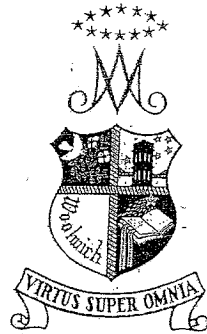Name _____   Teacher _____

Year 12 Mathematics
Assessment Task

Weight: 27%   *March 15th 2007*

Instructions to students

- Calculators may be used.
- Use black or blue pen only except for graphs.
- Write your name on EVERY page of your solution sheet.
- The phrase 'not attempted' MUST be written on the answer sheet for any question or parts of questions that are not attempted. Insert a blank sheet if an entire question is not attempted.
- Begin each question on a new page.

Time allowed: 1 hour 30 minutes

| OUTCOMES ASSESSED	TOPICS COVERED	Result
H1, H2, H4, H5, H6, H9	Probability	/15
	Geometrical Applications of Calculus	/33
	Total	/48

Question 1 Probability

Mark

(a)   A box contains 8 red and 11 green marbles. Shez randomly selects three marbles one at a time and without replacement. What is the probability that she selects

green, red, green

in that order?   1

(b)   In a large school, the student population is 42% male and 58% female. Two students are selected at random to take part in a survey. Find correct to 2 decimal places, the probability that:
    (i)   both are female   1
    (ii)   they are different sexes   2
    (iii)   both are of the same sex   2

(c)   Four girls' names and five boys' names are in a hat. Two names are drawn out without replacement.

Find the probability that the names of a boy and a girl are drawn out?   2

(d)   In a large barrel there is a mixture of Fundraiser chocolates. White Chocolate Freddos and Strawberry Freddos are in the ratio 2:3. Find the probability that if three chocolates are chosen from the mix, at least one is a Strawberry Freddo?   2

(e)   A coin is biased so that $P(H) = \frac{1}{3}$. How many times must the coin be tossed so that the probability of getting at least one head is more than 90%?   3

(f)   An urn contains W white and B black marbles.   2

Marbles are selected at random, without replacement.

If the probability of selecting 2 white marbles is $\frac{1}{3}$ while the probability of selecting 3 white marbles is $\frac{1}{6}$, find the number of white marbles in the urn.

Question 2 Geometrical Applications of Calculus (start a new page)   **Mark**

(a) For the function $y = 2x^3 - 15x^2 + 36x$ determine the values of x for which the function is increasing.   3

(b) For the curve $y = x^4 - 3x^3 + 9x$ find the value of $\dfrac{d^2y}{dx^2}$ at the point $(1, 7)$.   2

(c) Find the equation of the curve $y = f(x)$ if $f''(x) = 2(x-1)$ and the point $(3, -7)$ is a minimum point on the curve.   3

(d) Find the primitive functions for each of the following

(i) $3x^4 - 2$ 1

(i) $x^{-\frac{1}{3}}$ 2

(iii) $x^2\sqrt{x}$ 2

(e) The curve $f(x) = x^3 + 3x^2 - 9x - 1$ is defined in the domain $-4 \le x \le 2$.

(i) Find the coordinates of the two stationary points and determine their nature.   4

(ii) Show that a point of inflexion occurs at $x = -1$ 2

(iii) Sketch the curve.   3

(f)
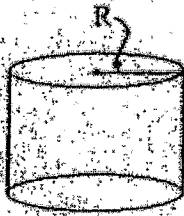
The sum of the radius R and height h of a cylinder is 60 cm.

(i) Express h in terms of R.   1

(ii) Show that the volume of the cylinder is given by $V = \pi\,(60R^2 - R^3)$ 2

(iii) Hence find the radius that will give the maximum volume possible. (leave your answer in exact form)   3

Question 2 continues over the page

- 3 -

(g) The graph of the function Y, defined in the interval $0 \le x \le 6$, passes through the points $(2, 10)$, $(3, 15)$ and $(5, 0)$.

The graphs of $\dfrac{dY}{dX}$ and $\dfrac{d^2Y}{dX^2}$ are shown below.

Without finding the equation of Y:

(i) Determine the maximum and minimum points of the graph of Y.

(ii) Sketch the graph of Y, labelling the important features.

- 4 -
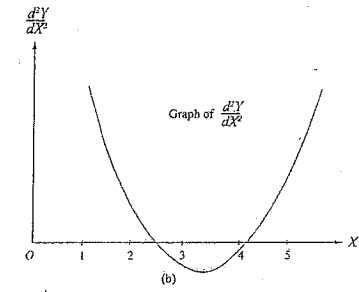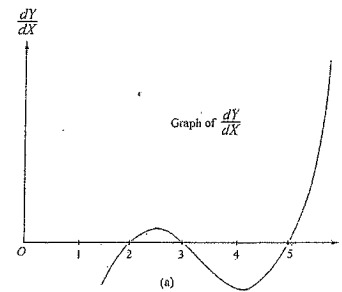
a) 8r, 11g choose 3

$$P(g, r, g) = \frac{11}{19} \times \frac{8}{18} \times \frac{10}{17} = \frac{440}{2907}$$ ①

b) 42%m, 58%of choose 2

I) $P(f, f) = 0.58^2 = 0.3364 \doteq 0.34$ (2dp). ①

II) $P(f, m \text{ or } m, f) = 2 \times 0.42 \times 0.58 = 0.4872 \doteq 0.49$ (2dp) ②

III) $P(f, f \text{ or } m, m) = 0.58^2 + 0.42^2 = 0.5128 \doteq 0.51$ (2dp) ②

c) 4g, 5G choose 2

$$P(G, g \text{ or } g, G) = \frac{5}{9} \times \frac{4}{8} + \frac{4}{9} \times \frac{5}{8} = 2 \times \frac{5}{9} \times \frac{4}{8 \cdot 4} = \frac{5}{9}$$ ✓ ②

d) W : D = 2 : 3 choose 3

P(at least one D) = 1 - P(no D)

$$= 1 - \left(\frac{2}{5}\right)^3 \checkmark$$

$$= 1 - \frac{8}{125}$$

$$= \frac{117}{125} \;(= 0.936). \checkmark$$ ②

e) $P(H) = \frac{1}{3} \Rightarrow P(T) = \frac{2}{3}$

P(at least one H) = 1 - P(no H)

$$= 1 - P(\text{all } T).$$

$$= 1 - \left(\frac{2}{3}\right)^n \checkmark$$

$$0.90 < 1 - \left(\frac{2}{3}\right)^n \checkmark$$

$$-0.1 < -\left(\frac{2}{3}\right)^n$$

$$0.1 > \left(\frac{2}{3}\right)^n$$

$$\frac{\ln 0.1}{\ln 2/3} < n$$

$$n > 5.7$$

$$\therefore n = 6 \checkmark$$ ③

f) $P(2w) = \frac{1}{3}$, $P(3w) = \frac{1}{6}$.

let no of marbles be n.

now $P(2w) = \frac{1}{3} = \frac{w}{n} \times \frac{w-1}{n-1}$

$$= \frac{w^2 - w}{n^2 - n}$$

$$n^2 - n = 3w^2 - 3w \quad — \text{①}$$

now $P(3w) = P(2w) \times \frac{w-2}{n-2}$

$$\frac{1}{6} = \frac{1}{3} \times \frac{w-2}{n-2}$$

$$\frac{1}{2} = \frac{w-2}{n-2}$$

$$n - 2 = 2w - 4$$

$$n = 2w - 2 \quad — \text{②} \checkmark$$

② into ①.

$$(2w-2)^2 - (2w-2) = 3w^2 - 3w$$

$$4w^2 - 8w + 4 - 2w + 2 = 3w^2 - 3w$$

$$w^2 - 7w + 6 = 0.$$

$$(w - 6)(w - 1) = 0.$$

$$\therefore w = 1, 6$$

but $w \neq 1$ $\therefore w = 6$.

The minimum no of white marble is 6. ✓ ②

∴ The no of white marbles is $6n$, $n \in \mathbb{Z}^+$.

Question 2.

$y = 2x^3 - 15x^2 + 36x$

$y' = 6x^2 - 30x + 36$

increasing when $y' > 0$

$6x^2 - 30x + 36 > 0$

$x^2 - 5x + 6 > 0$

$(x-3)(x-2) > 0$

$\therefore x < 2, x > 3$

b) $y = x^4 - 3x^3 + 9x$

$\dfrac{dy}{dx} = 4x^3 - 9x^2 + 9$

$\dfrac{d^2y}{dx^2} = 12x^2 - 18x$

at $x = 1$

$\dfrac{d^2y}{dx^2} = 12 - 18$

$= -6$

c) $f''(x) = 2x - 2$

$f'(x) = x^2 - 2x + C_1$

when $f'(x) = 0, \; x = 3$

$0 = 9 - 6 + C_1$

$-3 = C_1$

$\therefore f'(x) = x^2 - 2x - 3$

$f(x) = \dfrac{x^3}{3} - x^2 - 3x + C_2$

When $x = 3, \; y = -7$

$\therefore -7 = 9 - 9 - 9 + C_2$

$2 = C_2$

$\therefore f(x) = \dfrac{x^3}{3} - x^2 - 3x + 2$

d) i) $\displaystyle\int 3x^4 - 2 \; dx$

$= \dfrac{3x^5}{5} - 2x + C$

ii) $\displaystyle\int x^{-\frac{1}{3}} \, dx$

$= \dfrac{3}{2} x^{\frac{2}{3}} + C$

iii) $\displaystyle\int x^2\sqrt{x} \, dx$

$= \displaystyle\int x^{\frac{5}{2}} \, dx$

$= \dfrac{2}{7} x^{\frac{7}{2}} + C$

2e)

$f(x) = x^3 + 3x^2 - 9x - 1$

$-4 \le x \le 2$

i) for st pts $f'(x) = 0$

$f'(x) = 3x^2 + 6x - 9$

$0 = 3x^2 + 6x - 9$

$0 = x^2 + 2x - 3$

$0 = (x+3)(x-1)$

$\therefore x = -3 \text{ or } 1$

x	-4	-3	-2
$f'(x)$	$+15$	0	-9

$\therefore$ max at $(-3, 26)$

x	0	1	2
$f'(x)$	-9	0	$+15$

$\therefore$ min at $(1, -6)$
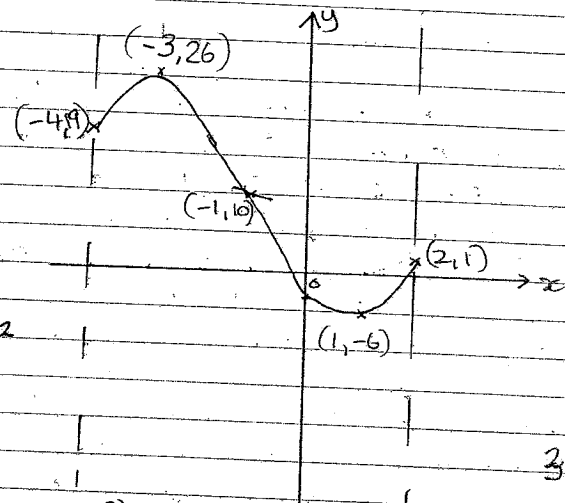
ii) For POI $f''(x) = 0$

$f''(x) = 6x + 6$

If $f''(x) = 0$

$x = -1$

$\therefore$ possible inf at $(-1, 10)$

x	-2	-1	0
$f''(x)$	-6	0	$+6$

$\therefore$ change of concavity

$\therefore$ inf at $(-1, 10)$

(iii) $x = -4, \; y = 19 \quad (-4, 19)$

$x = 2, \; y = 1 \quad (2, 1)$

$x = 0, \; y = -1 \quad (0, -1)$

f) i) $R + h = 60$

$h = 60 - R$

ii) $V = Ah$

$= \pi R^2 (60 - R)$

$= \pi (60R^2 - R^3)$

iii) $V = 60\pi R^2 - \pi R^3$

$V' = 120\pi R - 3\pi R^2$

for max/min $V' = 0$

$0 = 3\pi R(40 - R)$

$R = 40 \quad R \ne 0$

$V'' = 120\pi - 6\pi R$

if $R = 40$

$V'' < 0$

$\therefore$ max volume

when $R = 40$

Q2 ctd

g) $0 \leqslant x \leqslant 6$

   $(2,10)$ $(3,15)$ $(5,0)$

i) min $(2,10)$ 1
   max $(3,15)$ 1
   min $(5,0)$ 1

x		2		3		5	
$\dfrac{dy}{dx}$	−	0	+	0	−	0	+

ii)

$(3,15)$

$(2,10)$

2