Worksheet on logs

Month 5 DG

1. Evaluate: a) $\log_3 1$ b) $\log_2 \frac{1}{2}$ c) $\log_2(2\sqrt{2})$ d) $\log_3 \sqrt{27}$
e) $\log_4 8$ f) $\log_{10} 4 + \log_{10} 25$

2. If $\log 2 = a$ and $\log 3 = b$ find in terms of a and b:
(base is 10) a) $\log 0{\cdot}2$ b) $\log 15$ c) $\log 2\frac{1}{2}$ d) $\log 2^3$
e) $\frac{\log 4}{\log 9}$ f) $\log 2 \times \log 8$ g) $\log(2+8)$ h) $\log 10^{ab}$

3. Simplify as far as possible: (no calculators)
a) $\frac{\log_x 8}{\log_x 16}$ b) $\frac{\log a^2}{\log b^2}$ c) $\frac{\log 1}{\log 5}$ d) $\log x + \log \frac{1}{x}$
e) $\frac{\log x^2}{\log x}$ f) $\frac{\log 9 + \log 4}{\log 27 + \log 8}$ g) $\frac{2\log 4 + \log 3}{3\log 2 + \log 6}$ h) $2\log 3 + \log \frac{1}{9}$

Simplify: (base is 10)
a) $\log \frac{400}{4}$ b) $\log 5 + \log 12 - \log 6$ c) $\frac{\log 4 + \log 25}{\log 0{\cdot}01}$ d) $\frac{\log 400}{\log 4}$.
e) $\log 6 + 2\log 20 - \log 3 - 3\log 2$ f) $\log 16 + \log 30 + \log 5 - \log 24$

5. Find a: a) $\log_3 a = 4$ b) $\log_2 a = -3$ c) $2\log_5 a = -2$

6. Solve for x: a) $7^x = 5$ b) $5^{x+1} = 82$ c) $2^x \times 3^{x-1} = 22$

7. Simplify $\log_5 2 \times \log_7 5 \times \log_6 7$ giving your answer in terms of w if $w = \log_2 3$.

8. Find without using a calculator: $\log_4 8 - \log_{\frac{1}{4}} 32$

9. Solve for x: a) $\log_{10}(2x+1) - \log_{10}(x-1) = 1$ b) $\log_2(1-x) + \log_2(5+x) = 3$
c) $\log\sqrt{x} = 1{\cdot}1$ d) $10^x = 7{\cdot}23$

10. If $3^{x+1} = 6^{2x}$ show that $x = \log_{12} 3$.

11. Prove that $\log_a(a^4b^4) - \log_b(a^4b^4) = 4(\log_a b - \log_b a)$

12. If $y = 2\log_{10}(3x+7)$ find 10^y. Solve the equation
$2\log_{10}(3x+7) - \log_{10}(2x-1) = 2$

13. If $m^2 + n^2 = 23mn$, prove that $\log\frac{1}{5}(m+n) = \frac{1}{2}(\log m + \log n)$

14. If $\log_a(x-m) = \log_{\sqrt{a}} x - \log_{\sqrt{a}} m$, prove that $x^2 - xm^2 + m^3 = 0$.

15. Prove that $\log\frac{56}{81} - \log\frac{32}{63} = 2(\log 7 - \log 6)$

16. If $\log_8 9 = x$ and $\log_4 21 = y$, prove that $\log_2 7 = \frac{1}{2}(4y - 3x)$

17. If $\log_5 8 = a$ prove that $\log_{10} 2 = \frac{a}{a+3}$

18. If $\log_8 9 = x$ and $\log_4 21 = y$ prove that $\log_2 7 = \frac{1}{2}(4y - 3x)$

SOLUTIONS

$\log_3 \sqrt{27} = x$

$3^x = 3^{3/2}$

$x = 1\frac{1}{2}$

$\therefore \log_3 \sqrt{27} = 1\frac{1}{2}$

b) $\log_2 2 = 1$

c) $\log_2 2\sqrt{2}$

$2^x = 2 \cdot 2^{\frac{1}{2}}$

$x = 1\frac{1}{2}$

$\therefore \log_2 2\sqrt{2} = 1\frac{1}{2}$

) Let $\log_4 8 = x$

$4^x = 8$

$2^{2x} = 2^3$

$x = 1\frac{1}{2}$

$\therefore \log_4 8 = 1\frac{1}{2}$

f) $\log_{10} 4 + \log_{10} 25$

$= \log_{10} 100$

$= 2.$

a) $\log 0.2$

$= \log \frac{2}{10}$

$= \log 2 - \log 10$

$= a - 1$

b) $\log 15$

$= \log 5 \times 3$

$= \log \frac{10}{2} \times 3$

$= \log 10 \cdot \log 2 + \log 3$

$= 1 - a + b$

c) $\log 2\frac{1}{2}$

$= \log \frac{5}{2}$

$= \log 5 - \log 2$

$= \log 10 - \log 2 - \log 2$

$= 1 - 2a.$

) $\log 2^3$

$= 3 \log 2$

$= 3a$

e) $\frac{\log 4}{\log 9}$

$= \frac{2 \log 2}{2 \log 3}$

$= \frac{a}{b}$

f) $\log 2 \times \log 8$

$= \log 2 \times 3 \log 2$

$= a \times 3a$

$= 3a^2$

) $\log (2 + 8)$

$= \log 10$

$= 1$

k) $\log 10^{ab}$

$= ab \log 10$

$= ab.$

$\log_x 16$

$\frac{3\log_x 2}{4\log_x 2}$

$= \frac{3}{4}$

b) $\frac{\log a^2}{\log b^2}$

$= \frac{2\log a}{2\log b}$

$= \frac{\log a}{\log b}$

c) $\frac{\log 1}{\log 5}$

$= 0$

d) $\log 2 + \log 2$

$= \log 1$

$= 0$.

e) $\frac{\log x^2}{\log x}$

$= \frac{2\log x}{\log x}$

$= 2$

f) $\frac{\log 9 + \log 4}{\log 27 + \log 8}$

$= \frac{2\log 3 + 2\log 2}{3\log 3 + 3\log 2}$

$= \frac{2(\log 3 + \log 2)}{3(\log 3 + \log 2)}$

$= \frac{2}{3}$

g) $\frac{2\log 4 + \log 3}{3\log 2 + \log 6}$

$= \frac{4\log 2 + \log 3}{3\log 2 + \log 2 + \log 3}$

$= \frac{4\log 2 + \log 3}{4\log 2 + \log 3}$

$= 1$.

h) $2\log 3 + \log \frac{1}{9}$

$= \log 9 \times \frac{1}{9}$

$= \log 1$

$= 0$.

4 a) $\log \frac{400}{4}$

$= \log 100$

$= 2$.

b) $\log 5 + \log 12 - \log 6$

$= \log \frac{5 \times 12}{6}$

$= \log 10$

$= 1$

c) $\frac{\log 4 + \log 25}{\log 0.01}$

$= \frac{\log 100}{\log 0.01}$

$= \frac{2}{-2}$

$= -1$

d) $\frac{\log 400}{\log 4}$

$= \frac{\log 4 + \log 100}{2\log 2}$

$= \frac{2\log 2 + 2}{2\log 2}$

$= \frac{2(\log 2 + 1)}{2\log 2}$

$= \frac{\log 2 + 1}{\log 2}$

e) $\log 6 + 2\log 20 - \log 3 - 3\log 2$

$= \log \frac{6 \times 400}{3 \times 8}$

$= \log 100$

$= 2$.

f) $\log 16 + \log 30 + \log 5 - \log 24$

$= \log \frac{16 \times 30 \times 5}{24}$

$= \log 100$

$= 2$.

$= a$

$= 81$

b) $\log_2 a = -3$

$a = 2^{-3}$

$a = \frac{1}{8}$

c) $2\log_5 a = -2$

$\log_5 a = -1$

$a = 5^{-1}$

$= \frac{1}{5}$

6 a) $7^x = 5$

$x \log 7 = \log 5$

$x = \frac{\log 5}{\log 7}$

$= 0.827$

b) $5^{x+1} = 82$

$(x+1)\log 5 = \log 82$

$x \log 5 + \log 5 = \log 82$

$x \log 5 = \log 82 - \log 5$

$x = \frac{\log 82 - \log 5}{\log 5}$

$= 1.738$

c) $2^x \times 3^{x-1} = 22$

$x \log 2 + (x-1)\log 3 = \log 22$

$x \log 2 + x \log 3 - \log 3 = \log 22$

$x = \frac{\log 22 + \log 3}{\log 2 + \log 3}$

$= 3.338$

$\log_5 2 \times \log_7 5 \times \log_6 7$

$= \frac{\log_2 2}{\log_2 5} \times \frac{\log_2 5}{\log_2 7} \times \frac{\log_2 7}{\log_2 6}$

$= \frac{\log_2 2}{\log_2 6}$

$= \frac{1}{\log_2 6}$

$= \frac{1}{\log_2 2 \times 3}$

$= \frac{1}{\log_2 2 + \log_2 3}$

$= \frac{1}{1 + w}$.

8. $\log_4 8 - \log_{\frac{1}{4}} 32$

$= \frac{\log 8}{\log 4} - \frac{\log 32}{\log \frac{1}{4}}$

$= \frac{3\log 2}{2\log 2} - \frac{5\log 2}{-2\log 2}$

$= \frac{3}{2} + \frac{5}{2}$

$= 4$.

$\log(2x+1) - \log_{10}(x-1) = 1$

$\log_{10} \frac{2x+1}{x-1} = 1$

$\frac{2x+1}{x-1} = 10$.

$10x - 10 = 2x + 1$

$8x = 11$

$x = \frac{11}{8} = 1\frac{3}{8}$.

b) $\log_2(1-x) + \log_2(5+x) = 3$

$\log_2(1-x)(5+x) = 3$

$(1-x)(5+x) = 8$

$5 - 4x - x^2 = 8$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -3$ or -1.

c) $\log\sqrt{x} = 1.1$

$10^{1.1} = \sqrt{x}$

$\sqrt{x} = 12.59$

$x = 158.489$.

d) $10^x = 7.23$

$x = \log_{10} 7.23$

$= 0.859$.

10. $3^{x+1} = 6^{2x}$

$(x+1)\log 3 = 2x \log 6$

$x\log 3 + \log 3 = 2x(\log 2 + \log 3)$

$x\log 3 - 2x\log 2 - 2x\log 3 = -\log 3$.

$x(-\log 3 - \log 4) = -\log 3$

$x = \frac{\log 3}{\log 12}$

$x = \log_{12} 3$

11. RTP $\log_a(a^4b^4) - \log_b(a^4b^4) = 4(\log_a b - \log_b a)$

LHS $= 4\log_a a + 4\log_a b - 4\log_b a - 4\log_b b$

$= 4 + 4(\log_a b - \log_b a) - 4$

$= 4(\log_a b - \log_b a)$

$=$ RHS.

12. $y = 2\log_{10}(3x+7)$

$10^y = (3x+7)^2$

$2\log_{10}(3x+7) - \log_{10}(2x-1) = 2$

$\log_{10}\frac{(3x+7)^2}{2x-1} = 2$

$\frac{9x^2+42x+49}{=x-1} = 10$.

$9x^2 + 42x + 49 = 200x - 100$

$9x^2 - 158x + 149 = 0$

$(9x - 149)(x - 1) = 0$

$x = \frac{149}{9}$ or 1.

$x = 16.5$ or 1.

13. $m^2 + n^2 = 23mn$.

RTP: $\log \frac{1}{5}(m+n) = \frac{1}{2}(\log m + \log n)$

$m^2 + n^2 = 23mn$

$23mn = (m+n)^2 - 2mn$

$23mn = (m+n)^2 - 2mn$

$(m+n)^2 = 25mn$.

LHS $= 2 \times \frac{1}{2} \log \frac{1}{5}(m+n)$

$= \frac{1}{2} \log \frac{1}{5}^2 (m+n)^2$

$= \frac{1}{2} \log \frac{1}{25} \cdot 25mn$

$= \frac{1}{2} \log mn$

$= \frac{1}{2}(\log m + \log n)$

14. $\log_a (x-m) = \log_{\sqrt{a}} x - \log_{\sqrt{a}} m$

$\frac{\log (x-m)}{\log a} = \frac{\log x}{\frac{1}{2}\log a} - \frac{\log m}{\frac{1}{2}\log a}$

8. $\log (x-m) = 2\log x - 2\log m$

$\log (x-m) = \log \frac{x^2}{m^2}$

$x - m = \frac{x^2}{m^2}$

$xm^2 - m^3 = x^2$

$\therefore x^2 - xm^2 + m^3 = 0$

5. $\log \frac{56}{81} - \log \frac{32}{63}$

$= \log \frac{56^7}{81_9} \times \frac{63^7}{32_4}$

$= \log \frac{7^2}{3^2 \cdot 2^2}$

$= \log \left(\frac{7}{6}\right)^2$

$= 2(\log 7 - \log 6)$

$\log_8 9 = x$ $\quad$ $\log_4 21 = y$

$\frac{\log_2 9}{\log_2 8} = x$ $\quad$ $\frac{\log_2 21}{\log_2 4} = y$

$\frac{\log_2 9}{3} = x$ $\quad$ $\frac{\log_2 21}{2} = y$

$\log_2 9 = 3x$ $\quad$ $\log_2 21 = 2y$

$2\log_2 3 = 3x$

$\log_2 3 = \frac{3x}{2}$

$$\begin{aligned}\log_2 7 &= \log_2 \frac{21}{3} \\ &= \log_2 21 - \log_2 3 \\ &= 2y - \frac{3x}{2} \\ &= \frac{1}{2}(4y - 3x)\end{aligned}$$

7. $\log_5 8 = a$

$\frac{\log_{10} 8}{\log_{10} 5} = a$

$\frac{3\log_{10} 2}{\log_{10} 10 - \log_{10} 2} = a$

$\frac{3\log_{10} 2}{1 - \log_{10} 2} = a$

$3\log_{10} 2 = a - a\log_{10} 2$

$3\log_{10} 2 + a\log_{10} 2 = a$

$\log_{10} 2 (3 + a) = a$

$\log_{10} 2 = \frac{a}{3+a}$.

~~$\log_8 9 = x$ $\quad$ $\log_4 21 = y$~~