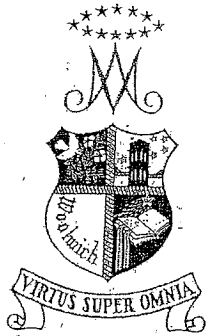
Name _____ Teacher _____



Year 12 Mathematics
Assessment Task

Weight: 27% *March 15th 2007*

Instructions to students

- Calculators may be used.
- Use black or blue pen only except for graphs.
- Write your name on EVERY page of your solution sheet.
- The phrase 'not attempted' **MUST** be written on the answer sheet for any question or parts of questions that are not attempted. Insert a blank sheet if an entire question is not attempted.
- Begin each question on a new page.

Time allowed: **1 hour 30 minutes**

OUTCOMES ASSESSED	TOPICS COVERED	Result
H1, H2, H4, H5, H6, H9	Probability	/15
	Geometrical Applications of Calculus	/33
	Total	/48

Question 1 Probability

Mark

(a) A box contains 8 red and 11 green marbles. Shez randomly selects three marbles one at a time and without replacement. What is the probability that she selects 1

green, red, green

in that order?

(b) In a large school, the student population is 42% male and 58% female. Two students are selected at random to take part in a survey. Find correct to 2 decimal places, the probability that:

(i) both are female 1
(ii) they are different sexes 2
(iii) both are of the same sex 2

(c) Four girls' names and five boys' names are in a hat. Two names are drawn out without replacement.

Find the probability that the names of a boy and a girl are drawn out? 2

(d) In a large barrel there is a mixture of Fundraiser chocolates. White Chocolate Freddos and Strawberry Freddos are in the ratio 2:3. Find the probability that if three chocolates are chosen from the mix, at least one is a Strawberry Freddo? 2

(e) A coin is biased so that $P(H) = \frac{1}{3}$. How many times must the coin be tossed so that the probability of getting at least one head is more than 90%? 3

(f) An urn contains W white and B black marbles. 2

Marbles are selected at random, without replacement.

If the probability of selecting 2 white marbles is $\frac{1}{3}$ while the probability of selecting 3 white marbles is $\frac{1}{6}$, find the number of white marbles in the urn.

Question 2 Geometrical Applications of Calculus (start a new page) **Mark**

(a) For the function $y = 2x^3 - 15x^2 + 36x$ determine the values of x for which the function is increasing. 3

(b) For the curve $y = x^4 - 3x^3 + 9x$ find the value of $\frac{d^2y}{dx^2}$ at the point (1 , 7). 2

(c) Find the equation of the curve $y = f(x)$ if $f''(x) = 2(x-1)$ and the point (3 , -7) is a minimum point on the curve. 3

(d) Find the primitive functions for each of the following

(i) $3x^4 - 2$ 1

(i) $x^{-\frac{1}{3}}$ 2

(iii) $x^2\sqrt{x}$ 2

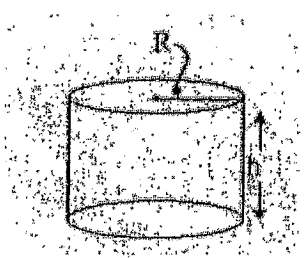
(e) The curve $f(x) = x^3 + 3x^2 - 9x - 1$ is defined in the domain $-4 \le x \le 2$

(i) Find the coordinates of the two stationary points and determine their nature. 4

(ii) Show that a point of inflexion occurs at $x = -1$ 2

(iii) Sketch the curve. 3

(f)



The sum of the radius R and height h of a cylinder is 60 cm.

(i) Express h in terms of R. 1

(ii) Show that the volume of the cylinder is given by $V = \pi\,(60R^2 - R^3)$ 2

(iii) Hence find the radius that will give the maximum volume possible. (leave your answer in exact form) 3

Question 2 continues over the page

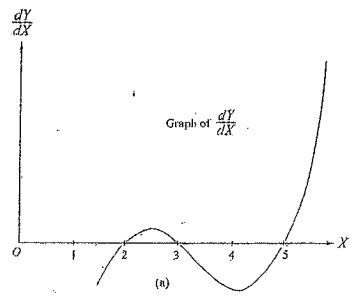
(g) The graph of the function Y, defined in the interval $0 \le x \le 6$, passes through the points (2, 10), (3, 15) and (5, 0).

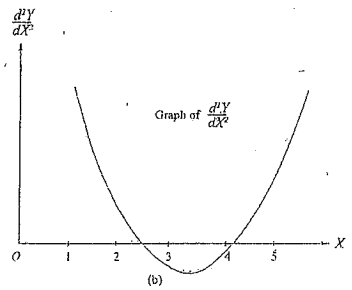
The graphs of $\frac{dY}{dX}$ and $\frac{d^2Y}{dX^2}$ are shown below.

Without finding the equation of Y:

(i) Determine the maximum and minimum points of the graph of Y. 3/3

(ii) Sketch the graph of Y, labelling the important features. 2/2





) 8r, 11g choose 3

$$P(g,r,g) = \frac{11}{19} \times \frac{8}{18} \times \frac{10}{17} = \frac{440}{2907} \quad (1)$$

) 42% m, 58% f choose 2

I) $P(f,f) = 0.58^2 = 0.3364 \doteq 0.34$ (2dp). (1) ✓

II) $P(f,m \text{ or } m,f) = 2 \times 0.42 \times 0.58 = 0.4872 \doteq 0.49$ (2dp) (2)

III) $P(ff \text{ or } m,m) = 0.58^2 + 0.42^2 = 0.5128 \doteq 0.51$ ✓ (2dp) (2)

4g, 5b choose 2

$P(b,g \text{ or } g,b) = \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{5}{8} = 2 \times \frac{5}{9} \times \frac{4}{8} = \frac{5}{9}$ ✓ (2)

W:D = 2:3 choose 3

$P(\text{at least one D}) = 1 - P(\text{no D})$

$= 1 - \left(\frac{2}{5}\right)^3$ ✓

$= 1 - \frac{8}{125}$

$= \frac{117}{125} \ (= 0.936)$. ✓ (2)

$P(H) = \frac{1}{3} \Rightarrow P(T) = \frac{2}{3}$

$P(\text{at least one H}) = 1 - P(\text{no H})$

$= 1 - P(\text{all T})$

$= 1 - \left(\frac{2}{3}\right)^n$ ✓

i.e. $0.90 < 1 - \left(\frac{2}{3}\right)^n$ ✓

$-0.1 < -\left(\frac{2}{3}\right)^n$

$0.1 > \left(\frac{2}{3}\right)^n$

$\frac{\ln 0.1}{\ln \frac{2}{3}} < n$

$n > 5.7$

$\therefore n = 6$ ✓ (3)

†) $P(2w) = \frac{1}{3}$, $P(3w) = \frac{1}{6}$.

Let no of marbles be n.

now $P(2w) = \frac{1}{3} = \frac{w}{n} \times \frac{w-1}{n-1}$

$= \frac{w^2-w}{n^2-n}$

$n^2 - n = 3w^2 - 3w$ — (1)

now $P(3w) = P(2w) \times \frac{w-2}{n-2}$

i.e. $\frac{1}{6} = \frac{1}{3} \times \frac{w-2}{n-2}$

$\frac{1}{2} = \frac{w-2}{n-2}$

$n - 2 = 2w - 4$

$n = 2w - 2$ — (2) ✓

) into (1).

$(2w-2)^2 - (2w-2) = 3w^2 - 3w$

$4w^2 - 8w + 4 - 2w + 2 = 3w^2 - 3w$

$w^2 - 7w + 6 = 0$.

$(w-6)(w-1) = 0$.

$\therefore w = 1, 6$

but $w \neq 1$ $\therefore w = 6$.

The minimum no of white marble is 6. ✓ (2)

$\therefore$ The no of white marbles is $6n$, $n \in \mathbb{Z}^+$.

Question 2

$y = 2x^3 - 15x^2 + 36x$

$y' = 6x^2 - 30x + 36$ 1

increasing when $y' > 0$

$6x^2 - 30x + 36 > 0$ 1

$x^2 - 5x + 6 > 0$

$(x-3)(x-2) > 0$

$\therefore x < 2, \quad x > 3$ 1

b) $y = x^4 - 3x^3 + 9x$

$\frac{dy}{dx} = 4x^3 - 9x^2 + 9$

$\frac{d^2y}{dx^2} = 12x^2 - 18x$ 1

at $x = 1$

$\frac{d^2y}{dx^2} = 12 - 18$

$= -6$ 1

c) $f''(x) = 2x - 2$

$f'(x) = x^2 - 2x + C_1$ 1

when $f'(x) = 0, \quad x = 3$

$0 = 9 - 6 + C_1$

$-3 = C_1$

$\therefore f'(x) = x^2 - 2x - 3$ 1

$f(x) = \frac{x^3}{3} - x^2 - 3x + C_2$

When $x = 3, \ y = -7$

$\therefore -7 = 9 - 9 - 9 + C_2$

$2 = C_2$

$\therefore f(x) = \frac{x^3}{3} - x^2 - 3x + 2$ 1

d) i) $\int 3x^4 + 2 \ dx$

$= \frac{3x^5}{5} + 2x + C$ 1

ii) $\int x^{-\frac{1}{3}} dx$

$= \frac{3}{2} x^{\frac{2}{3}} + C$ 2

iii) $\int x^2 \sqrt{x} \ dx$

$= \int x^{\frac{5}{2}} dx$

$= \frac{2}{7} x^{\frac{7}{2}} + C$ 2

2e)

$f(x) = x^3 + 3x^2 - 9x - 1$

$-4 \le x \le 2$

i) for st pts $f'(x) = 0$

$f'(x) = 3x^2 + 6x - 9$

$0 = 3x^2 + 6x - 9$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$\therefore x = -3$ or 1 1

x	-4	-3	-2
$f'(x)$	$+15$	0	-9

$\therefore$ max at $(-3, 26)$ 2

x	0	1	2
$f'(x)$	-9	0	$+15$

$\therefore$ min at $(1, -6)$ 2

ii) For POI $f''(x) = 0$

$f''(x) = 6x + 6$

If $f''(x) = 0$

$x = -1$

$\therefore$ possible inf at $(-1, 10)$ 1

x	-2	-1	0
$f''(x)$	-6	0	$+6$

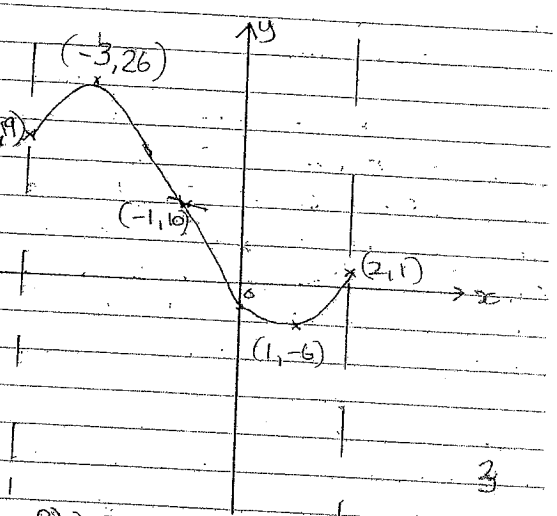
$\therefore$ Change of concavity 1

$\therefore$ inf at $(-1, 10)$

(iii) $x = -4, \ y = 19 \quad (-4, 19)$

$x = 2, \ y = 1 \quad (2, 1)$

$x = 0, \ y = -1 \quad (0, -1)$



3

f) i) $R + h = 60$

$h = 60 - R$ 1

ii) $V = Ah$

$= \pi R^2 (60 - R)$

$= \pi (60R^2 - R^3)$ 2

iii) $V = 60\pi R^2 - \pi R^3$

$V' = 120\pi R - 3\pi R^2$ 1

for max/min $V' = 0$

$0 = 3\pi R(40 - R)$

$\therefore R = 40 \quad R \neq 0$ 1

$V'' = 120\pi - 6\pi R$

if $R = 40$

$V'' < 0$

$\therefore$ max volume

when $R = 40$ 1

Q2 ctd

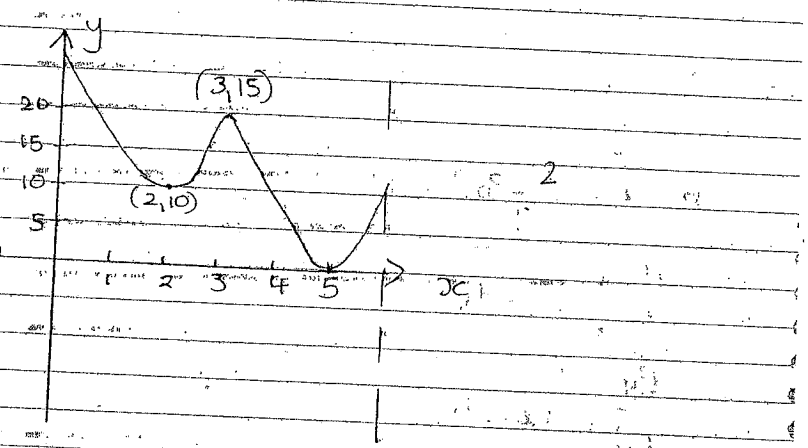
g) $0 \leq x \leq 6$

(2,10) (3,15) (5,0)

i) min (2,10) 1
max (3,15) 1
min (5,0) 1

x		2		3		5	
$\frac{dy}{dx}$	−	0	+	0	−	0	+

ii)



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