Western Region

2008
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (84)

- Attempt Questions 1-7
- All questions are of equal value

Question 1 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) Solve the inequality $\frac{4-2x}{x+5} \le 2$ **3**

(b) Find $\lim_{x \to 0} \frac{\sin\left(\frac{\pi}{4}x\right)}{2x}$ **2**

(c) Find the acute angle between the lines: $x - 2y + 1 = 0$, $y = 5x - 4$ **2**

(d) Differentiate $ln(\sin^{-1} 2x)$ **2**

(e) Find the Cartesian equation of the parabola given $x = t - 2$ and $y = 3t^2 - 1$. **1**

(f) How many arrangements of the word **CHARACTERISTIC** are there? **2**

End of Question 1

Question 2 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) i. Prove that $\sin\theta\sec\theta = \tan\theta$ **1**

ii. Hence solve $\sin\theta\sec\theta = \sqrt{3}$. (0 = ? = 2p) **1**

(b) Use the process of mathematical induction to show that: **4**

$$1 + 3 + 9 + \dots\dots\dots\dots + 3^{n-1} = \frac{1}{2}\left(3^n - 1\right)$$

(c) Find the coefficient of x^4 in the expansion of $\left(3x - \frac{4}{x^2}\right)^7$. **2**

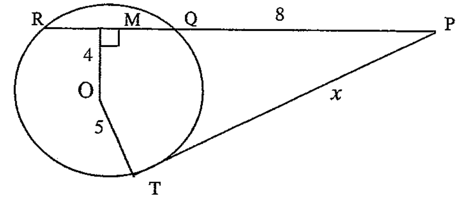
(d) From the top of a cliff an observer spots two ships out at sea. One is north east with an angle of depression of 6° while the other is south east with an angle of depression of 4°. If the two ships are 200 metres apart, find the height of the cliff, to the nearest metre. **3**

(e) Find the remainder when the polynomial $x^3 - 2x^2 - 4x + 7$ is divided by $(x + 3)$ **1**

End of Question 2

Question 3 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) **2**



PT is a tangent to the circle, centre O. OM is perpendicular to the secant RQ.
Find the value of x.

(b) If $\alpha = \sin^{-1}\left(\frac{2}{3}\right)$ and $\beta = \sin^{-1}\left(\frac{3}{5}\right)$, find the value of $\sin(a + \beta)$ **3**

(c) Evaluate $\int \cos^2 4x\, dx$ **2**

(d) Using the substitution $u = x - 2$, evaluate $\int_3^4 \frac{x^2}{(x-2)^2}\, dx$ **3**

(e) Find the value of $1 + \cos^2 x + \cos^4 x + \cos^6 x + \dots\dots\dots\dots$ $(0 < x < \frac{\pi}{2})$ **2**

End of Question 3

Question 4 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) Prove that $\binom{n}{r} \cdot {}^{r}P_{r} = {}^{n}P_{r}$ **2**

(b) A first approximation to the solution of the equation $x^3 - 3x^2 + 1 = 0$ is 0·5. Use one application of Newton's method to find a better approximation. Give your answer correct to two decimal places. **2**

(c) Let P$(2ap, 2ap^2)$ and Q$(2aq, 2aq^2)$ be points on the parabola $y = \frac{x^2}{2a}$. **6**

i. Find the equation of the chord PQ.

ii. If PQ is a focal chord, find the relationship between p and q.

iii. Show that the locus of the midpoint of PQ is a parabola.

(d) Find the area under the curve $y = \frac{1}{\sqrt{4 - x^2}}$ from $x = 1$ to $x = 2$. **2**

End of Question 4

Question 5 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) The polynomial $x^3 - 4x^2 + 5x - 1 = 0$ has 3 roots, namely a, ß and ?.

i. Find the value of a + ß + ? **1**

ii. Find the value of aß? **1**

iii. Find the equation of the polynomial with roots 2a, 2ß and 2?. **2**

(b) If $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots\ldots\ldots\ldots + \binom{n}{n}x^n$, show that: **2**

$$2\binom{n}{2} + (2\times 3)\binom{n}{3} + (3\times 4)\binom{n}{4} + \ldots\ldots\ldots\ldots + n(n-1)\binom{n}{n} = n(n-1)\,2^{n-2}$$

(c) Given that $v = \frac{dx}{dt}$, prove that $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d^2x}{dt^2}$ **3**

(d)

i. At the local high school, seven boys and eight girls have nominated for prefect. If three boys and three girls are selected, in how many ways can this be done. **3**

ii. At their first meeting the six prefects sit around a circular table. What is the probability that the two captains do not sit together?

End of Question 5

Question 7 (12 Marks) Start a fresh sheet of paper. **Marks**

a) A bomb is dropped from a helicopter hovering at a height of 800 metres. At the same time a projectile is fired from a gun located on the ground 800 metres to the west of the point directly beneath the helicopter. The intent is for the projectile to intercept the bomb at a height of exactly 400m. (Use acceleration due to gravity = 10 m / s^2) **8**

i. Show that the time taken for the bomb to fall to a height of 400m is $4\sqrt{5}$ seconds.

ii. Derive the formula for the horizontal and vertical components of the displacement of the projectile.

iii. Find the initial velocity and angle of inclination of the projectile if it is to successfully intercept the bomb as intended.

b) A particle moves so that its distance x centimetres from a fixed point O at time t seconds is $x = 8\sin 3t$. **4**

i. Show that the particle is moving in simple harmonic motion.

ii. What is the period of the motion?

iii. Find the velocity of the particle when it first reaches 4 centimetres to the right of the origin.

End of Examination

STANDARD INTEGRALS

$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$

$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$

$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$

$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$

$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$

$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$

$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$

$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$

$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$

$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$

NOTE : $\ln x = \log_e x, \quad x > 0$

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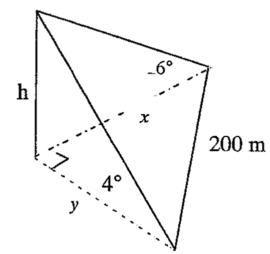
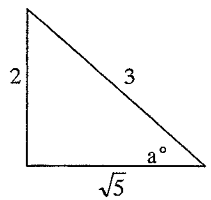
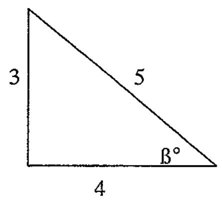
Mathematics Extension 1

Solutions

Solutions Question 1 2008	Marks/Comments
a. $\frac{4-2x}{x+5} \le 2$ x ? - 5 $\therefore 4 - 2x = 2x + 10$ $-6 = 4x$ $x = -1.5$ solution is $x < -5,\ x \ge -1\frac{1}{2}$	**1 for x ? - 5** **1 for other limit** **1 for correct inequality**
b. $\lim_{x\to 0} \frac{\sin\left(\frac{\pi}{4}x\right)}{2x} = \lim_{x\to 0} \frac{1}{2}.\frac{\pi}{4}.\frac{\sin\left(\frac{\pi}{4}x\right)}{\frac{\pi}{4}x}$ $= \frac{\pi}{8}$	**1 – working** **1- correct answer**
c. $x - 2y + 1 = 0$? $y = \frac{1}{2}x + \frac{1}{2}$? $m_1 = \frac{1}{2}$ $y = 5x - 4$ $m_2 = 5$ $\tan\theta = \left\|\frac{m_1 - m_2}{1 + m_1 m_2}\right\| = \left\|\frac{\frac{1}{2} - 5}{1 + 5(\frac{1}{2})}\right\| = \left\|\frac{-4\frac{1}{2}}{3\frac{1}{2}}\right\| = \frac{9}{7}$ θ = 52°8'	**1 – substituting gradients into correct formula** **1- correct answer**
d. $\frac{d}{dx}[ln(\sin^{-1} 2x)] = \frac{1}{\sin^{-1} 2x} \cdot \frac{2}{\sqrt{1-4x^2}}$	**1 – deriving $\sin^{-1} x$** **1 – fully correct**
e. $x = t - 2$ $y = 3t^2 - 1$ From (1) $t = x + 2$ sub in (2) $y = 3(x+2)^2 - 1$ $= 3x^2 + 12x + 12 - 1$ $y = 3x^2 + 12x + 11$	**1 – Cartesian equation**
f. Number of arrangements = $\frac{14!}{3!2!2!2!2!}$ = 908 107 200	**Full marks if correct – less one for each mistake**

Solutions Question 2 2008	Marks/Comments
a. i. $\sin\theta\sec\theta = \sin\theta.\dfrac{1}{\cos\theta}$ $= \tan ?$	**1 mark**
ii. $\sin\theta\sec\theta = \sqrt{3}$ i..e. $\tan\theta = \sqrt{3}$ $\therefore ? = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$	**1 mark for both answers**
b. $1+3+9+ \ldots\ldots\ldots\ldots\ldots + 3^{n-1} = \frac{1}{2}(3^n - 1)$	
Test n = 1. $3^{1-1} = \frac{1}{2}(3^1 - 1) = 1$ $\therefore$ true for $n = 1$	**1 mark for test $n = 1$**
$n = k$ *i.e.* $1+3+9+ \ldots\ldots\ldots\ldots\ldots + 3^{k-1} = \frac{1}{2}(3^k - 1)$	
Prove true for n=k + 1 i.e.	
$1+3+9+ \ldots\ldots\ldots\ldots\ldots + 3^{k-1} + 3^k = \frac{1}{2}(3^k - 1) + 3^{(k+1)-1}$	**1 mark**
$= \frac{1}{2}(3^k - 1) + 3^k$ $= \frac{1}{2}(3^k - 1 + 2.3^k)$ $= \frac{1}{2}(3.3^k - 1)$	
$= \frac{1}{2}(3^{k+1} - 1)$	**1 mark**
Which is in the form $\frac{1}{2}(3^n - 1)$ where $n = k + 1$	
$\therefore$ True for $n = k + 1$ when true for $n = k$, But it is true for $n = 1$ $\therefore$ True for $n = 1 + 1 = 2$ And $2 + 1 = 3$ Etc. Hence by Mathematical Induction $1+3+9+ \ldots\ldots\ldots\ldots\ldots + 3^{n-1} = \frac{1}{2}(3^n - 1)$	**1 mark**

Solutions Question 2 2008 (Cont'd)	
c. $\left(3x - \dfrac{4}{x^2}\right)^7$ $T_{k+1} = {}^7C_k\,(3x)^{7-k}\,(-4x^{-2})^k$ $= {}^7C_k\,3^{7-k}(-4)^k\,.x^{7-k}\,(x^{-2})^k$ $= {}^7C_k\,3^{7-k}(-4)^k\,.x^{7-3k}$	**1 mark for $k = 1$.**
Coefficient of x^4 when $7 - 3k = 4$ i.e. $k = 1$ Coeeficient of $x^4 = {}^7C_1\,3^{7-1}\,(-4)^1$ $= -20\,412$	**1 for correct coefficient**
d.  $\tan 6° = \dfrac{h}{x}$ $x = \dfrac{h}{\tan 6°}$ $\tan 4° = \dfrac{h}{y}$ $y = \dfrac{h}{\tan 4°}$	**1 mark for the expressions for x and y**
Now, $(200)^2 = \left(\dfrac{h}{\tan 6°}\right)^2 + \left(\dfrac{h}{\tan 4°}\right)^2$ $= h^2\left(\dfrac{1}{\tan^2 6°} + \dfrac{1}{\tan^2 4°}\right)$	**1 mark for the use of Pythagoras**
$h = \sqrt{(200)^2 \div \left(\dfrac{1}{\tan^2 6°} + \dfrac{1}{\tan^2 4°}\right)}$ $= 11{\cdot}64381501$ $= 12$ metres (nearest metre)	**1 Mark for correct answer**
e. $x^3 - 2x^2 - 4x + 7$ divisible by $(x+3)$ $\therefore f(-3) =$ remainder $\therefore$ Remainder $= (-3)^3 - 2(-3)^2 - 4(-3) + 7$ $= -27 - 18 + 12 + 7$ $= -26$	**1 mark**

Solutions Question 3 2008	Marks/Comments
a. By Pythagoras RM = $\sqrt{5^2-4^2}$ = 3 Since the line through the centre of a circle perpendicular to a chord bisect the chord , RQ = 6, so PR = 14 Now $(PT)^2$ = PQ . PR $x^2 = (8)(14)$ $x^2 = 112$ $x = \sqrt{112} = 10{\cdot}583\ldots\ldots\ldots$ = 10.6 units (1dp)	**1 for length RM** **1 – correct answer**
b. $\alpha = \sin^{-1}\left(\frac{2}{3}\right)$ $\beta = \sin^{-1}\left(\frac{3}{5}\right)$   $\sin\alpha = \frac{2}{3}$ $\sin\beta = \frac{3}{5}$ $\cos\alpha = \frac{\sqrt{5}}{3}$ $\cos\beta = \frac{4}{5}$ Sin (a + ß) = sin a cos ß + sin ß cos a $= \frac{2}{3}\cdot\frac{4}{5}+\frac{3}{5}\cdot\frac{\sqrt{5}}{3}$ $= \frac{8+3\sqrt{5}}{15}$	**1 for values of cos a and cos ß** **1 use of result** **1 answer**
c. $\int \cos^2 4x\,dx$ $\text{Cos } 2x = 2\cos^2 x - 1$ $2\cos^2 x = \text{Cos } 2x + 1$ $\text{Cos}^2 x = ½ (\cos 2x + 1)$ $\therefore \cos^2 4x = ½ (\cos 8x + 1)$ $\int \cos^2 4x\,dx = \frac{1}{2}\int(\cos 8x + 1)dx$ $= \frac{1}{2}\left[\frac{1}{8}(\sin 8x) + x\right] + c$ $= \frac{\sin 8x}{16} + \frac{x}{2} + c$	1 1
Solutions Question 3 2008 (Cont'd)	

d. $\int_3^4 \frac{x^2}{(x-2)^2}$ $u = x - 2$ $du = dx$ $x = u + 2$ $x = 3,\ u = 1$ $x = 4,\ u = 2$ $= \int_3^4 \frac{x^2}{(x-2)^2}$ $= \int_1^2 \frac{(u^2+4u+4)du}{u^2}$ $= \int_1^2 \left(1 + \frac{4}{u} + 4u^{-2}\right)du$ $= \left[u + 4\ln u - 4u^{-1}\right]_1^2$ $= 2 + 4\ln 2 - \frac{4}{2} - (1 + 4\ln 1 - \frac{4}{1})$ $= 4\ln 2 - (1 + 0 - 4)$ $= 4\ln 2 - (-3)$ $= 4\ln 2 + 3$	**1 integral using change of variable** 1 1
e. $1 + \cos^2 x + \cos^4 x + \cos^6 x + \ldots\ldots\ldots\ldots$ $a = 1, r = \cos^2 x$ $S_\infty = \frac{a}{1-r} = \frac{1}{1-\cos^2 x}$ $= \frac{1}{\sin^2 x}$ $= \text{cosec}^2 x$	1 1

Solutions Question 4 2008	Marks/Comments
a. $\binom{n}{r}.\,{}^rP_r = {}^nP_r$ LHS = $\binom{n}{r}.\,{}^rP_r$ $= \frac{n!}{(n-r)!r!} \cdot \frac{r!}{(r-r)!}$ $= \frac{n!}{(n-r)!}$ $= {}^nP_r$	1 1
b. $f(x) = x^3 - 3x^2 + 1 \quad f(0 \cdot 5) = 0 \cdot 5^3 - 3(0 \cdot 5)^2 + 1 = 0 \cdot 375$ $f'(x) = 3x^2 - 6x \quad f'(0 \cdot 5) = 3(0 \cdot 5)^2 - 6(0 \cdot 5) = -2 \cdot 25$ $a_1 = a_0 - \frac{f(a_0)}{f'(a_0)} = 0 \cdot 5 - \frac{0 \cdot 375}{-2 \cdot 25} = 0 \cdot 67 \quad (2\,dp)$	**1 mark sub into formula** **1 for answer**
c. P($2ap$, $2ap^2$) and Q($2aq$, $2aq^2$) i. $m = \frac{2ap^2 - 2aq^2}{2ap - 2aq} = \frac{2a(p+q)(p-q)}{2a(p-q)} = p + q$ $y - 2ap^2 = (p + q)(x - 2ap)$ $y - 2ap^2 = px + qx - 2ap^2 - 2apq$ $y = px + qx - 2apq$	1 1
ii. $y = \frac{x^2}{2a}$? $x^2 = 2ay \quad \therefore 4A = 2a$ $A = \frac{2a}{4} = \frac{a}{2}$ $\therefore$ focus is S$\left(0, \frac{a}{2}\right)$ $y = px + qx - 2apq$ *is a focal chord if passes through S.* i.e. $\frac{a}{2} = p(0) + q(0) - 2apq$ $\frac{a}{2} = -2apq$ $a = -4apq$ $pq = -\frac{1}{4}$	1 1

Question 6 (12 Marks) Start a fresh sheet of paper. **Marks**

(a) The probability of a person contracting influenza on exposure is 0·65. A family of six has come into contact with a person who has influenza. **4**

What is the probability that:

i. The entire family contracts the flu?

ii. Only two members of the family contract the flu?

iii. Less than half the family contracts the flu?

(b) Given P = 2000 + Ae^{kt},

i. Prove that it satisfies the equation $\frac{dP}{dt} = k\,(P - 2000)$ **1**

ii. Initially, P = 3000, and when t = 5, P = 8000. Use this information to find the values of 'A' and 'k'. **2**

iii. How long does it take the value of 'P' to double and what is the rate of change of 'P' at this time. **3**

(c) The two equal sides of an isosceles triangle are of length 6cm. If the angle between them is increasing at the rate of 0·05 radians per second, find the rate at which the area of the triangle is increasing when the angle between the equal sides is $\frac{\pi}{6}$ radians. **2**

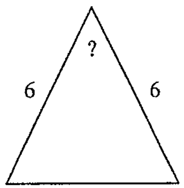
End of Question 6

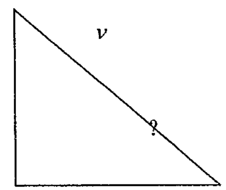
Solutions Question 4 2008 (Cont'd)	
iii. Midpoint PQ = $\left(\frac{2ap+2aq}{2}, \frac{2ap^2+2aq^2}{2}\right)$ $= \left(\frac{2a(p+q)}{2}, \frac{2a(p^2+q^2)}{2}\right)$ i.e. $x = a(p+q)$ $y = a(p^2+q^2)$ $p+q = \frac{x}{a}$ $= a[(p+q)^2 - 2pq]$ $= a\left[\left(\frac{x}{a}\right)^2 - 2\left(-\frac{1}{4}\right)\right]$	1 achieving this step
$y = \frac{x^2}{a} + \frac{a}{2}$ $ay = x^2 + \frac{a^2}{2}$ $ay - \frac{a^2}{2} = x^2$ $x^2 = a\left(y - \frac{a}{2}\right)$ *Which is a parabola with vertex* $\left(0, \frac{a}{2}\right)$, *focal length* $\frac{a}{4}$	1 for equation
d. Area = $\int_1^2 \frac{dx}{\sqrt{4-x^2}}$	1
$= \left[\sin^{-1}\frac{x}{2}\right]_1^2$ $= \sin^{-1}(1) - \sin^{-1}\left(\frac{1}{2}\right)$ $= \frac{\pi}{2} - \frac{\pi}{6}$ $= \frac{\pi}{3}$ square units	1

Solutions Question 5 2008	Marks/Comments
a. $x^3 - 4x^2 + 5x - 1 = 0$	
i. $\alpha + \beta + \gamma = -\frac{b}{a} = \frac{-(-4)}{1} = 4$	1
ii. $\alpha\beta\gamma = \frac{-d}{a} = \frac{-(-1)}{1} = 1$	1
iii. The equation with roots 2a, 2ß and 2? takes the form $x^3 - (2\alpha + 2\beta + 2\gamma)x^2 + (2\alpha 2\beta + 2\alpha 2\gamma + 2\beta 2\gamma)x - 2\alpha 2\beta 2\gamma$ i.e. $x^3 - 2(\alpha+\beta+\gamma)x^2 + 4(\alpha\beta + \alpha\gamma + \beta\gamma)x - 8\alpha\beta\gamma = 0$	
Now,	1
$\alpha + \beta + \gamma = 4$ $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} = 5$ $\alpha\beta\gamma = 1$	1
$\therefore$ Equation is $x^3 - 2(4)x^2 + 4(5)x - 8(1) = 0$ i.e. $x^3 - 8x^2 + 20x - 8 = 0$	
b. $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots\ldots + \binom{n}{n}x^n$ *Differentiate both sides* $n(1+x)^{n-1} = \binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 \ldots\ldots + n\binom{n}{n}x^{n-1}$ *Differentiating both sides again* $n(n-1)(x+1)^{n-2} = 2\binom{n}{2} + (2\times 3)\binom{n}{3}x + \ldots + n(n-1)x^{n-2}$ *Let* $x = 1$, $\therefore$ $2\binom{n}{2} + (2\times 3)\binom{n}{3} + (3\times 4)\binom{n}{4} + \ldots\ldots + n(n-1)\binom{n}{n} = n(n-1)2^{n-2}$	1 for differentiating twice 1 for substituting $x = 1$

Solutions Question 5 2008 (Cont'd)	Marks/Comments
c. $\frac{d^2x}{dt^2}=\frac{d}{dt}\left(\frac{dx}{dt}\right)$	
$=\frac{dv}{dt}$	1
$=\frac{dv}{dx}\times\frac{dx}{dt}$	
$=\frac{dv}{dx}\times v$	1
$=\frac{dv}{dx}\times\frac{d}{dv}\left(\frac{1}{2}v^2\right)$	
$=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	1
$\therefore\ \frac{d}{dx}\left(\frac{1}{2}v^2\right)=\frac{d^2x}{dt^2}$	
d.	
i. Total number of ways = ${}^7C_3\times{}^8C_3$ = 1960	1
ii. Total number of ways = 5! = 120	**1 total number of ways**
Number of ways captains together = 2! × 4! = 48	
Number of ways separated = 5! - 48 = 72	1
Probability = $\frac{72}{120}=\frac{3}{5}$	

Solutions Question 6 2008	Marks/Comments
a.	
i. P(entire family contracts flu) = $(0{\cdot}65)^6 = 0{\cdot}07541889$	1
ii. P(only 2 contract flu) = ${}^6C_2\,(0\cdot65)^2(0\cdot35)^4$ $= 0{\cdot}095102109$	1
iii. P(Less than half contract flu) = $(0{\cdot}35)^6$ + ${}^6C_1\,(0\cdot65)^1(0\cdot35)^5 + {}^6C_2\,(0\cdot65)^2(0\cdot35)^4$ $= 0{\cdot}117423906$	2
b.	
i. $P = 2000 + Ae^{kt}$ $\frac{dP}{dt}=kAe^{kt}$ $= k(2000 + Ae^{kt} - 2000)$ $= k(P - 2000)$	**1 mark**
ii. $P = 2000 + Ae^{kt}$ *when t = 0, P = 3000* $3000 = 2000 + Ae^0$ $\therefore A = 1000.$	**1 for 'A'**
$P = 2000 + 1000e^{kt}$ *when t = 5, P = 8000* $8000 = 2000 + 1000e^{5k}$ $6000 = 1000\,e^{5k}$ $6 = e^{5k}$ $\ln 6 = 5k$ $k = \ln 6 \div 5$ $= 0{\cdot}358351893$	**1 for 'k'**
iii. $P = 2000 + 1000e^{0\cdot358351893\,t}$ $6000 = 2000 + 1000e^{0\cdot358351893\cdot t}$ $4000 = 1000\,e^{0\cdot358351893\cdot t}$ $4 = e^{0\cdot358351893\cdot t}$	**1 correct substitution**
$\ln 4 = 0{\cdot}358351893t$ $t = \ln 4 \div 0{\cdot}358351893$ $t = 3{\cdot}868528072$ i.e. 3·9 years	**1 for number of years**
$\frac{dP}{dt}=k(P-2000)$ $\frac{dP}{dt}=0\cdot358351893\,(6000-2000)$ $= 1433{\cdot}407572$	**1 for rate**

Solutions Question 6 2008 (Cont'd)	Marks/Comments
(c)	
 $\frac{d\theta}{dt} = 0 \cdot 05$	
A = ½ *ab sin C* = ½ (6)(6) *sin ?* = 18 *sin ?* $\frac{dA}{d\theta} = 18 \cos\theta$	
$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$ $\frac{dA}{dt} = 18 \cos\theta \times 0 \cdot 05$	1
When ? = $\frac{\pi}{6}$	
$\frac{dA}{dt} = 18 \cos\frac{\pi}{6} \times 0 \cdot 05$ = 0·779422863 = 0·78 cm^2/s (2dp)	1

Solutions Question 7 2008	Marks/Comments
a.	
i. $\ddot{y} = -10$ $\dot{y} = -10t + C$ *When t = 0,* $\dot{y} = 0$, *so C = 0* $\therefore \dot{y} = -10t$ $y = -5t^2 + C$	1 for equation for y
When t = 0, y = 800, so C = 800. $\therefore y = -5t^2 + 800$ *When t = 400* $400 = -5t^2 + 800$ $-\ 400 = -\ 5t^2$ $t^2 = 80$ $t = \sqrt{80} = 4\sqrt{5}$	1 for time
ii.	
 $\dot{x}_h = v \cos ?$ $y_v = v \sin ?$	
$\ddot{x} = 0$ $\ddot{y} = -\ 10$ $\dot{x} = c$ $\dot{y} = -\ 10\,t + c$ *When t = 0,* x_h *= v cos ?* *When t = 0,* y_v *= v sin ?* $\therefore \dot{x}_v = v \cos ?$ $\therefore \dot{y}_v = -\ 10t + v \sin ?$	1 for working
x = vt cos ? $y = -\ 5t^2 + vt \sin ?$ *When t = 0, x = 0, ∴c = 0* *When t = 0, y = 0, ∴c = 0* ∴ *x = vt cos ?* $y = -\ 5t^2 + vt \sin ?$	1 mark for each equation (2)

Solutions Question 7 2008 (Cont'd)	Marks/Comments
iii. Successfully intercepts the bomb if when $t = 4\sqrt{5}$, $x = 800$ and $y = 400$. i.e. $800 = 4\sqrt{5}\,v\cos ?$ ---------(1) $400 = -5(4\sqrt{5})^2 + 4\sqrt{5}\,v\sin ?$ ---------(2) From (1) $v = \dfrac{800}{4\sqrt{5}\cos\theta} = \dfrac{200}{\sqrt{5}\cos\theta}$ ---- (3) Sub (3) in (2) $400 = -400 + 4\sqrt{5}\left(\dfrac{200}{\sqrt{5}\cos\theta}\right)\sin ?$ $800 = 800\tan ?$ $\tan ? = 1$ $? = 45°$ $v = \dfrac{200}{\sqrt{5}\cos 45°} = 126{\cdot}49$ m/s i.e. $126 m/s$	1 for working 1 for angle 1 for velocity
b. i. $x = 8\sin 3t.$ $\dot{x} = 24\cos 3t$ $\ddot{x} = -72\sin 3t$ $= -72x$ Which is in the form $-n^2x$ ii. Period of motion is $\dfrac{2\pi}{n}$ i.e. $\dfrac{2\pi}{3}$ iii. iii. $x = 8\sin 3t.$ When $x = 4$ $4 = 8\sin 3t$ $\dfrac{1}{2} = \sin 3t$ $3t = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}, \ldots\ldots$ $t = \dfrac{\pi}{18}, \dfrac{5\pi}{18}, \dfrac{13\pi}{18}, \dfrac{17\pi}{18}, \ldots\ldots$	1 mark 1 mark 1 for value of t

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Now, $\dot{x} = 24\cos 3t$ When $t = \dfrac{\pi}{18}$, $\dot{x} = 24\cos 3\left(\dfrac{\pi}{18}\right)$ $= 24\cos\dfrac{\pi}{6}$ $= 12\sqrt{3}$ cm/s	1 for velocity