h}{1!}+\frac{h^2}{2!}+\frac{h^3}{3!}+...$ $\frac{e^h-1}{h}=\frac{1}{1!}+\frac{h}{2!}+\frac{h^2}{3!}+...$ $\frac{e^h-1}{h}=1+\frac{h}{2!}+\frac{h^2}{3!}+...$	1 mark
(ii) $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $=\lim_{h\to 0}\frac{e^{x+h}-e^x}{h}$ $=\lim_{h\to 0}\frac{e^x(e^h-1)}{h}$ $=e^x\lim_{h\to 0}\frac{(e^h-1)}{h}$	1 mark for substitution and taking e^x out of limit.
$=e^x\lim_{h\to 0}\left(1+\frac{h}{2!}+\frac{h^2}{3!}+...\right)$ $=e^x\left(1+\frac{0}{2!}+\frac{0^2}{3!}+...\right)$ $=e^x(1)$ $=e^x$	1 for evaluating limit.

Solutions Question 7 2005	Marks/Comments
7(a) i) Given that $y = sin^{-1} x$ $\{-1\le x\le 1\}$ is equivalent to the function $x = sin y$ $\left\{-\frac{\pi}{2}\le y\le\frac{\pi}{2}\right\}$ we can derive x	**Alternate methods possible.**
$\frac{dx}{dy}=\cos y$, and using $sin^2 y+cos^2 y=1$ we get	1 for finding $\frac{dx}{dy}$
$cos^2 y = 1-sin^2 y$ $cos\, y=\sqrt{1-sin^2 y}$ $\therefore \frac{dx}{dy}=\sqrt{1-sin^2 y}$	1 for $\frac{dx}{dy}$ in terms of $\sin y$
$\frac{dx}{dy}=\sqrt{1-x^2}$ since $x = sin y$ $\therefore \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$	1 for $\frac{dy}{dx}$ in terms of x
ii) $\frac{d}{dx}\left(sin^{-1}(\sqrt{x})\right)=\frac{1}{\sqrt{1-(\sqrt{x})^2}}.\frac{d}{dx}(\sqrt{x})$ $=\frac{1}{\sqrt{1-x}}.\frac{d}{dx}\left(x^{\frac{1}{2}}\right)$ $=\frac{1}{\sqrt{1-x}}.\frac{1}{2}x^{-\frac{1}{2}}$	1 for derivative in terms of x
$=\frac{1}{\sqrt{1-x}}.\frac{1}{2\sqrt{x}}$ $=\frac{1}{\sqrt{1-\frac{1}{2}}}.\frac{1}{2\sqrt{\frac{1}{2}}}$ $=\frac{1}{\sqrt{\frac{1}{2}}}.\frac{1}{2\sqrt{\frac{1}{2}}}$ $=\frac{1}{2\sqrt{\frac{1}{4}}}$ $=\frac{1}{2.\frac{1}{2}}$ $=1$	1 for substitution.

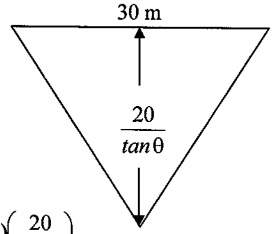
7(b)

$$\tan\theta = \frac{20}{h}$$

$$h = \frac{20}{\tan\theta}$$



Shadow



$$\text{Area} = \frac{1}{2}(30)\left(\frac{20}{\tan\theta}\right)$$
$$= \frac{300}{\tan\theta}$$

1 mark

ii) $\frac{dA}{dt} = \frac{dA}{d\theta}\cdot\frac{d\theta}{dt}$

So need to find $\frac{dA}{d\theta}$

$$\frac{dA}{d\theta} = \frac{d}{d\theta}\left(\frac{300}{\tan\theta}\right)$$
$$= \frac{d}{d\theta}\left(300(\tan\theta)^{-1}\right)$$
$$= -300(\tan\theta)^{-2}\left(\sec^2\theta\right)$$
$$= -300\left(\frac{\cos^2\theta}{\sin^2\theta}\right)\left(\frac{1}{\cos^2\theta}\right)$$
$$= -300\left(\frac{1}{\sin^2\theta}\right)$$
$$= -300\,cos\,ec^2\theta$$

1 for $\frac{dA}{d\theta}$

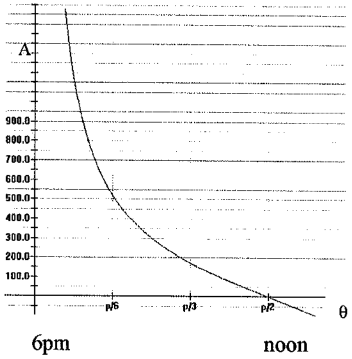
At 3 p.m. angle = $\frac{\pi}{2} - 3\frac{\pi}{12} = \frac{\pi}{4}$

$$\frac{dA}{dt} = \frac{dA}{d\theta}\cdot\frac{d\theta}{dt}$$
$$= -300\cos ec^2\theta.\left(-\frac{\pi}{12}\right)$$
$$= 25\pi\cos ec^2\theta$$
$$= 25\pi\cos ec^2\left(\frac{\pi}{4}\right)$$
$$= 50\pi \quad \text{m}^2/\text{h} \quad \text{since } \cos ec^2\left(\frac{\pi}{4}\right) = 2$$

1 for $\frac{dA}{dt} = \frac{dA}{d\theta}\cdot\frac{d\theta}{dt}$

1 for substitution to obtain answer..

7 b) iii) Graph is equivalent to $A = 300\cot x$



At noon, sun is overhead, so angle of elevation is $\theta = \frac{\pi}{2}$

At 6.p.m., sun is setting, so angle of elevation is $\theta = 0$

1 for graph

As the afternoon progresses from noon to 6 pm, the length and hence the area of the shadow increases, at first gradually from zero at noon, to approach infinity as 6 p.m. is neared. The graph gets steeper from noon to 6 pm, so the rate of increase in area also increases as the afternoon progresses, also approaching infinity as 6 p.m. is neared.

1 for area going from 0 to approach infinity.

1 for Rate of increase increasing as afternoon progresses.