

# WORKSHEET : PIECE-WISE DEFINED FUNCTIONS

For each of the following piece-wise defined functions:

- sketch its graph
- state its domain and range
- state continuous or discontinuous
- state odd or even or neither

1.  $f(x) = \begin{cases} -2, & \text{for } x < 0 \\ 2, & \text{for } x \geq 0 \end{cases}$

2.  $f(x) = \begin{cases} 3, & x < 0 \\ x, & x \geq 0 \end{cases}$

3.  $f(x) = \begin{cases} -x, & -3 \leq x < 1 \\ -1, & x \geq 1 \end{cases}$

4.  $f(x) = \begin{cases} \frac{1}{2}x, & x < 2 \\ 0, & x = 2 \\ 1, & x > 2 \end{cases}$

5.  $f(x) = \begin{cases} x, & x \leq 0 \\ -x^2, & 0 < x \leq 2 \end{cases}$

6.  $f(x) = \begin{cases} x+2, & x \leq -1 \\ 1, & -1 < x < 1 \\ 2-x, & x \geq 1 \end{cases}$

7.  $f(x) = \begin{cases} 0, & x < -3 \\ \sqrt{9-x^2}, & -3 \leq x \leq 3 \\ 0, & x > 3 \end{cases}$

8.  $f(x) = \begin{cases} x-1, & x < 2 \\ 1, & 2 \leq x \leq 4 \end{cases}$

9.  $f(x) = \begin{cases} -8, & -4 \leq x < -2 \\ x^3, & -2 \leq x \leq 2 \\ 8, & 2 < x \leq 4 \end{cases}$

10.  $f(x) = \begin{cases} x+4, & -4 \leq x < -2 \\ |x|, & -2 \leq x \leq 4 \\ 8-x, & 4 < x \leq 8 \end{cases}$

## Further Exercises

11. Make a minor change to the definition of  $f(x)$  in No 1 above, so that  $f(x)$  is odd.

12. Make a minor change to the definition of  $f(x)$  in No 4 above, so that  $f(x)$  is continuous.

13.  $f(x) = \begin{cases} \sqrt{16-x^2}, & -4 \leq x \leq 0 \end{cases}$

Complete this definition so that  $f(x)$  is odd. Sketch  $f(x)$ .

14.  $f(x) = \begin{cases} 2^x, & x \geq 0 \end{cases}$

Complete this definition so that  $f(x)$  is even. Sketch  $f(x)$ .

15.  $f(x) = \begin{cases} -x-1, & x \leq -1 \\ \log_2 x, & x \geq 1 \end{cases}$

Complete this definition so that  $f(x)$  is continuous. Sketch  $f(x)$ .

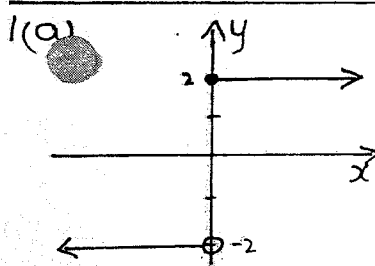
16.  $f(x) = \begin{cases} 2, & x \leq 2 \\ 1, & x \geq 0 \end{cases}$

- Is  $f(x)$  a function?
- Make a minimal change to its definition so that it becomes a function. Sketch it.
- State its domain and range.

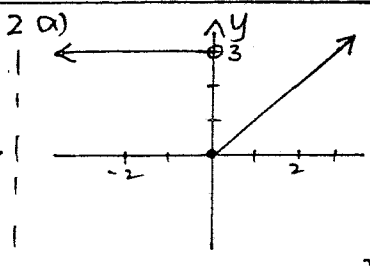
17. Write definitions for six piece-wise defined functions  $f(x)$  so that:
- $f(x)$  is continuous
  - $f(x)$  is even and continuous
  - $f(x)$  has domain  $\{x: -4 \leq x \leq 4, x \in \mathbb{R}\}$
  - $f(x)$  has range  $\{y: y \geq 0, y \in \mathbb{R}\}$
  - $f(x)$  has two points of discontinuity
  - $f(x)$  involves an absolute value function.

Note a) to f) are six different functions. Each must have at least two "pieces" to it. (2 steps of definition).

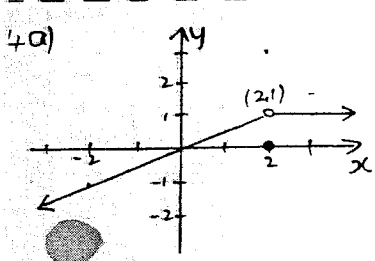
### ANSWERS



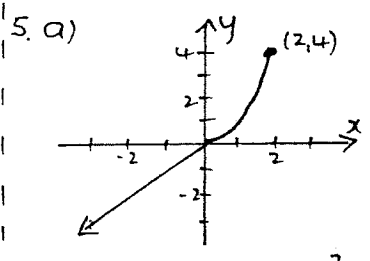
- b)  $\mathbb{R}, \{2, -2\}$   
 c) discontinuous  
 d) neither



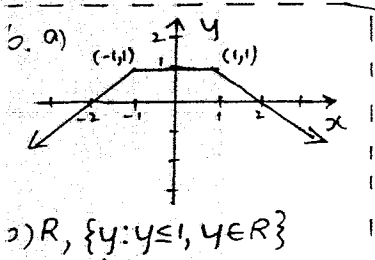
- b)  $\mathbb{R}, \{y: y \geq 0, y \in \mathbb{R}\}$   
 c) discontinuous  
 d) neither



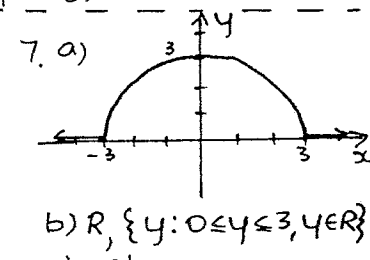
- b)  $\mathbb{R}, \{y: y \leq 1, y \in \mathbb{R}\}$   
 c) discontinuous  
 d) neither



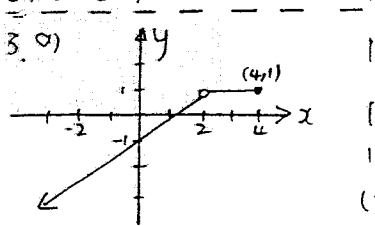
- b)  $\{x: x \leq 2, x \in \mathbb{R}\}$   
 $\{y: y \leq 4, y \in \mathbb{R}\}$   
 c) continuous  
 d) neither



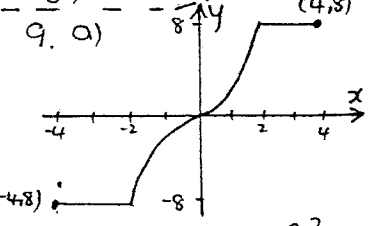
- b)  $\mathbb{R}, \{y: y \leq 1, y \in \mathbb{R}\}$   
 c) continuous  
 d) even



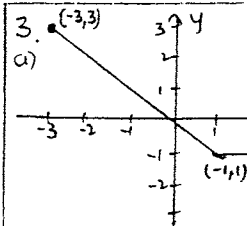
- b)  $\mathbb{R}, \{y: 0 \leq y \leq 3, y \in \mathbb{R}\}$   
 c) continuous  
 d) even



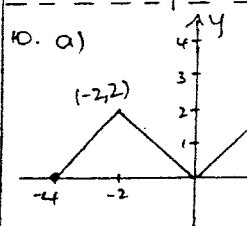
- b)  $\{x: x \leq 4, x \neq 2, x \in \mathbb{R}\}$   
 $\{y: y \leq 1, y \in \mathbb{R}\}$



- b)  $\{x: -4 \leq x \leq 4, x \in \mathbb{R}\}$   
 $\{y: -8 \leq y \leq 8, y \in \mathbb{R}\}$



- b) Domain  $\{x: x > -3, x \in \mathbb{R}\}$   
 Range:  $\{y: -1 \leq y \leq 3, x \in \mathbb{R}\}$   
 c) continuous  
 d) neither



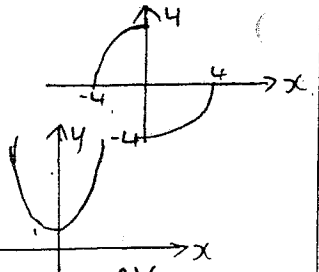
- b)  $\{x: -4 \leq x \leq 8, x \in \mathbb{R}\}$   
 $\{y: 0 \leq y \leq 4, x \in \mathbb{R}\}$   
 c) continuous  
 d) neither

### Further Exercises

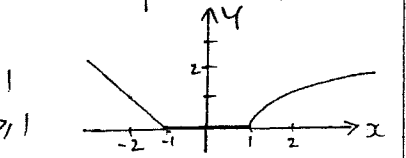
11.  $f(x) = \begin{cases} -2, & x < 0 \\ 2, & x > 0 \end{cases}$

12.  $f(x) = \begin{cases} \frac{1}{2}x, & x \leq 2 \\ 1, & x > 2 \end{cases}$

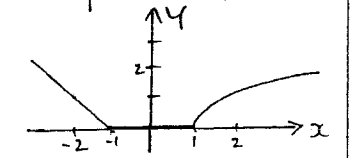
13.  $f(x) = \begin{cases} \sqrt{16-x^2}, & -4 \leq x \leq 0 \\ -\sqrt{16-x^2}, & 0 < x \leq 4 \end{cases}$



14.  $f(x) = \begin{cases} 2^x, & x < 0 \\ 2^{-x}, & x \geq 0 \end{cases}$

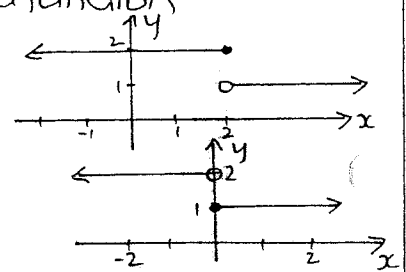


15.  $f(x) = \begin{cases} -x-1, & x \leq -1 \\ 0, & -1 < x < 1 \\ \log_2 x, & x \geq 1 \end{cases}$



16. a)  $f(x)$  is not a function

b)  $f(x) = \begin{cases} 2, & x \leq 2 \\ 1, & x > 2 \end{cases}$   
 or  $f(x) = \begin{cases} 2, & x < 0 \\ 1, & x \geq 0 \end{cases}$



c) For both, domain =  $\mathbb{R}$ , range =  $\{1, 2\}$

### TRY THESE

- For  $f(x)$  in No 4, find  $f(-6) + f(6)$
- For  $f(x)$  in No 5, find  $f(-2) + f(2)$
- For  $f(x)$  in No 9, find  $f(-3) + f(1.5) + f(-4)$
- For  $f(x)$  in No 10, find  $f(-3) + f(-1) + 2f(3) + f(7)$
- For  $f(x)$  in No 10, find  $f(-4) - f(3) + f(5)$

20.  $-8 + 3.75 + 8 = 3.75$   
 22.  $0 - 3 + 3 = 0$   
 19.  $-2 + 4 = 2$   
 18.  $-3 + 1 = -2$   
 21.  $1 + 1 + 2 \times 3 + 1 = 9$