WESTERN REGION

2005
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics
Extension 2

General Instructions

- Reading Time- 5 minutes
- Working Time – 3 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.

Total marks (**120**)

- Attempt Questions 1-8
- All questions are of equal value
- Start a fresh sheet of paper for each question.
- Put your name and the question number at the top of each sheet.

Question 1 (15 marks) **Marks**

(a) Find $\int x\cos(x^2)e^{\sin(x^2)}dx$ **2**

(b) By completing the square in the denominator, find

$$\int\frac{3dx}{x^2-6x+13}$$ **3**

(c) Apply the method of partial fractions to show that

$$\int_3^5\frac{3x+2}{x^2-4}dx=\ell n\left(\frac{63}{5}\right)$$ **4**

(d) Using integration by parts, evaluate

$$\int_0^1 \sin^{-1}x\ dx$$ **3**

(e) Find $\int\frac{2x^2+4x-3}{x+1}dx$ **3**

Question 2 (15 marks)

(a) Given the complex numbers $A = 3 - 4i$ and $B = 1 + i$, determine the following in the form $x + iy$

(i) $A - B$ **1**

(ii) $\frac{A}{B}$ **2**

(iii) $\sqrt{A}$ **3**

(b) Given $C = 1 + \sqrt{3}i$

(i) Write C in mod-arg form **2**

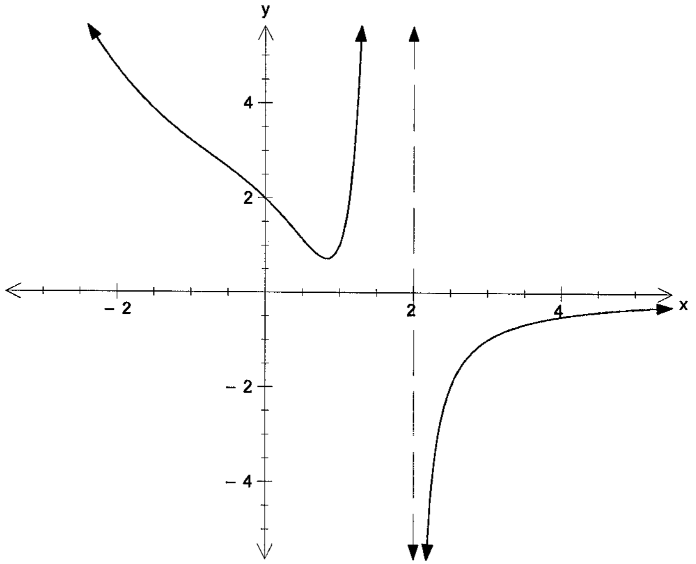
(ii) Hence, using De Moivre's theorem, find C^6 **2**

(c) Determine the Cartesian equation of the locus of $Z = x + iy$ given that $\arg(Z - 1) - \arg(Z + 1) = \frac{\pi}{2}$ **3**

(d) On an Argand diagram, sketch the region where the inequalities $1 \le \operatorname{mod} Z \le 3$ and $0 \le \arg Z \le \frac{\pi}{4}$ both hold. **2**

Question 3 (15 marks)

(a) The diagram below shows the graph of $y = f(x)$



Draw separate sketches of the following (indicate important features)

(i) $y = \frac{1}{f(x)}$ **2**

(ii) $y = [f(x)]^2$ **2**

(iii) $y = \int f(x)dx$ given that when $x = 0, y = 0$ **2**

Question 3 is continued on the next page

Question 3 continued. **Marks**

(b) Given that α, β and γ are the roots of the equation $3x^3+2x^2-5x+1=0$, find equations whose roots are:

(i) α^2, β^2 and γ^2 **3**

(ii) $\frac{1}{\alpha}, \frac{1}{\beta}$ and $\frac{1}{\gamma}$ **2**

(c) Using your answer to (b) (i) above, or otherwise, evaluate $\alpha^3+\beta^3+\gamma^3$. **4**

Question 4 (15 marks)

(a) A solid has its base in the XY plane being the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$
Cross-sections perpendicular to the major axis are squares.

(i) Show that the area of the cross-section at $x=p$ is given by $36-\frac{9p^2}{4}$ units ^{2} **3**

(ii) Hence by using the method of slicing, calculate the volume of the solid. **2**

(b) P is a point on the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ which has a foci at S and S^{l}

(i) Determine the eccentricity of the ellipse. **1**

(ii) Find the coordinates of S and S^{l} and also the equation of the directrices **2**

(iii) Prove that PS + PS^{l} = 4 **2**

(iv) Prove that the normal at P bisects the angle S^{l}PS **5**

Question 5 (15 marks) **Marks**

(a) A solid of mass m kg is falling under gravity in a fluid in which the resistance is proportional to the velocity ($V\ ms^{-1}$).

(i) Write down the equation for this motion. **1**

(ii) If the solid is initially at rest show that, after t seconds the velocity is given by $V = \frac{g}{k}(1 - e^{-kt})$ **4**

(iii) What would be the terminal velocity of this solid? **1**

(iv) Find an expression for the distance fallen, in terms of V **4**

(b) A solid of mass 2kg is attached to an inextensible string of length 1•5 metres, the other end of the string being fixed. The mass rotates in a horizontal circle with an angular velocity of $\pi\ rad\ \ s^{-1}$, forming a conical pendulum. (Take $g = 10ms^{-2}$)

(i) Calculate the tension in the string **3**

(ii) Determine the angle between the string and the vertical axis **1**

(iii) Find the radius of the rotation **1**

Question 6 (15 marks)

(a) The sequence of numbers $U_1, U_2, U_3, \ldots$ is defined by $U_{n+2} = 5U_{n+1} - 6U_n$ **5**
If $U_1 = 1$ and $U_2 = 5$, prove by Induction that $U_n = 3^n - 2^n$

(b) If Z_1, Z_2, Z_3, Z_4 and Z_5 are the roots of $Z^5 = 1$

(i) Determine the values of all of these roots, in the form $\cos\theta + i\sin\theta$ **2**

(ii) Factorise $Z^5 = 1$ in terms of quadratic and linear factors. **2**

(iii) Hence show that $\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = \frac{-1}{2}$ **2**

(c) If $Z = \cos\theta + i\sin\theta$

(i) Show that $Z^n + \frac{1}{Z^n} = 2\cos n\theta$ **1**

(ii) Hence by expanding $(Z + \frac{1}{Z})^4$, **3**
find an expression for $\cos^4\theta$ in the form $a\cos 4\theta + b\cos 2\theta + c$

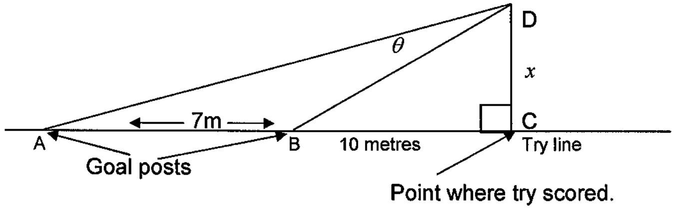
Question 7 (15 marks)

(a) A corner on a racing circuit is the arc of a circle of radius 50 metres. Calculate the angle at which is corner must be inclined so that a vehicle of mass 500kg can take this corner at a speed of $30ms^{-1}$ without any tendency to slip sideways up or down the track. **4**
(Take $g = 9\cdot8ms^{-2}$)

(b) Given that the roots of the equation $x^3 + ax^2 + bx + c = 0$ form a geometric sequence, determine the relationship between a, b and c **4**

(c) In a game of rugby, the goal posts (A and B below) are approximately 7 metres apart.
When a try is scored, a kick at goal is allowed from any point on the line which is perpendicular to the goal line at the point where the try is scored (the point C below)..

A try is scored 10 metres to the side of the goal post B. The kicker takes the ball back a distance of x metres, giving an angle of θ within which he must kick the ball to go between the posts.

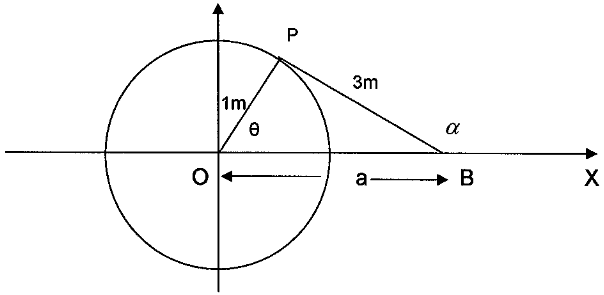


(i) Show that $\tan\theta = \dfrac{7x}{170 + x^2}$ **4**

(ii) Hence show that the kicker must take the ball approximate 13 metres to maximise the angle between the posts for a successful kick. **3**

Question 8 (15 marks) **Marks**

(a)



A rod OP is such that P is free to move in a circle as shown. Rod PB is joined to OP with a flexible joint at P with B restricted to move along the X axis. P is moving about O with angular velocity of π rad s^{-1}. The length of OP is 1 metre and PB is 3 metres.

(i) By letting $\angle XBP = \alpha$ find an expression for the angular velocity of P about B **3**

(ii) Hence find this angular velocity when $\theta = \dfrac{\pi}{4}$ **3**

(iii) If the length of OB is 'a' metres, find an expression for the length a and hence the velocity of Point B along the X axis ($\dfrac{da}{dt}$) when $\theta = \dfrac{\pi}{4}$ **5**

(b) If $\alpha\beta\gamma$ represents a 3 digit number and $\alpha + \beta + \gamma$ is divisible by three show that this number is divisible by 3. **2**

(c) By taking the log of both sides of $y = U(x).V(x)$ verify the differentiation of a product rule **2**

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0 \ \text{ if } \ n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a}\sin ax, a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax, a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax, a \neq 0$$

$$\int \sec ax \tan ax dx = -\frac{1}{a}\sec ax, a \neq 0$$

$$\int \frac{1}{a \ + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, a > 0, -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

Note: $\ln x = \log_e x, x > 0$

Western Region

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Mathematics
Extension 2

Solutions

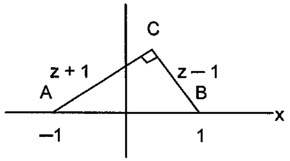
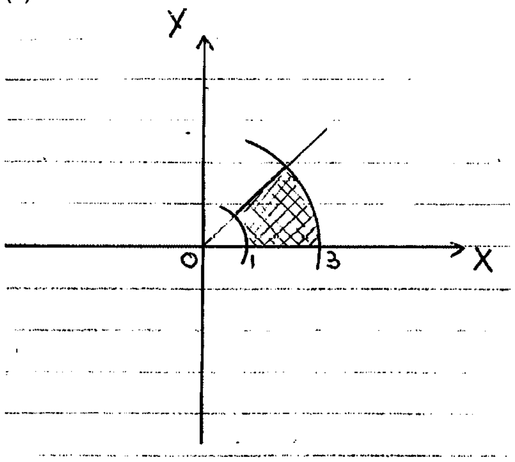
Course: EXTENSION 2 2004

Solutions	Marks	Comments
Question 1		
(a) $\int x\cos(x^2)e^{\sin(x^2)}dx = \frac{1}{2}e^{\sin(x^2)} + C$	2	
(b) $\int\frac{3dx}{x^2-6x+13} = \int\frac{3dx}{x^2-6x+9+4}$		1 – completing the square
$= \int\frac{3dx}{(x-3)^2+4}$		1
$= \frac{3}{2}\tan^{-1}\left(\frac{x-3}{4}\right)+C$	3	
(c) Let $\frac{A}{x+2}+\frac{B}{x-2}=\frac{3x+2}{x^2-4}$ $\frac{A(x-2)+B(x+2)}{x^2-4}=$ $\therefore A(x-2)+B(x+2)=3x+2$ When $x = 2$ $4B = 8$ $\therefore B = 2$ When $x = -2$ $-4A = -4$ $\therefore A = 1$		2 – correct values of A and B
$\therefore \int_3^5\frac{3x+2}{x^2-4}dx = \int_3^5\left[\frac{1}{x+2}+\frac{2}{x-2}\right]dx$		
$= [\ln(x+2)+2\ln(x-2)]_3^5$		1 – correct integration
$= [\ln(x+2)(x-2)^2]_3^5$ $= ln\ 63 - ln\ 5$		1 – correct answer
$= ln\left(\frac{63}{5}\right)$	4	
(d) $\int_0^1\sin^{-1}xdx = \left[x\sin^{-1}x - \int x\frac{1}{\sqrt{1-x^2}}dx\right]_0^1$		1 – use of integration by parts
$= \left[x\sin^{-1}x+\sqrt{1-x^2}\right]_0^1$		1 – correct integration
$= \frac{\pi}{2}-1$	3	1 – correct answer
(e) $\begin{array}{r} 2x+2 \\ x+1\overline{)2x^2+4x-3} \\ \underline{2x^2\ \ +2x} \\ 2x-3 \\ \underline{2x+2} \\ -5 \end{array}$		2 – correct transformation
$\therefore \int\frac{2x^2+4x-3}{x+1}dx = \int\left[2x+2-\frac{5}{x+1}\right]dx$ $= x^2 + 2x - 5ln(x + 1) + C$	3	1 – correct integration

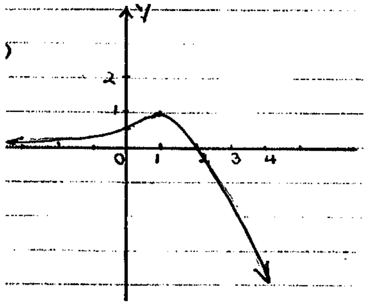
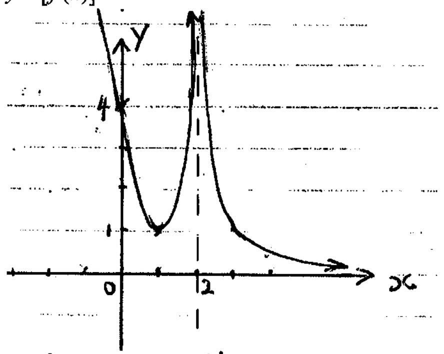
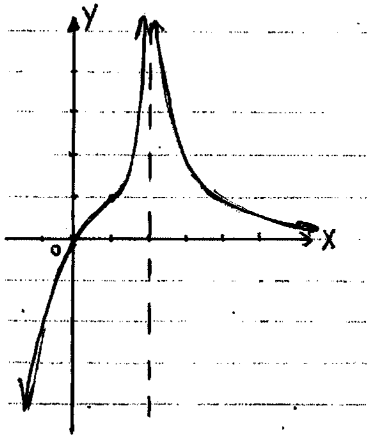
Western Region Trial H.S.C. Marking Scheme

Course: EXTENSION 2

Solutions	Marks	Comments
Question 2		
(a) (i) A − B = (3 − 4i) − (1 + i) = 2 − 5i	1	
(ii) $\frac{A}{B}=\frac{3-4i}{1+i}\times\frac{1-i}{1-i}$ $=\frac{3-7i-4}{1+1}$ $=-\frac{1}{2}-\frac{7}{2}i$	2	
(iii) $\sqrt{A}=\sqrt{3-4i}$ Let $x + iy = \sqrt{3-4i}$ (x & y real) Squaring $x^2 - y^2 + 2xyi = 3 - 4i$ Equating parts $x^2 - y^2 = 3$ $2xy = -4$ Solving $x = \pm 2, \quad y = \pm 1$ $\therefore \sqrt{A} = \pm(2-i)$	3	Any correct method
(b) (i) $C = 1 + \sqrt{3}i$ $= 2(\frac{1}{2}+\frac{\sqrt{3}}{2}i)$ $= 2(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3})$	2	
(ii) $C^6 = 2^6\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^6$ $= 64(\cos 2\pi + i\sin 2\pi)$ = 64	2	

Question 2 Cont'd		
(c) Geometrically, Let = (−1, 0), B(1, 0) and C(x, y) Arg (z − 1) − Arg (z + 1) = ∠ ACB $= \frac{\pi}{2}$  ∴ Circle, centre (0, 0), radius 1 $\therefore x^2 + y^2 = 1$	3	Algebraic method is OK.
(d) 	2	1 – for each inequality

Course: EXTENSION 2 2004

Solutions	Marks	Comments
Question 3 (a) (i) $y=\frac{1}{f(x)}$ 	2	1 – x intercept 1 – General shape
(ii) $y=[f(x)]^2$ 	2	2 – Overall graph
(iii) $y=\int f(x)dx$ 	2	1 – inflexion at (1, 1) 1 – General shape

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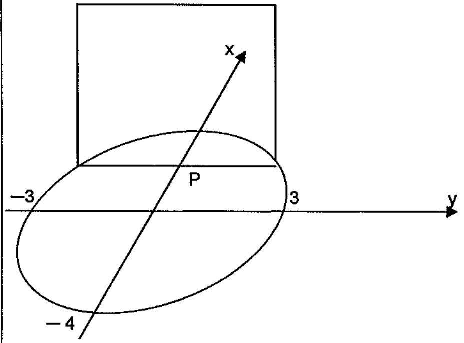
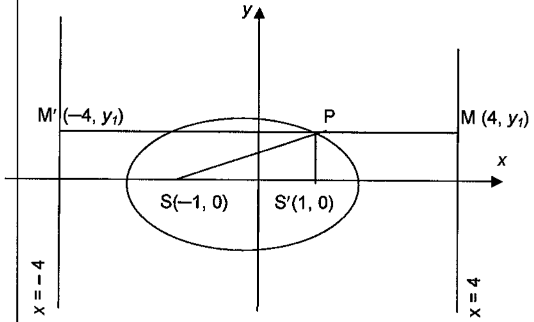
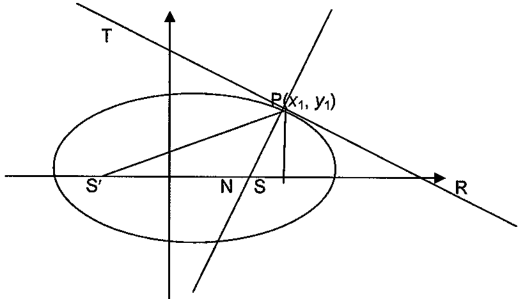
Question 3 Cont'd (b) $3x^3+2x^2-5x+1=0$ (i) For roots $\alpha^2, \beta^2 and \gamma^2$ Let $y=x^2$ $\therefore x=\sqrt{y}$ $\therefore 3(\sqrt{y})^3+2(\sqrt{y})^2-5(\sqrt{y})+1=0$ $3y\sqrt{y}+2y-5\sqrt{y}+1=0$ $2y+1=\sqrt{y}\,(5-3y)$ $4y^2+4y+1=25y-30y^2+9y^3$ $0=9y^3-34y^2+21y-1$ $\therefore$Required Equation is $9x^3-34x^2+21x-1=0$	3	Any correct method
(ii) For roots $\frac{1}{\alpha},\frac{1}{\beta} and \frac{1}{\gamma}$ Let $y=\frac{1}{x}$ $\therefore x=\frac{1}{y}$ $\therefore \frac{3}{y^3}+\frac{2}{y^2}-\frac{5}{y}+1=0$ $3+2y-5y^2+y^3=0$ $\therefore$Required Equation is $x^3-5x^2+2x+3=0$	2	Any correct method
(c) As $x=\alpha, \beta, \gamma$ $\therefore 3\alpha^3+2\alpha^2-5\alpha+1=0$ $3\beta^3+2\beta^2-5\beta+1=0$ $3\gamma^3+2\gamma^2-5\gamma+1=0$ Adding, $3(\alpha^3+\beta^3+\gamma^3)+2(\alpha^2+\beta^2+\gamma^2)-5(\alpha+\beta+\gamma)+3=0$	4	Any method
from original equation $\alpha+\beta+\gamma=-\frac{2}{3}$		1
from (b) (i) $\alpha^2+\beta^2+\gamma^2=-\frac{b}{a}=\frac{34}{9}$		1
$\therefore 3(\alpha^3+\beta^3+\gamma^3)+2(\frac{34}{9})-5(-\frac{2}{3})+3=0$		1
$\therefore \alpha^3+\beta^3+\gamma^3=-\frac{125}{27}$		1

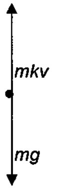
Western Region Trial H.S.C. Marking Scheme

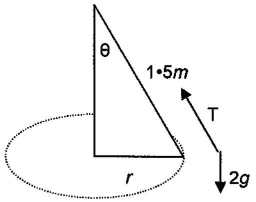
Course: EXTENSION 2 2004

Solutions	Marks	Comments
Question 4 		
(a)(i) When $x = p$ $y=\pm\sqrt{9-\frac{9p^2}{16}}$ ∴ Sidelength of square = $2\sqrt{9-\frac{9p^2}{16}}$ ∴ Area of square = $\left[2\sqrt{9-\frac{9p^2}{16}}\right]^2$ $= 36-\frac{9p^2}{4}$ units^2	3	
(ii) $\lim_{\delta_p=0}\sum_{p=-4}^{4}(36-\frac{9p^2}{4})\delta p=\int_{-4}^{4}\left(36-\frac{9p^2}{4}\right)dp$ $= 2\left[36p-\frac{3p^3}{4}\right]_0^4$ = 2[(144 − 48) − 0] ∴ Volume = 192 units^3	2	
(b) (i) For $\frac{x^2}{4}+\frac{y^2}{3}=1$, $a^2 = 4$, $b^2 = 3$ as $e^2=1-\frac{b^2}{a^2}$ $e^2=1-\frac{3}{4}$ $=\frac{1}{4}$ ∴ Eccentricity = $\frac{1}{2}$	1	

Question 4 Cont'd		
(ii) Foci ($\pm ae$, 0) $S' = (-1, 0)$ $S = (1, 0)$ Directrices $x=\pm\frac{a}{e}$ ∴ $x = 4$ and $x = -4$	2	
(iii) Let P have coordinates (x_1, y_1) Also let M and M′ on the directrices have Coordinates $(4, y_1)$ and $(-4, y_1)$  By definition $\frac{PS}{PM}=e$ and $\frac{PS'}{PM'}=e$ ∴ $PS = ePM$ ∴ $PS' = ePM'$ ∴ $PS + PS' = e(PM + PM')$ $= eMM'$ $=\frac{1}{2}\times 8$ = 4	2	**This may be done differently.**
(iv) Let PN be the normal at P and TR be the tangent at P as shown.  For tangent at P $\frac{2x}{4}+\frac{2y}{3}\cdot\frac{dy}{dx}=0$ $\therefore\frac{dy}{dx}=\frac{-3x}{4y}$		

Question 4 Cont'd		
$\therefore$ Gradient of Normal at P = $\frac{4y_1}{3x_1} = m_1$		
Grad PS = $\frac{y_1}{x_1 - 1} = m_2$		
Grad PS′ = $\frac{y_1}{x_1 + 1} = m_3$		
$\therefore$ tan ∠SPN = $\left\lvert\frac{m_2 - m_1}{1 + m_2 m_1}\right\rvert$		
= $\left\lvert\frac{\frac{y_1}{x_1-1} - \frac{4y_1}{3x_1}}{1 + \frac{y_1}{x_1-1}\cdot\frac{4y_1}{3x_1}}\right\rvert$		
= $\left\lvert\frac{3x_1y_1 - 4y_1(x_1-1)}{3x_1(x_1-1) + 4y_1^2}\right\rvert$		
= $\left\lvert\frac{4y_1 - x_1y_1}{3x_1^2 + 4y_1^2 - 3x_1}\right\rvert$		
As $\frac{x_1^2}{4} + \frac{y_1^2}{3} = 1$ then $3x_1^2 + 4y_1^2 = 12$		
$\therefore$ tan ∠SPN = $\left\lvert\frac{4y_1 - x_1y_1}{12 - 3x_1}\right\rvert = \frac{y_1(4-x_1)}{3(4-x_1)} = \frac{y_1}{3}$		
tan ∠S′PN = $\left\lvert\frac{m_3 - m_1}{1 + m_3 m_1}\right\rvert$		
= $\left\lvert\frac{\frac{y_1}{x_1+1} - \frac{4y_1}{3x_1}}{1 + \frac{y_1}{x_1+1}\cdot\frac{4y_1}{3x_1}}\right\rvert$		
= $\left\lvert\frac{3x_1y_1 - 4y_1(x_1+1)}{3x_1(x_1+1) + 4y_1^2}\right\rvert$		**This may be done by other correct methods.**
= $\left\lvert\frac{-4y_1 - x_1y_1}{3x_1^2 + 4y_1^2 + 3x_1}\right\rvert$		
$\therefore$ tan ∠S′PN = $\left\lvert\frac{-4y_1 - x_1y_1}{12 + 3x_1}\right\rvert = \left\lvert\frac{-y_1(4+x_1)}{3(4+x_1)}\right\rvert = \frac{y_1}{3}$		
$\therefore$ As both angles are acute ∠SPN = ∠S′PN	5	
$\therefore$ The normal at P bisects the angle S′PS		

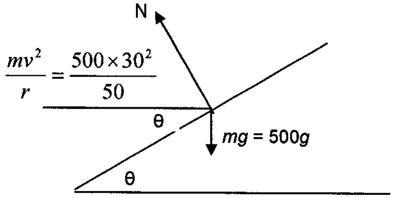
Solutions	Marks	Comments
Question 5		
(a) (i)  $m\ddot{x} = m(g - kv)$		
$\therefore\ \ddot{x} = g - kv$	1	
(ii) $\ddot{x} = g - kv$		
$\frac{dv}{dt} = g - kv$		
$\therefore \frac{dt}{dv} = \frac{1}{g - kv}$		1
$\therefore t = -\frac{1}{k}\ln(g - kv) + C$		
When $t = 0$, $v = 0$		
$\therefore C = \frac{1}{k}\ln g$		1
$\therefore t = -\frac{1}{k}\ln(g - kv) + \frac{1}{k}\ln g$		1
$kt = \ln\left(\frac{g}{g - kv}\right)$		
$e^{kt} = \frac{g}{g - kv}$		
$\frac{g - kv}{g} = e^{-kt}$		
$g - kv = ge^{-kt}$		
$kv = g - ge^{-kt}$		
$\therefore v = \frac{g}{k}\left(1 - e^{-kt}\right)$	4	1
(iii) For terminal velocity, $\ddot{x} = 0$		**Can be done from (ii) as t→ ∞**
$\therefore\ g - kv = 0$		
$\therefore v = \frac{g}{k}$		
$\therefore$ Terminal velocity = $\frac{g}{k}ms^{-1}$	1	

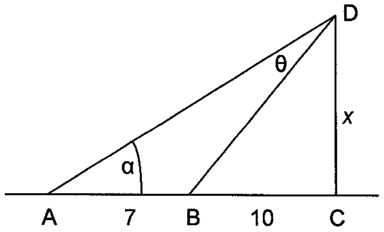
Question 5 cont'd (iv) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = g - kv$ $\frac{d}{dv}\left(\frac{1}{2}v^2\right)\frac{dv}{dx} = g - kv$ $v\frac{dv}{dx} = g - kv$ $\frac{dv}{dx} = \frac{g-kv}{v}$		1
$\therefore \frac{dx}{dv} = \frac{v}{g-kv}$ $\frac{dx}{dv} = -\frac{1}{k} + \frac{g/k}{g-kv}$		1
$\therefore x = -\frac{v}{k} - \frac{g}{k^2}\ln(g-kv) + C$ When $x = 0$, $v = 0$ $\therefore C = \frac{g}{k^2}\ln g$		1
$\therefore x = -\frac{v}{k} - \frac{g}{k^2}\ln(g-kv) + \frac{g}{k^2}\ln g$ $= -\frac{v}{k} + \frac{g}{k^2}\ln\left(\frac{g}{g-kv}\right)$	4	1
(b) (i)  Vertically *T*Cos θ = 2g = 20 ……………(α) Horizontally *T*Sin θ = $mr\omega^2$ ……………(β) But *r* = 1•5Sin θ ∴ *T*Sin θ = 2 × 1•5 Sin θ × π² ∴ *T* = 3π² ∴ Tension in string = 3π² N	3	**1 – Sketch** **1 – resolving** **1 – for T**
(ii) Using (α) Cos θ = $\frac{20}{3\pi^2}$ ∴ θ = 47° 31′	1	
(iii) *r* = 1•5Sin θ ∴ *r* = 1•12*m* (2 *dp*) ∴ Radius = 1•12*m* (nearest *cm*)	1	

Course: EXTENSION 2

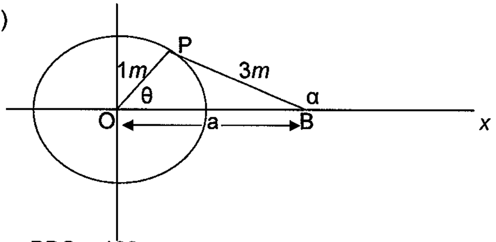
Solutions	Marks	Comments
Question 6 (a) $u_1 = 1$ $3^1 - 2^1 = 1$ $u_2 = 5$ $3^2 - 2^2 = 5$ $u_3 = 5 \times 5 - 6 \times 1 = 19$ $3^3 - 2^3 = 19$ ∴ True up to *n* = 3 Assume true up to *n* = *k* i.e. $u_k = 5u_{k-1} - 6u_{k-2} = 3^k - 2^k$ When *n* = *k* + 1 $U_{k+1} = 5u_k - 6u_{k-1}$ $= 5(3^k - 2^k) - 6(3^{k-1} - 2^{k-1})$ $= 15.3^{k-1} - 10.2^{k-1} - 6.3^{k-1} + 6.2^{k-1}$ $= 9.3^{k-1} - 4.2^{k-1}$ $= 3^{k+1} - 2^{k+1}$ ∴ True for *n* = *k* + 1 If true for *n* = 1 then true for *n* = 2 If true for *n* = 2 then true for *n* = 3 Etc. ∴ True for all *n*	5	
(b) (i) Let *Z* = cos θ + *i*sin θ ∴ $Z^5 = 1$ By De Moivre's Theorem cos 5θ + *i*sin 5θ = 1 ∴ Equating real parts cos 5θ = 1 5θ = 0, 2π, 4π, 6π, 8π $\theta = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}$ ∴ $z_1 = \cos 0 = 1$, $z_2 = \text{cis}\,\frac{2\pi}{5}$, $z_3 = \text{cis}\,\frac{4\pi}{5}$, $z_4 = \text{cis}\,\frac{6\pi}{5}$, $z_5 = \text{cis}\,\frac{8\pi}{5}$	2	
(ii) Noting $z_5 = \bar{z}_2$ and $z_4 = \bar{z}_3$ ∴ $z^5 - 1 = (z - z_1)(z - z_2)(z - \bar{z}_2)(z - z_3)(z - \bar{z}_3)$ $= (z - z_1)[z^2 - (z_2 + \bar{z}_2)z + z_2\bar{z}_2][z^2 - (z_3 + \bar{z}_3)z + z_3\bar{z}_3]$ $= (z - z_1)(z^2 - 2\cos\frac{2\pi}{5}z + 1)(z^2 - 2\cos\frac{4\pi}{5}z + 1)$	2	

Question 6 Cont'd		
(iii) Noting that $z_1 + z_2 + z_3 + z_4 + z_5 = 0$ i.e. coefficient of z^4 in $z^5 - 1$ $\therefore 1 + z_2 + \bar{z}_2 + z_3 + \bar{z}_3 = 0$ $1 + 2\cos\frac{2\pi}{5} + 2\cos\frac{4\pi}{5} = 0$ $2\left(\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5}\right) = -1$ $\therefore \cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$	2	
(c) (i) $z^n = \cos n\theta + i\sin n\theta$ $\frac{1}{z^n} = z^{-1} = \cos n\theta - i\sin n\theta$ $\therefore z^n + \frac{1}{z^n} = 2\cos n\theta$	1	
(ii) $\left(z+\frac{1}{z}\right)^4 = (2\cos\theta)^4 = 16\cos^4\theta$ By expansion, $\left(z+\frac{1}{z}\right)^4 = z^4 + 4.z^3.\frac{1}{z} + 6.z^2\frac{1}{z^2} + 4.z.\frac{1}{z^3} + \frac{1}{z^4}$ $= z^4 + \frac{1}{z^4} + 4\left(z^2 + \frac{1}{z^2}\right) + 6$ $\therefore 16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$ $\therefore \cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$	3	

Solutions	Marks	Comments
Question 7 (a)  Resolving along the slope $mg\sin\theta = m\frac{v^2}{r}\cos\theta$ $\tan\theta = \frac{v^2}{rg}$ $= \frac{30^2}{50\times 9\cdot 8}$ $\therefore \theta = 61° 26'$ (nearest minute) $\therefore$ Angle of banking 61°26′	4	
(b) Let the roots be $x = \frac{\alpha}{k}, \alpha, \alpha k$ $\therefore \frac{\alpha}{k} + \alpha + \alpha k = -a$ $\alpha\left(\frac{1}{k} + 1 + k\right) = -a$ ....(1) $\frac{\alpha}{k}\cdot\alpha + \frac{\alpha}{k}\cdot\alpha k + \alpha\cdot\alpha k = b$ $\alpha^2\left(\frac{1}{k} + 1 + k\right) = b$ ....(2) $\frac{\alpha}{k}\cdot\alpha\cdot k\alpha = -c$ $\therefore \alpha^3 = -c$ ....(3) (2) ÷ (1) $\alpha = -\frac{b}{a}$ $\therefore \alpha^3 = -\frac{b^3}{a^3}$ i.e. $-c = -\frac{b^3}{a^3}$ $\therefore b^3 = a^3c$	4	**Any equivalent answer**

Question 7 Cont'd (c) (i)  Let ∠DAB = ∠DAC = α Tan α = $\frac{x}{17}$ ∠DBC = θ + α (ext ∠ of ΔABD) tan (θ + α) = $\frac{x}{10}$ $\therefore x$ = 10tan (θ + α) $= \frac{10(\tan\theta + \tan\alpha)}{1 - \tan\theta\tan\alpha}$ $= \frac{10\left(\tan\theta + \frac{x}{17}\right)}{1 - \tan\theta \cdot \frac{x}{17}}$ $= \frac{170\tan\theta + 10x}{17 - x\tan\theta}$ $17x - x^2 \tan\theta = 170\tan\theta + 10x$ $7x = (170 + x^2)\tan\theta$ $\therefore \tan\theta = \frac{7x}{170 + x^2}$	4	
(ii) Diffn $\sec^2\theta \frac{d\theta}{dx} = \frac{(170+x^2)7 - 7x.2x}{(170+x^2)^2}$ $\therefore \frac{d\theta}{dx} = \frac{1190 - 7x^2.\cos^2\theta}{(170+x^2)^2}$ for max θ, $\frac{d\theta}{dx} = 0$ $\therefore$ 1190 – 7x^2 = 0 x^2 = 170 $x = \sqrt{170} \approx 13$ $\therefore$ Approximate distance = 13*m.*	3	

Course: EXTENSION 2

Solutions	Marks	Comments
Question 8 (a)  (i) ∠PBO = 180 – α By Sine Rule $\frac{\sin(180-\alpha)}{1} = \frac{\sin\theta}{3}$ $\therefore \sin\alpha = \frac{\sin\theta}{3}$ diff w.r.t. time $\cos\alpha.\frac{d\alpha}{dt} = \frac{\cos\theta}{3}\frac{d\theta}{dt}$ $\therefore \frac{d\alpha}{dt} = \frac{\cos\theta}{3\cos\alpha} \cdot \frac{d\theta}{dt}$ $\frac{d\theta}{dt} = \pi$ $\therefore \frac{d\alpha}{dt} = \frac{\cos\theta.\pi}{3\cos\alpha}$	3	
(ii) When θ = $\frac{\pi}{4}$ $\sin\alpha = \frac{\sin\frac{\pi}{4}}{3}$ $= \frac{1}{3\sqrt{2}}$ $\therefore \cos\alpha = \sqrt{1 - \frac{1}{18}} = \sqrt{\frac{17}{18}} = \frac{\sqrt{17}}{3\sqrt{2}}$ $\therefore \frac{d\alpha}{dt} = \frac{\frac{1}{\sqrt{2}}\pi}{3\frac{\sqrt{17}}{3\sqrt{2}}} = \frac{\pi}{\sqrt{17}} rad/s$	3	

Question 8 cont'd

(iii) ∠OPB = α – θ

$$\therefore \frac{a}{\sin(\alpha-\theta)} = \frac{3}{\sin\theta}$$

$$a = \frac{3\sin(\alpha-\theta)}{\sin\theta}$$

$$= \frac{3(\sin\alpha\cos\theta - \cos\alpha\sin\theta)}{\sin\theta}$$

$$= \frac{3\sin\alpha\cos\theta}{\sin\theta} - 3\cos\alpha$$

= 3sin α cot θ –3cos α

2

$$\frac{da}{dt} = 3\left(-\sin\alpha\cos ec^2\theta\cdot\frac{d\theta}{dt} + \cos\alpha\cot\theta\cdot\frac{d\alpha}{dt}\right) + 3\sin\alpha\cdot\frac{d\alpha}{dt}$$

When $\theta = \frac{\pi}{4}$

$$\frac{da}{dt} = 3\left(-\frac{1}{3\sqrt{2}}.2\pi + \frac{\sqrt{17}}{3\sqrt{2}}.1.\frac{\pi}{\sqrt{17}}\right) + 3.\frac{1}{3\sqrt{2}}.\frac{\pi}{\sqrt{17}}$$

$$= \frac{(1-\sqrt{17})}{\sqrt{34}}\pi \quad m\,s^{-1}$$

3

(b) αβγ represents

100α + 10β + γ

as α + β + γ is divisible by 3

α + β + γ 3*K* (*K integer)*

∴99α +9β + α + β + γ = 3[33α +3β] +3K

= 3[33α +3β +K]

which is divisible by 3.

2

(c) y = *u*(*x*) . *v*(*x*)

∴*ln y* = *ln u*(*x*) + *ln v*(*x*)

$$\therefore \frac{1}{y}\frac{dy}{dx} = \frac{1}{u(x)}\cdot u'(x) + \frac{1}{v(x)}\cdot v'(x)$$

$$\frac{1}{u(x).v(x)}\cdot\frac{dy}{dx} = \frac{u'(x)}{u(x)} + \frac{v'(x)}{v(x)}$$

$$\therefore \frac{dy}{dx} = \frac{u'(x)}{u(x)}\cdot u(x).v(x) + \frac{v'(x)}{v(x)}\cdot u(x).v(x)$$

$$= v(x).u'(x) + u(x).v'(x)$$

2