WESTERN REGION

2007
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading Time- 5 minutes
- Working Time – 2 hours
- Write using a black or blue pen
- Approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (84)

- **Attempt Questions 1-7**
- **All questions are of equal value**

Total Marks – 84
Attempt Questions 1-7
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

Question 1 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) State the domain and range of $y=\sin^{-1}\left(\frac{2x}{3}\right)$. **2**

b) The point P (-2, 5) divides the interval joining A (-4, 1) and B (x, y) internally in the ratio 2 : 3. Find the coordinates of the point B. **2**

c) Using the substitution $u=2x^2-3x$, or otherwise, find $\int\frac{(4x-3)dx}{\sqrt{2x^2-3x}}$ **3**

d) Find the Cartesian equation of the curve defined by the parametric equations $x=\sin\theta$ and $y=\cos^2\theta-3$ **2**

e) Evaluate $\lim_{x\to 0}\frac{\sin\left(\frac{3x}{4}\right)}{2x}$ **3**

End of Question 1

Question 2 (**12 marks)** Use a SEPARATE sheet of paper. **Marks**

a) Find $\frac{d}{dx}\left(x\cos^{-1}x\right)$ **2**

b) Find the coefficient of x^8 in the expansion of $\left(\frac{1}{3}x^2+2\right)^5$ **2**

c) Use the table of standard integrals to evaluate $\int_0^{\frac{\pi}{3}} \sec 2x \tan 2x \; dx$ **2**

d) A bottle of medicine which is initially at a temperature of 10° C is placed into a room which has a constant temperature of 25° C. The medicine warms at a rate proportional to the difference between the temperature of the room and the temperature (T) of the medicine. That is, T satisfies the equation

$$\frac{dT}{dt}=-k(T-25)$$

i) Show that $T=25+Ae^{-kt}$ satisfies this equation. **1**

ii) If the temperature of the medicine after ten minutes is 16° C, find its temperature after 40 minutes. **3**

e) Find $\int \cos^2 9x \, dx$ **2**

End of Question 2

Question 3 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) For the function $f(x)=\sin x-\cos^2 x$

i) Show that $f(x)$ has a root between $x=2$ and $x=3$. **1**

ii) Starting with $x_1=2.2$ use one application of Newton's method to find a better approximation for the root. Answer correct to 2 significant figures. **3**

b) How many distinct eight letter arrangements can be made using the letters of the word PARALLEL? **2**

c) The probability of a grommet being faulty after manufacture is 0.09. Find the probability that there are more than two faulty grommets in a batch of ten. **2**

d) A particle P is moving in a straight line with its position in metres from a fixed origin at a time t seconds being given by

$$x=4\cos\left(2t-\frac{\pi}{6}\right).$$

i) Show that P is moving in simple harmonic motion. **2**

ii) What is the amplitude of the motion? **1**

iii) What is the maximum speed of the particle? **1**

End of Question 3

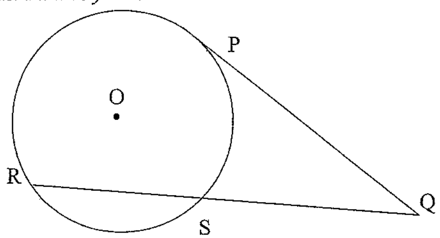
Question 4 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) Use mathematical induction to prove that **3**

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\dots\dots\frac{1}{n(n+1)}=\frac{n}{n+1}$$

for all positive integers n.

b) In the circle centre O, the tangent PQ is 4 cm. The secant RQ is x cm and the chord RS is y cm.



i) Show that $y = x - \frac{16}{x}$ **1**

ii) Show that as x increases, so does y. **2**

iii) What is the geometric significance of the case where $x = 4$? **1**

c) For the polynomial equation $p(x) = x^3 - 2x^2 + 4x - 5 = 0$, with roots α, β, and γ, find the value of

i) $\alpha + \beta + \gamma$ **1**

ii) $\alpha^2 + \beta^2 + \gamma^2$ **1**

iii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ **1**

d) A particle is moving in simple harmonic motion about a fixed point O. Its period is 4π seconds and its amplitude is 3 m. Give an equation that relates its velocity v to its position x. **2**

End of Question 4

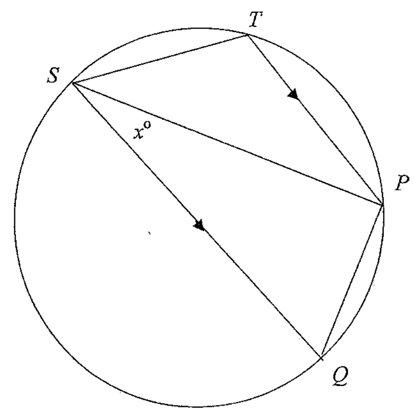
Question 5 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) Experience shows there is a probability of $\frac{2}{3}$ that Josie will choose the winner of any one game in football tipping competitions.

i) In a football tipping competition in which there are 8 games in a round, what is the probability that she will pick 5 winners? **1**

ii) In a competition in which there are 10 games in a round what is the probability that she will pick at least 8 winners? **2**

b) The points P, Q, S and T lie on the circumference of a circle. SQ is a diameter of the circle and TP $\parallel$ SQ.
$\angle PSQ = x^o$ **3**
Find an expression for $\angle TSP$ in terms of x.

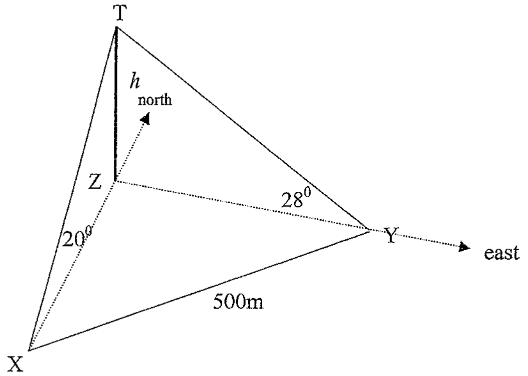


c) Find an expression for $\sin 5x$ in terms of $\sin x$ and $\cos x$ **3**

Question 5 continues on page 7

Question 5 (Continued) **Marks**

d) A conservationist observes the angle of elevation of the top of a tree, which is h metres tall, from two positions. From a point X, due south of the tree, it is 20° and from point Y, due east of the tree, it is 28°. The distance XY is 500 m.



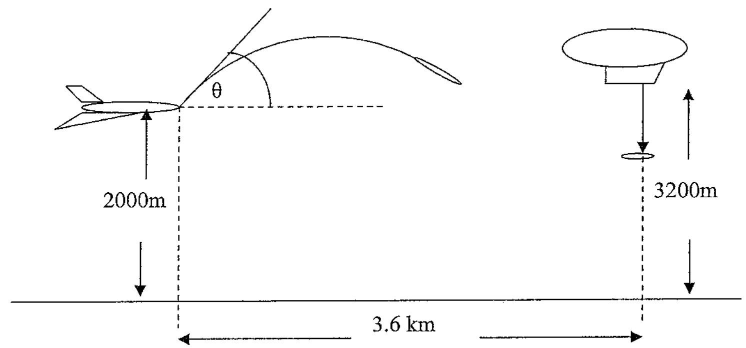
i) Write expressions for XZ and YZ in terms of h. **1**

ii) Calculate the value of h. **2**

End of Question 5

Question 6 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) A plane flying at a height of 2000 m observes a stationary blimp at a height of 3200 metres drop an object. The moment the object is released, the plane fires a projectile at an angle θ to the horizontal in the direction of the object at a velocity of 240 m/s. The horizontal distance between the plane and the blimp is 3.6 km at the time the projectile is fired.



The equations of motion of the projectile are :

$$x = 240t\cos\theta$$
$$y = 2000 + 240t\sin\theta - gt^2$$

The equations of motion of the dropped object (relative to a point below the plane) are :

$$x = 3600$$
$$y = 3200 - gt^2$$

(Use $g = 10\text{ms}^{-2}$)

i) What is the angle at which the projectile must be fired to intercept the object, and how long does it take to reach it? **3**

ii) At what height does the projectile intercept the object? **1**

Question 6 continues on page 9

Marks

Question 6 (Continued)

b) i) Use the expansion of the equation $(1+x)^{n+1} = (1+x)(1+x)^n$ to show that : **2**

$$\binom{n+1}{2} = \binom{n}{1} + \binom{n}{2}$$

ii) By differentiation of $(1+x)^{2n}$ show that **2**

$$\binom{2n}{1} + 2\binom{2n}{2} + 3\binom{2n}{3} + \ldots\ldots\ldots + n\binom{2n}{n} = n.4^n$$

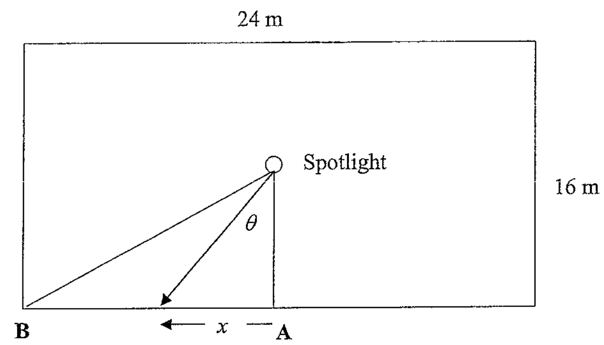
c) i) Sketch the function $f(x) = 2 + \frac{4}{(x-3)}$ for $x > 3$, indicating any asymptotes. **2**

ii) Find the inverse function $f^{-1}(x)$. **2**

End of Question 6

Question 7 (12 marks) Use a SEPARATE sheet of paper. **Marks**

a) A spotlight is in the centre of a rectangular nightclub which measures 24 m by 16 m. It is spinning at a rate of 20 rev/min. Its beam throws a spot which moves along the walls as it spins.



i) Write the rate of rotation $\frac{d\theta}{dt}$ in radians/sec **1**

ii) Find an expression for the velocity $\frac{dx}{dt}$ in terms of x at which the spot appears to be moving along the wall from A to B. **2**

iii) What is the difference in the velocities at which the spot appear to be moving at the points A, nearest to the light and B, furthest from the light? **2**

b) i) The polynomial $P(x) = x^4 + Ax^3 + 9x^2 + 4x - 12 = 0$ has a root at $x = 3$. Find the value of A. **1**

ii) The polynomial has another root at $x = -1$ and a double root. Fully factorise $P(x)$. **1**

iii) Sketch $y = P(x)$. **2**

c) A tangent to the parabola $x^2 = 4ay$ at the point T $(2at, at^2)$ meets the parabola $x^2 = -4ay$ in two points P and Q. Show that the locus of M, the midpoint of PQ, is also a parabola and give its equation. **3**

End of Question 7
End of Examination

WESTERN REGION

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EXAMINATION

Mathematics
Extension 1

SOLUTIONS

Question 1	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(a)	$y=\sin^{-1}\left(\frac{2x}{3}\right)$ $\frac{2x}{3}=\sin y$ $x=\frac{3}{2}\sin y$ Domain is $-\frac{3}{2}\le x\le\frac{3}{2}$ Range is $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$	2	1 each for domain and range
(b)	Ends are at A(-4, 1) and B(x, y) P (-2, 5) divides AB in the ratio 2 : 3. $\frac{2x+3(-4)}{5}=-2$ and $\frac{2y+3(1)}{5}=5$ $2x-12=-10$ $2y+3=25$ $2x=2$ $2y=22$ $x=1$ $y=11$ B is the point (1, 11)	2 marks	1 mark for equations 1 for solution
(c)	Using $u=2x^2-3x$ find $\int\frac{(4x-3)}{\sqrt{2x^2-3x}}$ $u=2x^2-3x$ $\frac{du}{dx}=4x-3$ $du=(4x-3)\,dx$ $\int\frac{(4x-3)dx}{\sqrt{2x^2-3x}}=\int\frac{du}{\sqrt{u}}$ $=\int u^{-\frac{1}{2}}du$ $=2u^{\frac{1}{2}}+c$ $=2\left(2x^2-3x\right)^{\frac{1}{2}}+c$ $=2\sqrt{2x^2-3x}+c$	3 marks	3 marks for final solution. 2 marks if small error made in any stage but other wise okay 1 mark if du found or a start made

Question 1	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(d)	$x=\sin\theta$ and $y=\cos^2\theta-3$ $y=\left(1-\sin^2\theta\right)-3$ $y=\left(1-x^2\right)-3$ $y=-2-x^2$	2 marks	2 marks for correct solution. 1 mark if method correct, but single error made.
(e)	$\lim_{x\to0}\frac{\sin\left(\frac{3x}{4}\right)}{2x}=\lim_{x\to0}\frac{\sin\left(\frac{3x}{4}\right)}{\frac{8}{3}\times\frac{3\times2x}{8}}$ $=\frac{3}{8}\times\lim_{x\to0}\frac{\sin\left(\frac{3x}{4}\right)}{\frac{3x}{4}}$ $=\frac{3}{8}$	3 marks	3 marks for final solution. 2 marks if small error made in any stage but other wise okay 1 mark if an attempt made to get standard limit

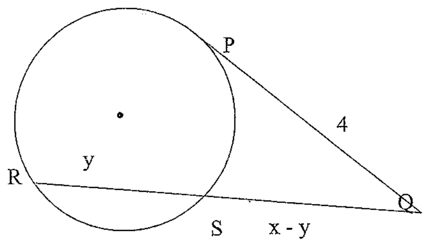
Question 2	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(a)	$\frac{d}{dx}\left(x\cos^{-1}x\right)=x.\left(\frac{-1}{\sqrt{1-x^2}}\right)+1.\cos^{-1}x$ $=\frac{-x}{\sqrt{1-x^2}}+\cos^{-1}x$	2	1 for use of product rule 1 for individual derivatives
(b)	Required term of $\left(\frac{1}{3}x^2+2\right)^5$ is $\binom{5}{4}\left(\frac{1}{3}x^2\right)^4(2)^1=\frac{10}{81}x^8$ Required coefficient is $\frac{10}{81}$	2	2 for correct result 1 if state the general term correctly but don't simplify.
(c)	$\int_0^{\frac{\pi}{3}}\sec 2x\tan 2x\ dx=\left[\frac{1}{2}\sec 2x\right]_0^{\frac{\pi}{3}}$ $=\left(\frac{1}{2}\sec\frac{2\pi}{3}\right)-\left(\frac{1}{2}\sec 0\right)$ $=-1-\frac{1}{2}$ $=-1\frac{1}{2}$	2	1 for correct use of standard integrals 1 for substitution
(d) i)	$T=25+Ae^{-kt}\Rightarrow T-25=Ae^{-kt}$ $\frac{dT}{dt}=-kAe^{-kt}$ $\frac{dT}{dt}=-k(T-25)$	1	Mark only if deriviative found and result shown

Question 2	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(d) ii)	$T=25+Ae^{-kt}$ When $t=0,\ T=10$ $10=25+A(1)$ $A=-15$ When $t=10,\ T=16$ $16=25-15e^{-10k}$ $\frac{9}{15}=e^{-10k}$ $\ln\left(\frac{9}{15}\right)=-10k$ $k=0.051$ (2 sig fig) When $t=40$ $T=25-15e^{-0.051(40)}$ $=23$ (2 sig fig) The temperature is about $23°C$	3	3 marks for obtaining final answer. 2 marks if error made in calculation of A or of k. 1 mark if only A is found or if method is correct, but there are multiple errors..
(e)	$\int \cos^2 9x\,dx=\int \frac{1}{2}(1+\cos 18x)\,dx$ $=\frac{1}{2}\left(x+\frac{1}{18}\sin 18x\right)+c$ $=\frac{x}{2}+\frac{1}{36}\sin 18x+c$	1 1	

Question 3	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(a)i)	$f(x)=\sin x-\cos^2 x$ has a root between $x=2$ and $x=3$ if it changes sign. $f(2)=\sin 2-\cos^2 2$ ≈ 0.74 (2 sig fig) $f(3)=\sin 3-\cos^2 3$ ≈ -0.84 (2 sig fig) So a root exists between $x=2$ and $x=3$	1	
a)ii)	$f(x)=\sin x-\cos^2 x$ $f'(x)=\cos x-2\cos x.(-\sin x)$ $=\cos x+2\cos x\sin x$ $x_2=x_1-\frac{f(x_1)}{f'(x_1)}$ $=2.2-\frac{f(2.2)}{f'(2.2)}$ $=2.2-\frac{\sin 2.2-\cos^2 2.2}{\cos 2.2+2\cos 2.2\sin 2.2}$ $=2.2-(-0.30)$ $=2.5$ (to 2 sig fig)	3	1 for derivative 1 for correct use of Newtons Method 1 for evaluating
(b)	Arrangements $=\frac{8!}{3!2!}=\frac{40320}{12}=3360$ ways	2	1 for 8! 1 for division
(c)	Probability of more than two faulty grommets = 1 – P(two or less faulty) =1- [P(0 f) + P(1 f) +P(2 f)] $=1-[\binom{10}{0}(0.09)^0(0.91)^{10}+\binom{10}{1}(0.09)^1(0.91)^9+\binom{10}{2}(0.09)^2(0.91)^8]$ $=1-[0.95]$ $=0.05$	 1 1	

Question 3	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(d) (i)	$x=4\cos\left(2t-\frac{\pi}{6}\right)$ $\dot{x}=-8\sin\left(2t-\frac{\pi}{6}\right)$ $\ddot{x}=-16\cos\left(2t-\frac{\pi}{6}\right)$ $\ddot{x}=-4\left[4\cos\left(2t-\frac{\pi}{6}\right)\right]$ $\ddot{x}=-2^2\left[4\cos\left(2t-\frac{\pi}{6}\right)\right]$ $\ddot{x}=-2^2x$ which is of the form $\ddot{x}=-n^2x$ so it is in simple harmonic motion.	2	1 for $\ddot{x}$ 1 for statement of SHM
(d) ii)	Amplitude is 4 units	1	
(d) iii)	Maximum speed is 8 m/s	1	

Question 4	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(a)	Prove $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\dots\dots\frac{1}{n(n+1)}=\frac{n}{n+1}$ Assume for $n=k$ $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\dots\dots\frac{1}{k(k+1)}=\frac{k}{k+1}$ Show that when $n=k+1$ $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\dots\dots\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}=\frac{k+1}{k+2}$ $LHS=\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\dots\dots\frac{1}{k(k+1)}+\frac{1}{(k+1)(k+2)}$ $=\frac{k}{(k+1)}+\frac{1}{(k+1)(k+2)}$ $=\frac{k(k+2)+1}{(k+1)(k+2)}$ $=\frac{k^2+2k+1}{(k+1)(k+2)}$ $=\frac{(k+1)^2}{(k+1)(k+2)}$ $=\frac{k+1}{k+2}$ $=RHS$ $\therefore$ if true for $n=k$, is also true for $n=k+1$ When $n=1$ LHS $=\frac{1}{1(1+1)}=\frac{1}{2}$ RHS $=\frac{1}{1+1}=\frac{1}{2}$ $\therefore$ true for $n=1$, and by induction true for all integers $n\geq1$	3	1 mark for stating the assumption. 1 for proving case for k+1 1 for n=1 and conclusion. Adjust accordingly if done in different order.

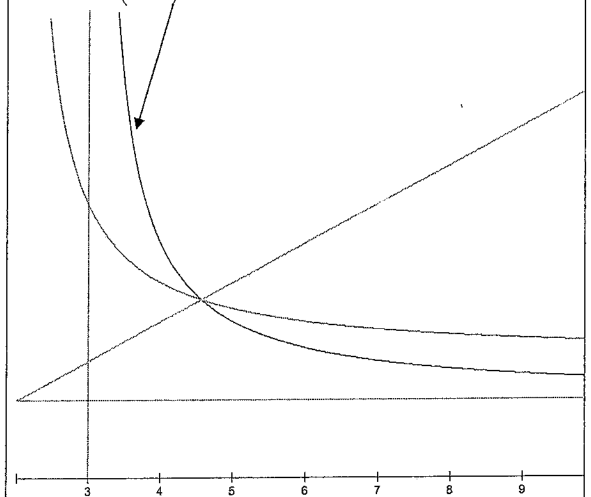
Question 4	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(b)	a) In the circle centre O, the tangent PQ is 4 cm. The secant RQ is x cm and the chord RS is y cm.  (i) $PQ^2 = RQ . QS$ $4^2 = x(x-y)$ $16 = x^2 - xy$ $xy = x^2 - 16$ $y = x - \frac{16}{x}$	1	
	(ii) $y = x - 16x^{-1}$ $\frac{dy}{dx} = 1 + 16x^{-2}$ $= 1 + \frac{16}{x^2}$ As $x^2 \geq 0$, $\frac{dy}{dx} > 0$ $\therefore$ y is an increasing function for $x > 0$	2	Other statements about graph which show it is increasing are acceptable.
	(iii) If $x = 4$ then $y = 0$ This means that QS = RQ =PQ =4 RQ (SQ) becomes a tangent to the circle. (Tangents from an external point are equal)	1	Either statement relating to both lines being tangents okay for the mark.
(c) i)	$\alpha + \beta + \gamma = \frac{-b}{a} = 2$	1	
(c) ii)	$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = 4$ $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \alpha\gamma)$ $= 2^2 - 2(4)$ $= -4$	1	

Question 4	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(c) iii)	$\alpha\beta\gamma = \frac{-d}{a} = 5$ $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \alpha\gamma + \beta\gamma}{\alpha\beta\gamma} = \frac{4}{5}$	1	
(d)	Amplitude $= a = 3$ Period $= 4\pi = \frac{2\pi}{n} \rightarrow n = \frac{1}{2}$ $v^2 = n^2(a^2 - x^2)$ $v^2 = \left(\frac{1}{2}\right)^2 (3^2 - x^2)$ $v^2 = \frac{(9 - x^2)}{4}$	2	1 for correct values of a and n 1 for equation

Question 5	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(a) i)	$P(X=5)=\binom{8}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^3=\frac{1792}{6561}=0.273$	1	Full mark if left as product.
(a) ii)	$P(X\geq 8)=\binom{10}{8}\left(\frac{2}{3}\right)^8\left(\frac{1}{3}\right)^2+\binom{10}{9}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)^1+\binom{10}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^0$ $=0.299$	2	2 marks if left as sum of terms
(b)	$\angle SQP=90^\circ$ (angle in a semicircle) $\angle TPS=x^\circ$ (alternate angles on || lines) $\angle TSQ=\angle TSP+x^\circ$ (adjacent angles) $\angle QPT=90^\circ+x^\circ$ (adjacent angles) $\angle TSQ+\angle QPT=180^\circ$ (opposite angles in cyclic quadrilateral) $\angle TSP+x^\circ+90^\circ+x^\circ=180^\circ$ $\angle TSP=90^\circ-2x^\circ$	3	Alternate solutions possible. 3 marks for complete solution 2 marks if a step is missing 1 if a start made with a correct relevant statement
(c)	$\sin 5x=\sin(4x+x)$ $=\sin 4x\cos x+\cos 4x\sin x$ $=2\sin 2x\cos 2x\cos x+(\cos^2 2x-\sin^2 2x)\sin x$ $=4\sin x\cos x(\cos^2 x-\sin^2 x)\cos x+((\cos^2 x-\sin^2 x)^2-(2\sin x\cos x)^2)\sin x$ $=4\sin x\cos^4 x-4\sin^3 x\cos^2 x+\cos^4 x\sin x-2\sin^3 x\cos^2 x+\sin^5 x-4\sin^3 x\cos^2 x$ $=5\sin x\cos^4 x-10\sin x^3\cos^2 x+\sin^5 x$	3	Three marks for any form that includes only powers of sinx & cosx 2 marks for incomplete expansion 1 mark if started using any valid breakup of sin5x

Question 5	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(d) i)	$\tan 20^\circ=\frac{h}{XZ}$ $\quad\tan 28^\circ=\frac{h}{YZ}$ $XZ=\frac{h}{\tan 20^\circ}$ $\quad YZ=\frac{h}{\tan 28^\circ}$	1	1 mark if both expressions given
(d) ii)	$XZ^2+YZ^2=500^2$ $\frac{h^2}{\tan^2 20^\circ}+\frac{h^2}{\tan^2 28^\circ}=500^2$ $h^2\left(\frac{\tan^2 28^\circ+\tan^2 20^\circ}{\tan^2 28^\circ\tan^2 20^\circ}\right)=250000$ $h^2=\frac{250000\tan^2 28^\circ\tan^2 20^\circ}{\tan^2 28^\circ+\tan^2 20^\circ}$ $h^2=22551.44$ $h=150$ m	2	2 marks for use of Pythagoras and final answer. 1 mark if started using Pyth or trig correctly, but not finished

Question 6	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(a) i)	$x = 240t\cos\theta \qquad x = 3600$ $y = 2000 + 240t\sin\theta - gt^2 \qquad y = 3200 - gt^2$ To intercept, the x and y values must be equal. $240t\cos\theta = 3600$ and $2000 + 240t\sin\theta - gt^2 = 3200 - gt^2$ $t\cos\theta = 15$ and $t\sin\theta = 5$ $\frac{t\sin\theta}{t\cos\theta} = \frac{1}{3}$ $\tan\theta = \frac{1}{3}$ $\theta = 18°26'$ $t = \frac{5}{\sin\theta} = \frac{5}{\sin 18°26'} = 15.8$ sec	3	3 for full solution obtained 2 if equated x and y and attempted to solve 1 if equated x and y only
ii)	$y = 3200 - gt^2$ $= 3200 - 10 \times 15.8^2$ $= 700$ metres	1	
(b) i)	$(1+x)^{n+1} = \binom{n+1}{0} + \binom{n+1}{1}x + \binom{n+1}{2}x^2 + \binom{n+1}{3}x^3 + \ldots\ldots$ $(1+x)(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots.$ $+\binom{n}{0}x + \binom{n}{1}x^2 + \binom{n}{2}x^3 + \binom{n}{3}x^4 + \ldots.$ $= \binom{n}{0} + \left(\binom{n}{1} + \binom{n}{0}\right)x + \left(\binom{n}{1} + \binom{n}{2}\right)x^2 + \ldots.$ Equating coefficients of x^2 $\binom{n+1}{2} = \binom{n}{1} + \binom{n}{2}$	2	2 for full solution 1 if wrote out expansion but not equated coeff or mistake in expansion

Question 6	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(b) ii)	$(1+x)^{2n} = \binom{2n}{0} + \binom{2n}{1}x + \binom{2n}{2}x^2 \ldots\ldots \binom{2n}{2n-1}x^{2n-1} + \binom{2n}{2n}x^{2n}$ Differentiating both sides gives: $2n(1+x)^{2n-1} = \left(\binom{2n}{1} + 2\binom{2n}{2}x + \ldots..(2n-1)\binom{2n}{2n-1}x^{2n-2} + (2n)\binom{2n}{2n}x^{2n-}\right.$ Let $x = 1$ $2n(2)^{2n-1} = \left(\binom{2n}{1} + 2\binom{2n}{2} + \ldots..(2n-1)\binom{2n}{2n-1} + (2n)\binom{2n}{2n}\right)$ $n(2)^{2n} = \binom{2n}{1} + 2\binom{2n}{2} + 3\binom{2n}{3} \ldots.(2n-1)\binom{2n}{2n-1} + (2n)\binom{2n}{2n}$ $n4^n = \binom{2n}{1} + 2\binom{2n}{2} + 3\binom{2n}{3} \ldots..(2n-1)\binom{2n}{2n-1} + (2n)\binom{2n}{2n}$	2	2 for full solution 1 if differentiation done correctly but not finished or mistake in diff then followed on okay
(c)	$f(x) = 2 + \frac{4}{(x-3)}$ 	2	1 for sketch 1 for asymptotes x = 3 and y = 2

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Part	Solution	Marks	Comment
(c) ii)	Inverse function comes from $x=2+\frac{4}{y-3}$ $x-2=\frac{4}{y-3}$ $\frac{y-3}{4}=\frac{1}{x-2}$ $y-3=\frac{4}{x-2}$ $y=3+\frac{4}{x-2}$ Inverse function is $f^{-1}(x)=3+\frac{4}{x-2}$ Sketch of inverse shown but not required.	2	2 for full solution 1 if substituted x and y correctly but mistake made after

Question 7	HSC Trial Examination- Extension 1	2007	
Part	Solution	Marks	Comment
(a) i)	20 rev/min $=20\times 2\pi$ rad/min $=\frac{40\pi}{60}=\frac{2\pi}{3}$ rad/sec	1	
(a) ii)	$\tan\theta=\frac{x}{8}$ $x=8\tan\theta$ $\frac{dx}{d\theta}=8\sec^2\theta$ $\frac{dx}{dt}=\frac{dx}{d\theta}.\frac{d\theta}{dt}$ $=8\sec^2\theta.\frac{2\pi}{3}$ $=\frac{16\pi}{3}\sec^2\theta$ $=\frac{16\pi}{3}(1+\tan^2\theta)$ $=\frac{16\pi}{3}(1+\tan^2\theta)$ $=\frac{16\pi}{3}\left(1+\left(\frac{x}{8}\right)^2\right)$	2	2 marks for full solution 1 mark if done in terms of θ or otherwise incomplete
(a) iii)	At A, $x=0$ $\frac{dx}{dt}=\frac{16\pi}{3}\left(1+\left(\frac{0}{8}\right)^2\right)$ $=\frac{16\pi}{3}$ At B, $x=12$ $\frac{dx}{dt}=\frac{16\pi}{3}\left(1+\left(\frac{12}{8}\right)^2\right)$ $=\frac{16\pi}{3}\left(\frac{13}{4}\right)$ $=\frac{52\pi}{3}$ Difference $=\frac{52\pi-16\pi}{3}$ $=\frac{9\pi}{4}$ m/s	2	1 mark if only one found correctly or if subtraction incorrect.

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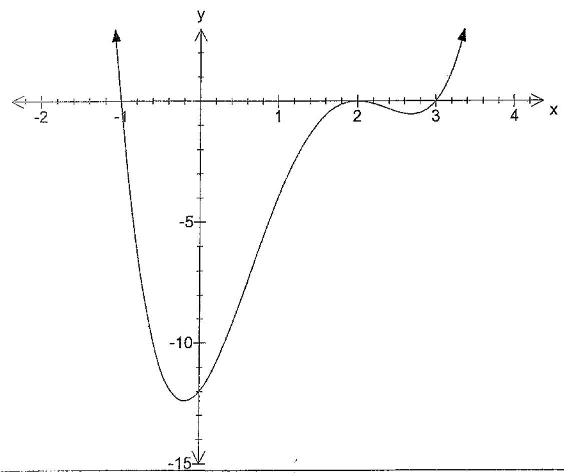
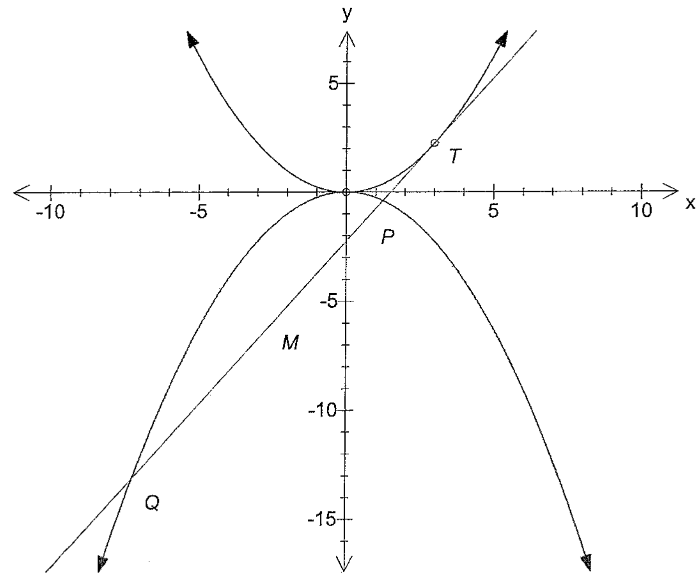
Part	**Solution**	**Marks**	**Comment**
(b) i)	$P(x)=x^4+Ax^3+9x^2+4x-12=0$ $P(3)=3^4+A.3^3+9.3^2+4.3-12=0$ $27A+162=0$ $27A=-162$ $A=-6$	1	
(b) ii)	Sum of roots $=\frac{-b}{a}=\frac{-(-6)}{1}=6$ $(3)+(-1)+2\gamma=6$ $2\gamma=4$ $\gamma=2$ Roots are 3, -1, 2, 2 $P(x)=(x-3)(x+1)(x-2)^2$	1	
(b) iii)		2	1 mark for correct roots including double root 1 mark for correct orientation and y intercept

Question 7	HSC Trial Examination- Extension 1	2007	
Part	**Solution**	**Marks**	**Comment**
(c)	 Tangent has equation $y=tx-at^2$ Intersects 2nd parabola where $x^2=-4a(tx-at^2)$ $x^2+4atx-4a^2t^2=0$ $x=\frac{-4at\pm\sqrt{(4at)^2-4.1(-4a^2t^2)}}{2}$ $x=\frac{-4at\pm\sqrt{16a^2t^2+16a^2t^2}}{2}$ $x=\frac{-4at\pm\sqrt{32a^2t^2}}{2}$ $x=\frac{-4at\pm4\sqrt{2}at}{2}$ $x=\frac{4at(-1\pm\sqrt{2})}{2}$ $y=t\left(\frac{-4at\pm4\sqrt{2}at}{2}\right)-at^2$ $y=-2at^2\pm2\sqrt{2}at^2-at^2$ $y=-3at^2\pm2\sqrt{2}at^2$	3	Diagram not needed. 2 for coordinates of end points of PQ

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Part	**Solution**	**Marks**	**Comment**
	Find coordinates of M $x=\dfrac{\dfrac{4at\left(-1+\sqrt{2}\right)}{2}+\dfrac{4at\left(-1-\sqrt{2}\right)}{2}}{2}$ $x=-2at \rightarrow t=\dfrac{x}{-2a}$ $y=\dfrac{-3at^2+2\sqrt{2}at^2+-3at^2-2\sqrt{2}at^2}{2}=-3at^2$ $y=-3a\left(\dfrac{x}{-2a}\right)^2$ $y=-\dfrac{3x^2}{4a}$ Which is a parabola.		1 for equatio n of locus.