

ALEXANDRIA PARK COMMUNITY SCHOOL

(WESTERN REGION 2013 PAPER)



Year 12 2013

HSC TRIAL EXAMINATION Mathematics Extension 2

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

1. A square root of $8 + 6i$ is :

(A) $3 - i$ (B) $5 - 3i$

(C) $-3 - i$ (D) $-3 + i$

2. The equation of a curve is given by $x^2 + xy + y^2 = 9$. Which of the following expressions will provide the value of $\frac{dy}{dx}$ at any point on the curve?

(A) $\frac{-2x - y}{2y}$ (B) $\frac{-2x - y}{x + 2y}$

(C) $\frac{-2x + y}{2y}$ (D) $\frac{-2x + y}{x + 2y}$

3. The equation $x^3 + 2x^2 - 4x + 5 = 0$ has roots α , β and γ . Find the value of $\alpha^2 + \beta^2 + \gamma^2$.

(A) -12 (B) -4 (C) 4 (D) 12

4. The area bounded by the curves $y = x^2$ and $x = y^2$ is rotated about the x -axis. The volume of the solid of revolution formed is:

(A) $\frac{9\pi}{70}$ (B) $\frac{3\pi}{10}$

(C) $\frac{7\pi}{10}$ (D) $\frac{3\pi}{2}$

General Instructions

- Reading Time - 5 minutes
- Working Time - 3 hours
- Write using a blue or black pen. Black pen is preferred
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11-16
- Write your student number at the top of every page

Total marks (100)

Section I

Total marks (10)

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided
- Allow about 15 minutes for this section

Section II

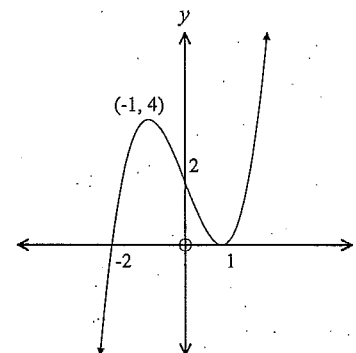
Total marks (90)

- Attempt questions 11 – 16
- Answer on the blank paper provided, unless otherwise instructed
- Start a new page for each question
- All necessary working should be shown for every question
- Allow about 2 hours 45 minutes for this section

5. The equation of an hyperbola is given by $9x^2 - 4y^2 = 36$. The foci and the directrices of this hyperbola are:

- (A) $(\pm\sqrt{13}, 0)$ and $x = \pm\frac{4\sqrt{13}}{13}$.
- (B) $(0, \pm\sqrt{13})$ and $x = \pm\frac{4\sqrt{13}}{13}$.
- (C) $(\pm\sqrt{13}, 0)$ and $y = \pm\frac{4\sqrt{13}}{13}$.
- (D) $(0, \pm\sqrt{13})$ and $y = \pm\frac{4\sqrt{13}}{13}$.

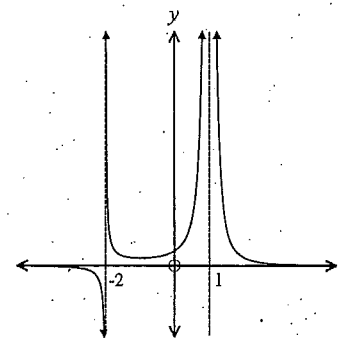
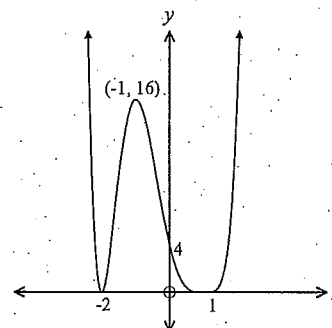
6. The graph of the function $y = f(x)$ is drawn below:



Which of the following graphs best represents the graph $y = \sqrt{f(x)}$?

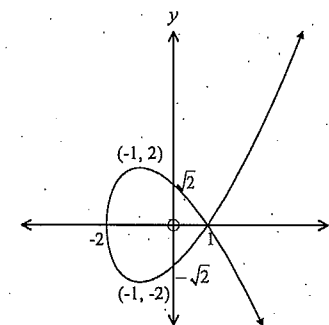
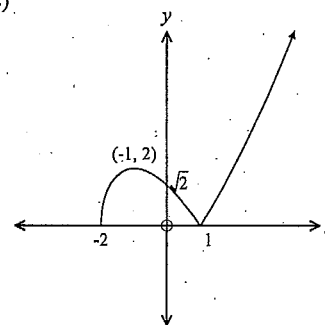
(A)

(B)



(C)

(D)



$$7. \int \frac{x^3 - 1}{(x^4 - 4x)^{\frac{2}{3}}} dx =$$

- (A) $\frac{3}{4\sqrt[3]{x^4 - 4x}} + C$ (B) $\frac{3}{4(x^4 - 4x)} + C$
 (C) $\frac{3\sqrt[3]{x^4 - 4x}}{4} + C$ (D) $\frac{3(x^4 - 4x)}{4} + C$

8. A point is moving in a circular path about a centre O with angular velocity ω .
 An expression for the normal acceleration of the point at a time t is:

- (A) $\frac{dv}{dt}$
 (B) $r \frac{d\theta}{dt}$
 (C) $r \omega$
 (D) $r \omega^2$

9. The general solution of the equation $\sin 4\theta - \sin 2\theta = \cos 3\theta$ is:

- (A) $\theta = \frac{n\pi}{3} \pm \frac{\pi}{6}$ or $\theta = n\pi + (-1)^n \frac{\pi}{3}$.
 (B) $\theta = \frac{n\pi}{2} \pm \frac{\pi}{4}$ or $\theta = n\pi + (-1)^n \frac{\pi}{6}$.
 (C) $\theta = \frac{2n\pi}{3} \pm \frac{\pi}{6}$ or $\theta = n\pi + (-1)^n \frac{\pi}{3}$.
 (D) $\theta = \frac{2n\pi}{3} \pm \frac{\pi}{6}$ or $\theta = n\pi + (-1)^n \frac{\pi}{6}$.

10. Thirteen students in a Year 10 PDHPE class are to be divided into two teams of six to play touch football, with the remaining person acting as the referee. If two particular students are not to be in the same team, the number of different ways the teams can be formed is:

- (A) ${}^{11}C_5 \times {}^6C_5 + {}^{12}C_6 \times 2$
 (B) ${}^{11}C_5 \times {}^6C_5 \times 2 + {}^{12}C_6$
 (C) ${}^{11}C_5 \times {}^6C_5 + {}^{12}C_6$
 (D) ${}^{13}C_6 \times {}^7C_6 + 2$

End of Section I

Section II

Total marks (90).

Attempt Questions 11-16

Allow about 2 hours 45 minutes for this section

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page. Do not write on the back of sheets.

Question 11 (15 marks)	Use a separate sheet of paper	Marks
a) i)	If $\frac{x}{(x-2)^2(x-1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x-1}$, find the values of A , B and C .	2
ii)	Hence evaluate $\int \frac{x}{(x-2)^2(x-1)} dx$.	2
b) i)	Derive the reduction formula	1
	$\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx.$	
ii)	Hence or otherwise, find $\int_0^2 x^3 e^x dx$	2
c)	Find	
i)	$\int \frac{5x-3}{x^2+6x-14} dx$	3
ii)	$\int \frac{dx}{(25+x^2)^{3/2}}$ using the trigonometric substitution $x = 5 \tan \theta$.	3
iii)	$\int \frac{dx}{\sqrt{4+2x-x^2}}$	2

End of Question 11

Question 12 (15 marks) Use a separate sheet of paper

Marks

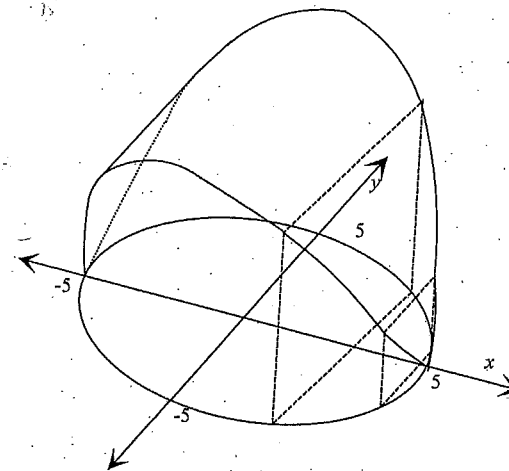
a)	If $z = \frac{1+\sqrt{3}i}{\sqrt{3}+i}$, find:	
i)	$ z $	1
ii)	$\text{Arg}(z)$	1
iii)	z^4	1
b)	Sketch the region in the Argand plane consisting of those points z for which:	3
	$ \arg(z+1) < \frac{\pi}{6}$, $z + \bar{z} \leq 6$ and $ z+1 > 2$.	
c)		
i)	Use De Moivre's theorem to express $\cos 4\theta$ in terms of $\cos \theta$.	2
ii)	Hence express $\cot 4\theta$ as a rational function of x where $x = \cot \theta$.	1
iii)	By considering the roots of $\cot 4\theta = 0$, prove that	2
	$\cot \frac{\pi}{8} \cdot \cot \frac{3\pi}{8} \cdot \cot \frac{5\pi}{8} \cdot \cot \frac{7\pi}{8} = 1.$	
d)	Prove that $\binom{n-1}{k-1} = \frac{k}{n} \binom{n}{k}$.	2
e)	If a and b are positive real numbers, show that $a^2 - ab + b^2 \geq \left(\frac{a+b}{2}\right)^2$	2

End of Question 12

- Question 13 (15 marks)** Use a separate sheet of paper **Marks**
- a)
- i) Show that if $x = a$ is a double root of the polynomial $P(x) = 0$, then $P'(a) = P(a) = 0$. 2
 - ii) Find the roots of the equation $f(x) = x^4 - 2x^3 + x^2 + 12x + 8$, given that it has a double root. 3
 - iii) Given that one root of the equation $x^4 - 5x^3 + 5x^2 + 25x - 26 = 0$ is $3 + 2i$, solve the equation. 3
- b) The equation $2x^3 - x^2 + 3x - 1 = 0$ has roots α, β , and γ . Find the equation which has roots:
- i) $2\alpha, 2\beta$ and 2γ . 2
 - ii) α^2, β^2 and γ^2 . 2
- c) The region bounded by $y = \sin x$ and the x -axis between $x = 0$ and $x = \pi$ is rotated about the y -axis. 3
- Find the volume of the resulting solid, using the method of Cylindrical Shells.

End of Question 13

- Question 14 (15 marks)** Use a separate sheet of paper **Marks**
- a) An ellipse has equation $\frac{x^2}{16} + \frac{y^2}{25} = 1$.
- i) Show that this is the equation of the locus of a point $P(x, y)$ that moves such that the sum of its distances from $A(0, 3)$ and $B(0, -3)$ is 10 units. 4
 - ii) Find the equations of the tangents to the ellipse when $y = 4$. 4
- b) The hyperbola H has equation $\frac{x^2}{4} - \frac{y^2}{9} = 1$. 3
- P is an arbitrary point $(2\sec \theta, 3\tan \theta)$. Show that P lies on H and show that the equation of the tangent at P is $\frac{x \sec \theta}{2} - \frac{y \tan \theta}{3} = 1$.
- c) Let S be the solid having for its base the region bounded by the circle $x^2 + y^2 = 16$. 4



Every vertical plane section of the solid perpendicular to the x -axis is a square.

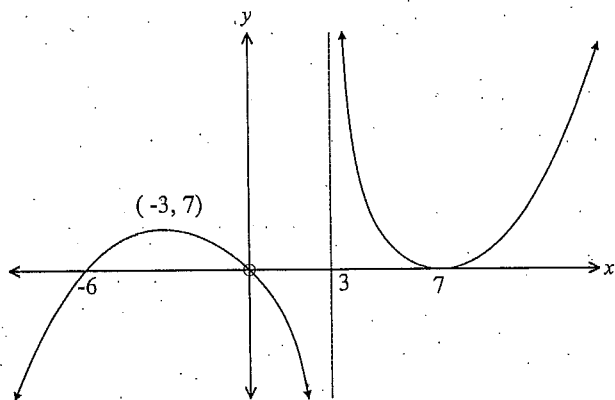
Find the volume of the solid S .

End of Question 14

Question 15 (15 marks) Use a separate sheet of paper

Marks

a) The graph of $y = f(x)$ is shown below:



Draw separate half page sketches of the following (indicate important features).

i) $y = \frac{1}{f(x)}$

2

ii) $y = |f(x)|$

2

iii) $y = e^{f(x)}$

3

b) i) Use the principle of Mathematical induction to prove that:

3

$$\sum_{i=1}^n i^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

ii) Hence evaluate

$$\lim_{x \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$$

2

c) To create the daily passcode for entry to an animal enclosure at the zoo, four letters from the word FERRETS are chosen in a random order. e.g. the code might be RRES or TERF.

3

How many distinct four letter passcodes are possible?

End of Question 15

Question 16 (15 marks) Use a separate sheet of paper

Marks

a) A body of mass 50 kg falls from a height at which gravitational acceleration is g . Assuming that air resistance is proportional to the speed v with a constant of proportion being $\frac{1}{10}$, find:

i) The velocity after time t .

3

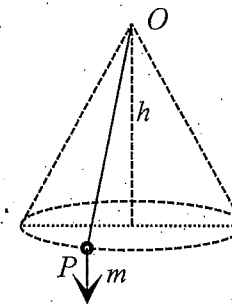
ii) The terminal velocity.

1

iii) The distance the object has fallen after time t .

2

b) A particle P , of mass m kg, is suspended by a light inextensible string from a point O . It describes a circle with constant speed in a horizontal plane whose vertical distance below O is h metres.



i) By resolving forces find an equation for the period of revolution.

2

ii) Find the period if the distance below the point of suspension is 50 cm and $g = 10$ m/s.

1

Question 16 continues on page 13

Question 16 continued.

Marks

- c) A circular track has a radius of 100 metres.
Using $g = 10\text{ms}^{-2}$, calculate the angle at which the track should be banked in order to allow cars to travel at 80 kilometres per hour with no sideways force on the tyres? **3**
- d) An object of mass 5kg is attached to a fixed point on a smooth table by a string of length 1.5 metres. If the object moves with a velocity of 3m/s:
- i) Find the tension in the string. **1**
- ii) If the maximum tension that the string can withstand is 150N, find the greatest number of revolutions per second that the particle can make without breaking the string. **2**

End of Examination

Western Mathematics Exams

Trial HSC Examination – Mathematics Extension 2
2013

2013
TRIAL HSC
EXAMINATION

Mathematics Extension 2

SOLUTIONS

Section I – Multiple Choice Answer Sheet

Name _____

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
A B C D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A B C D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A B ^{correct} C D

- Start Here →
1. A B C D
 2. A B C D
 3. A B C D
 4. A B C D
 5. A B C D
 6. A B C D
 7. A B C D
 8. A B C D
 9. A B C D
 10. A B C D

Multiple Choice Worked Solutions.	
<p>1. Let $(x + iy)^2 = 8 + 6i$ Then $x^2 + 2xyi - y^2 = 8 + 6i$ $\therefore x^2 - y^2 = 8$ --- (1) And $2xy = 6$</p> $(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$ $= 8^2 + (6)^2$ $= 100$ $\therefore x^2 + y^2 = 10$ <p>(1) + (2) $2x^2 = 18$ $x^2 = 9$</p> <p>(2) - (1) $2y^2 = 2$ $y^2 = 1$</p> <p>Since $2xy = 6$ Square root is $\pm(3 + i)$</p> <p style="text-align: center;">$x = \pm 3$</p> <p style="text-align: center;">$y = \pm 1$</p>	C
<p>2. $x^2 + xy + y^2 = 9$</p> $2x + y + x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$ $\frac{dy}{dx}(x + 2y) = -2x - y$ $\frac{dy}{dx} = \frac{-2x - y}{x + 2y}$	B
<p>3. $x^3 + 2x^2 - 4x + 5 = 0$</p> $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$ $= \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$ $= \left(\frac{-2}{1}\right)^2 - 2\left(\frac{-4}{1}\right)$ $= 4 + 8$ $= 12$	D
<p>4. $Volume = \pi \int_0^1 (\sqrt{x^2 - x^2}) dx = \pi \int_0^1 (x - x^4) dx$</p> $= \pi \left[\frac{x^2}{2} - \frac{x^5}{5} \right]_0^1$ $= \pi \left[\left(\frac{1}{2} - \frac{1}{5}\right) - 0 \right] = \frac{3\pi}{10}$	B

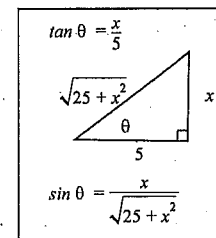
<p>5. $9x^2 - 4y^2 = 36$ $\frac{x^2}{4} - \frac{y^2}{9} = 1$</p> <p>$a = 2, b = 3$ $a^2e^2 = 2^2 + 3^2 = 13$ $ae = \sqrt{13}$</p> <p style="text-align: right;">Focus = $(\pm\sqrt{13}, 0)$</p> <p>$e = \frac{\sqrt{13}}{2}$</p> <p>Directrix $x = \pm \frac{a}{e} = \pm \frac{4}{\sqrt{13}} = \pm \frac{4\sqrt{13}}{13}$</p>	A
<p>6. Answer = C</p>	C
<p>7. $\int \frac{x^3 - 1}{(x^4 - 4x)^{\frac{2}{3}}} dx$ Let $u = x^4 - 4x$</p> $= \frac{1}{4} \int \frac{4x^3 - 4}{(x^4 - 4x)^{\frac{2}{3}}} dx$ <p style="text-align: right;">$du = 4x^3 - 4$</p> $= \frac{1}{4} \int u^{-\frac{2}{3}} du$ $= \frac{1}{4} \frac{u^{\frac{1}{3}}}{\frac{1}{3}} + c$ $= \frac{3}{4} \sqrt[3]{x^4 - 4x} + c$	C
<p>8. Normal component of acceleration</p> $= -\ddot{x} \cos \theta - \ddot{y} \sin \theta$ $= \dot{i} \cos \theta (-r\omega^2 \cos \theta - r\dot{\omega} \sin \theta) - \sin \theta (-r\omega^2 \sin \theta - r\dot{\omega} \cos \theta)$ $= r\omega^2 (\cos^2 \theta + \sin^2 \theta)$ $= r\omega^2$	D
<p>9. $\sin 4\theta - \sin 2\theta = \cos 3\theta$</p> $2 \cos \left(\frac{4\theta + 2\theta}{2}\right) \sin \left(\frac{4\theta - 2\theta}{2}\right) = \cos 3\theta$ $2 \cos 3\theta \sin \theta - \cos 3\theta = 0$ $\cos 3\theta (2 \sin \theta - 1) = 0$ <p>$\cos 3\theta = 0$ or $\sin \theta = \frac{1}{2}$</p> $3\theta = 2n\pi \pm \cos^{-1} 0$ $\therefore \theta = \frac{2n\pi}{3} \pm \frac{\pi}{6}$ <p style="text-align: right;">$\theta = n\pi + (-1)^n \sin^{-1} \frac{1}{2}$ or $n\pi + (-1)^n \frac{\pi}{6}$</p>	D

<p>10. Call the students who cannot be on the same team A and B. Case 1 students A and B are in separate teams. This leaves ${}^{11}C_5 \times {}^6C_5$ ways the remaining students can be placed with the last student being the referee. Case 2 Student A is the referee which leaves ${}^{12}C_6 \times {}^6C_6 = {}^{12}C_6$ ways the team 1 and team 2 can be formed. However those chosen for team 1 and team 2 could be interchanged, and it is still the same arrangement, so the number of ways = ${}^{12}C_6 + 2$ ways. Similarly if student B is the referee this again leaves ${}^{12}C_6 + 2$ ways the teams can be formed. Total Ways = ${}^{11}C_5 \times {}^6C_5 + {}^{12}C_6 + 2 \times 2 = {}^{11}C_5 \times {}^6C_5 + {}^{12}C_6$</p>	C
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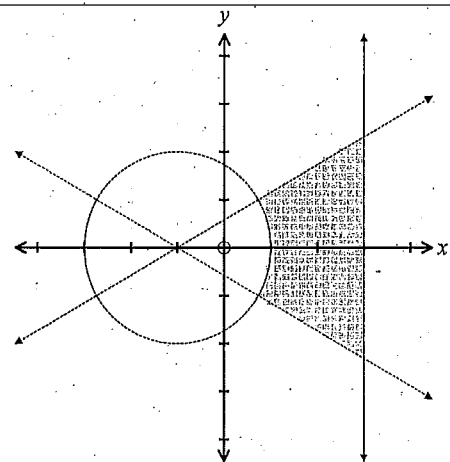
Question 11		Trial HSC Examination		2013	
Part	Solution	Marks	Comment		
(a) i)	$\frac{x}{(x-2)^2(x-1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x-1}$ $x = A(x-2)(x-1) + B(x-1) + C(x-2)^2$ <p>Let $x=1 \quad \therefore C=1$ Let $x=2 \quad \therefore B=2$ This now means:</p> $x = A(x-2)(x-1) + 2(x-1) + (x-2)^2$ <p>Let $x=0$ $0 = 2A - 2 + 4$ $2A + 2 = 0$ $2A = -2$ $A = -1$ Therefore $A = -1, B = 2$ and $C = 1$.</p>	1	For both B and C		
ii)	$\frac{x}{(x-2)^2(x-1)}$ $= -\frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{x-1}$ <p>$\therefore \int \frac{x}{(x-2)^2(x-1)} dx$</p> $= \int -\frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{x-1} dx$ $= \int -\frac{1}{x-2} dx + 2 \int (x-2)^{-2} dx + \int \frac{1}{x-1} dx$ $= -\ln(x-2) - 2(x-2)^{-1} + \ln(x-1)$ $= \ln\left(\frac{x-1}{x-2}\right) - \frac{2}{x-2} + c$	1	For the value of A		
		1	For the integral		
		1	Answer		

Question 11		Trial HSC Examination		2013	
Part	Solution	Marks	Comment		
(b)	<p>i) $x^n e^x dx$ Let $u = x^n$ $v' = e^x$ $\therefore u' = nx^{n-1}$ $v = e^x$ $\int x^n e^x dx = uv - \int v u'$ $= x^n e^x - \int e^x nx^{n-1} dx$ <i>i.e.</i> $\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$</p>	1	Reduction Formula		
	<p>ii) $\int_0^2 x^3 e^x dx = [x^3 e^x]_0^2 - 3 \int x^2 e^x dx$ $= [x^3 e^x]_0^2 - 3\{[x^2 e^x]_0^2 - 2 \int x e^x dx\}$ $= [x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x]_0^2$ $= [8e^2 - 12e^2 + 12e^2 - 6e^2] - [0 + 0 + 0 - 6]$ $= 2e^2 + 6$</p>	1	Substitution		
		1	Answer		

Question 11		Trial HSC Examination		2013	
Part	Solution	Marks	Comment		
(c)	<p>i. $\int \frac{5x-3}{x^2+6x-14} dx$ $= \frac{5}{2} \int \frac{2x+6}{x^2+6x-14} dx - \int \frac{18}{x^2+6x-14} dx$ $= \frac{5}{2} \int \frac{2x+6}{x^2+6x-14} dx - 18 \int \frac{dx}{x^2+6x+9-23}$ $= \frac{5}{2} \ln(x^2+6x-14) - 18 \int \frac{dx}{(x+3)^2-23}$ $= \frac{5}{2} \ln(x^2+6x-14) - \frac{18}{2\sqrt{23}} \ln \frac{x+3-\sqrt{23}}{x+3+\sqrt{23}} + c$ $= \frac{5}{2} \ln(x^2+6x-14) - \frac{9}{\sqrt{23}} \ln \frac{x+3-\sqrt{23}}{x+3+\sqrt{23}} + c$</p>	1	Splitting integral		
		1	First integral		
		1	Other integral		
	<p>ii. $I = \int \frac{dx}{(25+x^2)^{3/2}} \quad x = 5 \tan \theta$ $dx = 5 \sec^2 \theta d\theta$ $I = \int \frac{1}{(25+25 \tan^2 \theta)^{3/2}} 5 \sec^2 \theta d\theta$ $(25+25 \tan^2 \theta)^{3/2} = 125 (1+\tan^2 \theta)^{3/2}$ $= 125 (\sec^2 \theta)^{3/2}$ $= 125 \sec^3 \theta$ $I = \int \frac{5 \sec^2 \theta d\theta}{125 \sec^3 \theta}$ $= \frac{1}{25} \int \frac{1}{\sec \theta} d\theta$ $= \frac{1}{25} \int \cos \theta d\theta$ $= \frac{1}{25} \sin \theta + c$ $= \frac{1}{25} \frac{x}{\sqrt{25+x^2}} + c = \frac{x}{25\sqrt{25+x^2}} + c$</p>	1	Initial Substitution		
		1	Manipulation		
	<p>iii. $\int \frac{dx}{\sqrt{4+2x-x^2}} = \int \frac{dx}{\sqrt{4-(x^2-2x)}}$ $= \int \frac{dx}{\sqrt{5-(x^2-2x+1)}}$ $= \int \frac{dx}{\sqrt{5-(x-1)^2}}$ Let $u = x-1$ and so $du = dx$ $= \int \frac{du}{\sqrt{5-u^2}}$ $= \sin^{-1} \frac{u}{\sqrt{5}} + c$ $= \sin^{-1} \left[\frac{x-1}{\sqrt{5}} \right] + c$</p>	1	Integral		
		1	Splitting integral		
		1	Answer		



Question 12		Trial HSC Examination		2013	
Part	Solution	Marks	Comment		
(a)	<p>i. $z = \frac{1 + \sqrt{3}i}{\sqrt{3} + i}$ or $z = \frac{1 + \sqrt{3}i}{\sqrt{3} + i}$</p> <p>$z = \frac{ 1 + \sqrt{3}i }{ \sqrt{3} + i }$ $z = \frac{1 + \sqrt{3}i}{\sqrt{3} + i} \times \frac{\sqrt{3} - i}{\sqrt{3} - i}$</p> <p>$= \frac{\sqrt{1^2 + (\sqrt{3})^2}}{\sqrt{(\sqrt{3})^2 + 1^2}}$ $z = \frac{\sqrt{3 - i + 3i - \sqrt{3}i^2}}{3 - i^2}$</p> <p>$= \frac{2}{2} = 1$ $z = \frac{2\sqrt{3} + 2i}{4} = \frac{\sqrt{3} + i}{2}$</p> <p>ii. $z = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{1} = 1$</p> <p>iii. $\text{Arg } z = \frac{\text{Arg}(1 + \sqrt{3}i)}{\text{Arg}(\sqrt{3} + i)}$ Or $\text{Arg}\left(\frac{\sqrt{3} + i}{2}\right) = \frac{\pi}{6}$</p> <p>$= \text{Arg}(1 + \sqrt{3}i) - \text{Arg}(\sqrt{3} + i)$</p> <p>$= \frac{\pi}{3} - \frac{\pi}{6}$</p> <p>$= \frac{\pi}{6}$</p> <p>iv. $z = \text{cis } \frac{\pi}{6}$</p> <p>$z^4 = \left(\text{cis } \frac{\pi}{6}\right)^4$</p> <p>$z^4 = \text{cis } \frac{2\pi}{3}$</p> <p>v. $z^5 = \text{cis } \frac{\pi}{5}$</p> <p>$z = \frac{\text{cis } \frac{\pi}{5} + 2k\pi}{5}, k = 0, 1, 2, 3, 4$</p> <p>$z = \text{cis}\left(\frac{\pi}{30} + \frac{2k\pi}{5}\right), k = 0, 1, 2, 3, 4$</p> <p>$Z_0 = \text{cis } \frac{\pi}{30}$</p> <p>$Z_1 = \text{cis}\left(\frac{\pi}{30} + \frac{2\pi}{5}\right) = \text{cis}\left(\frac{13\pi}{30}\right)$</p> <p>$Z_2 = \text{cis}\left(\frac{\pi}{30} + \frac{4\pi}{5}\right) = \text{cis}\left(\frac{25\pi}{30}\right) = \text{cis}\left(\frac{5\pi}{6}\right)$</p> <p>$Z_3 = \text{cis}\left(\frac{\pi}{30} + \frac{6\pi}{5}\right) = \text{cis}\left(\frac{37\pi}{30}\right)$</p> <p>$Z_4 = \text{cis}\left(\frac{\pi}{30} + \frac{8\pi}{5}\right) = \text{cis}\left(\frac{49\pi}{30}\right)$</p>	1	Mod z		
		1	Arg z		
		1	z^4		
		1	Working		
		1	Roots		


Question 12		Trial HSC Examination		2013	
Part	Solution	Marks	Comment		
(b)	 <p>$\arg(z + 1) < \frac{\pi}{6}, z + \bar{z} \leq 6 \text{ and } z + 1 > 2.$</p>	3	One for each graph section		

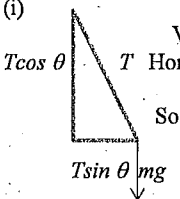
Question 13		Trial HSC Examination		2013	
Part	Solution	Marks	Comment		
(b)	<p>i.</p> $2x^3 - x^2 + 3x - 1 = 0$ <p>Let $x = 2X$, $\therefore X = \frac{x}{2}$</p> $\therefore 2\left(\frac{x}{2}\right)^3 - \left(\frac{x}{2}\right)^2 + 3\left(\frac{x}{2}\right) - 1 = 0$ $2\left(\frac{x^3}{8}\right) - \left(\frac{x^2}{4}\right) + \frac{3x}{2} - 1 = 0$ $\therefore 2x^3 - 2x^2 + 12x - 8 = 0$ $\therefore x^3 - x^2 + 6x - 4 = 0$ <p>ii.</p> <p>Let $x = \sqrt{x}$, $\therefore X = \sqrt{x}$</p> $\therefore 2(\sqrt{x})^3 - (\sqrt{x})^2 + 3(\sqrt{x}) - 1 = 0$ $2x\sqrt{x} - x + 3\sqrt{x} - 1 = 0$ $\sqrt{x}(2x + 3) = x + 1$ $x(2x + 3)^2 = (x + 1)^2$ $x(4x^2 + 12x + 9) = x^2 + 2x + 1$ $4x^3 + 12x^2 + 9x - x^2 - 2x - 1 = 0$ $4x^3 + 11x^2 + 7x - 1 = 0$	1	Substituting new variable		
		1	Correct Equation		
		1	Substituting new variable		
		1	Correct Equation		
(c)	$V = 2\pi \int_0^{\pi} xy \, dx$ $= 2\pi \int_0^{\pi} x \sin x \, dx$ <p>$u = x \quad v' = \sin x \quad uv - \int vu'$</p> <p>$u' = 1 \quad v = -\cos x \quad -x \cos x + \cos x$</p> $= 2\pi \{ [-x \cos x]_0^{\pi} + \int_0^{\pi} \cos x \, dx \}$ $= 2\pi [-x \cos x + \sin x]_0^{\pi}$ $= 2\pi [\pi - 0]$ $= 2\pi^2$	1	Correct Integral		
		1	Integrating by parts		
		1	Answer		

Question 14		Trial HSC Examination		2013	
Part	Solution	Marks	Comment		
(a)	<p>i.</p> <p>$P(x, y), A(0, 3), B(0, -3)$</p> $PA + PB = 10$ $\sqrt{x^2 + (y-3)^2} + \sqrt{x^2 + (y+3)^2} = 10$ $\sqrt{x^2 + (y-3)^2} = 10 - \sqrt{x^2 + (y+3)^2}$ $x^2 + y^2 - 6y + 9$ $= 100 - 20\sqrt{x^2 + (y+3)^2} + x^2 + y^2 + 6y + 9$ $20\sqrt{x^2 + (y+3)^2} = 100 + 12y$ $400(x^2 + y^2 + 6y + 9) = 10000 + 2400y + 144y^2$ $400x^2 + 400y^2 + 2400y + 3600 = 10000 + 2400y + 144y^2$ $400x^2 + 256y^2 = 6400$ <p>Dividing by 6400 to get</p> $\frac{x^2}{16} + \frac{y^2}{25} = 1$ <p>ii.</p> $\frac{x^2}{16} + \frac{y^2}{25} = 1$ $25x^2 + 16y^2 = 400$ <p>$y = 4$</p> $25x^2 + 16(4)^2 = 400$ $25x^2 + 256 = 400$ $25x^2 = 144$ $x = \pm \frac{12}{5}$ <p>$50x + 32y \frac{dy}{dx} = 0$</p> $\frac{dy}{dx} = \frac{-50x}{32y} = \frac{-25x}{16y}$ <p>At $x = \frac{12}{5}, m = \frac{-60}{64} = \frac{-15}{16}$</p> $y - 4 = \frac{-15}{16} \left(x - \frac{12}{5}\right)$ $16y - 64 = -15x + 36$ $15x + 16y - 100 = 0$ <p>At $x = -\frac{12}{5}, m = \frac{60}{64} = \frac{15}{16}$</p> $y - 4 = \frac{15}{16} \left(x + \frac{12}{5}\right)$ $16y - 64 = 15x + 36$ $15x - 16y - 100 = 0$	1	Substitution		
		1	Working		
		1	Working		
		1	Answer		
		1	x values		
		1	Gradients of tangents		
		1	Equation		
		1	Equation		

Question 15	Trial HSC Examination	2013	
Part	Solution	Marks	Comment
(b)	$\sum_{i=1}^n i^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$		
i)	<p>Test $n=1$ $1 = \frac{1}{3} + \frac{1}{2} + \frac{1}{6} = 1 \therefore$ True for $n = 1$ Assume true for $n = k$</p> $\sum_{i=1}^k i^2 = \frac{k^3}{3} + \frac{k^2}{2} + \frac{k}{6}$ <p>Test $n = k + 1$</p> $\sum_{i=1}^{k+1} i^2 = \frac{k^3}{3} + \frac{k^2}{2} + \frac{k}{6} + (k+1)^2$ $= \frac{2k^3 + 3k^2 + k + 6(k+1)^2}{6}$ $= \frac{2k^3 + 3k^2 + k + 6k^2 + 12k + 6}{6}$ $= \frac{2k^3 + 9k^2 + 13k + 6}{6}$ <p>By division, taking out a factor of $(k+1)$ LHS</p> $= \frac{(k+1)(2k^2 + 7k + 6)}{6}$ <p>Now, $\frac{(k+1)^3}{3} + \frac{(k+1)^2}{2} + \frac{k+1}{6}$</p> $= \frac{2(k+1)^3 + 3(k+1)^2 + (k+1)}{6}$ $= \frac{(k+1)[2(k+1)^2 + 3(k+1) + 1]}{6}$ $= \frac{(k+1)(2k^2 + 4k + 2 + 3k + 3 + 1)}{6}$ $= \frac{(k+1)(2k^2 + 7k + 6)}{6}$ <p>\therefore True for $n = k + 1$ if true for $n = k$, But true for $n = 1$ \therefore true for $n = 1 + 1 = 2$ etc</p> <p>Hence by Mathematical induction, true for all $n \geq 1$</p>	1	Tests $n = 1$ Assume $n = k$
		1	Case for $n = k + 1$
		1	Conclusion
ii)	$\lim_{x \rightarrow \infty} \frac{1^2 + 2^2 + 3^2 \dots \dots + n^2}{n^3}$ $= \lim_{x \rightarrow \infty} \frac{1}{n^3} \left(\frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \right)$ $= \lim_{x \rightarrow \infty} \left(\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2} \right) = \frac{1}{3}$	1	
		1	

Question 15	Trial HSC Examination	2013	
Part	Solution	Marks	Comment
(c)	<p>Possible passcodes could contain</p> <p>All different letters chosen from F,E,R,T,S in ${}^5C_4 \times 4! = 120$ ways.</p> <p>2 R's and no E's (ie R,R and two of F, T and S) in ${}^3C_2 \times \frac{4!}{2!} = 36$ ways</p> <p>2 E's and no R's (ie E,E and two of F, T and S) in ${}^3C_2 \times \frac{4!}{2!} = 36$ ways</p> <p>2 R's and one E (ie R,R,E and one of F, T and S) in ${}^3C_1 \times \frac{4!}{2!} = 36$ ways</p> <p>2 E's and one R (ie R,E,E and one of F, T and S) in ${}^3C_1 \times \frac{4!}{2!} = 36$ ways</p> <p>2 E's and 2 R's (ie R,R,E, E) in $\frac{4!}{2!2!} = 6$ ways</p> <p>Total possible passcodes = $120 + 36 \times 4 + 6 = 270$ ways.</p>	1	For identifying the different cases.
		1	For calculating the correct result

Question 16		Trial HSC Examination		2013	
Part	Solution	Marks	Comment		
(a)	<p>i.</p>  $R \propto v$ $R = kv$ $R = \frac{1}{10} v$ $F = ma$ $ma = mg - kv$ $a = g - \frac{kv}{m}$ $a = g - \frac{500}{m} v$ $\frac{dv}{dt} = g - \frac{500}{m} v$ $\frac{dt}{dv} = \frac{500}{500g - v}$ $\frac{dt}{dv} = \frac{500}{500g - v}$ $\therefore t = -500 \ln(500g - v) + c$ $v = 0, t = 0. 0 = -500 \ln(500g) + c$ $\therefore c = 500 \ln(500g)$ $\therefore t = -500 \ln(500g - v) + 500 \ln(500g)$ $= 500 [\ln 500g - \ln(500g - v)]$ $t = 500 \ln \frac{500g}{500g - v}$ $e^{\frac{t}{500}} = \frac{500g}{500g - v}$ $500g - v = 500g e^{-\frac{t}{500}}$ $\therefore v = 500g \left(1 - e^{-\frac{t}{500}}\right)$ <p>ii. Terminal velocity when $t \rightarrow \infty$</p> $\text{As } t \rightarrow \infty, e^{-\frac{t}{500}} \rightarrow 0$ $\therefore v \rightarrow 500g$ <p>iii.</p> $v = \frac{dx}{dt} = 500g \left(1 - e^{-\frac{t}{500}}\right)$ $x = \int (500g - 500g e^{-\frac{t}{500}}) dt$ $= 500gt + 250000g e^{-\frac{t}{500}} + c$ $t = 0, x = 0$ $0 = 0 + 250000g + c$ $\therefore c = -250000g$ $\therefore x = 500gt + 250000g \left(e^{-\frac{t}{500}} - 1\right)$	1	Expression for a		
		1	Equation for t		
		1	Finding v		
		1	Terminal Velocity		
		1	Equation for x		
		1	Result		

Question 16		Trial HSC Examination		2013	
Part	Solution	Marks	Comment		
(b)	<p>(i)</p>  $\text{Vertical: } T \cos \theta = mg \quad \text{---(1)}$ $\text{Horizontal: } T \sin \theta = mr\omega^2$ $\text{Now } r = h \tan \theta$ $\text{So } T \sin \theta = m \cdot h \tan \theta \omega^2 \quad \text{---(2)}$ <p>Eliminating T we obtain:</p> $\frac{(2)}{(1)} = \frac{T \sin \theta}{T \cos \theta} = \frac{m \cdot h \tan \theta \omega^2}{mg}$ $\tan \theta = \frac{h \tan \theta \omega^2}{g}$ $\therefore \omega^2 = \frac{g}{h}$ $\text{i.e. } \omega = \sqrt{\frac{g}{h}} \text{ rad/s}$ $\therefore \text{Time for 1 revolution} = \frac{2\pi}{\sqrt{\frac{g}{h}}}$ $\therefore \text{Period} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{h}{g}}$ <p>(ii) When $h = 50 \text{ cm}$ (i.e. 0.5 m) and $g = 10 \text{ m/s}^2$.</p> $P = 2\pi \sqrt{\frac{0.5}{10}} = 1.4 \text{ seconds}$	1	Resolving forces to obtain value of $\tan \theta$		
		1	Period		
		1	Value		
(c)	$v = \frac{80 \times 1000}{60^2} = 22 \frac{2}{9} \text{ m/s}$ $\tan \alpha = \frac{v^2}{gr}$ $= \frac{\left(22 \frac{2}{9}\right)^2}{\frac{10 \times 100}{9}} = 0.49 \dots$ $\alpha = 27^\circ 17'$	1	Velocity		
		1	Sub into equation		
		1	Angle		

Question 16		Trial HSC Examination		2013	
Part	Solution	Marks	Comment		
(d)	i. $T = \frac{mv^2}{r} = \frac{5(3)^2}{1.5}$ $= 30 \text{ N}$	1	Tension		
	ii. $T = \frac{mv^2}{r}$ $150 = \frac{5v^2}{1.5}$ $45 = v^2$ $v = 3\sqrt{5} \text{ m/s}$	1	Velocity		
	$\omega = \frac{v}{r}$ $= \frac{3\sqrt{5}}{1.5}$ $= \frac{3\sqrt{5}}{1.5 \times 2\pi}$ $= \frac{3\sqrt{5}}{3\pi} \text{ r/s}$	1	Number of Revolutions		