WESTERN REGION

2008

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- **Reading Time- 5 minutes**
- **Working Time – 3 hours**
- **Write using a black or blue pen**
- **Approved calculators may be used**
- **A table of standard integrals is provided at the back of this paper.**
- **All necessary working should be shown for every question.**
- **Begin each question on a fresh sheet of paper.**

Total marks (120)

- **Attempt Questions 1-8**
- **All questions are of equal value**

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x > 0$

Total Marks – 120
Attempt Questions 1-8
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

Question 1 (15 marks) Use a SEPARATE sheet of paper. **Marks**

a) Find $\int \frac{dx}{\sqrt{16-9x^2}}$ **2**

b) Find $\int 5\cos x \sin^2 x \, dx$ **2**

c) Evaluate $\int_1^e x \ln x \, dx$ **3**

d) Evaluate $\int_2^3 \frac{dx}{x^2-1}$ **4**

e) Using the substitution $t = tan\frac{\theta}{2}$ or otherwise find **4**

$$\int_0^{\frac{\pi}{2}} \frac{d\theta}{2+\cos\theta}$$

End of Question 1

Question 2 (15 marks) Use a SEPARATE sheet of paper. **Marks**

a) Let $A = 3+4i$ and $B = 2-2i$. Find in the form $x+iy$ (x and y real).

i) $\frac{A}{B}$ **2**

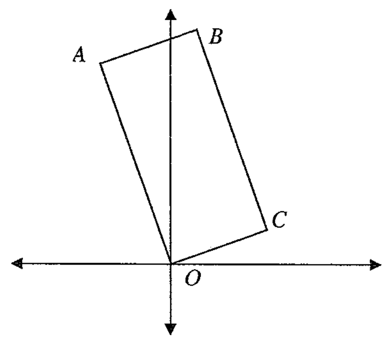
ii) $\sqrt{A}$ **3**

iii) $A-\overline{B}$ **1**

b) i) Write $1+\sqrt{3}i$ in the form $r(cos\,\theta + i\,sin\,\theta)$ **2**

ii) Hence write $\left(1+\sqrt{3}i\right)^6$ showing that it is totally real. **2**

c)



The points $OABC$ are the vertices of a rectangle on the Argand diagram with $|OA| = 2|OC|$. If OC represents the complex number $p+iq$, write down the complex numbers represented by

i) OA **1**

ii) OB **1**

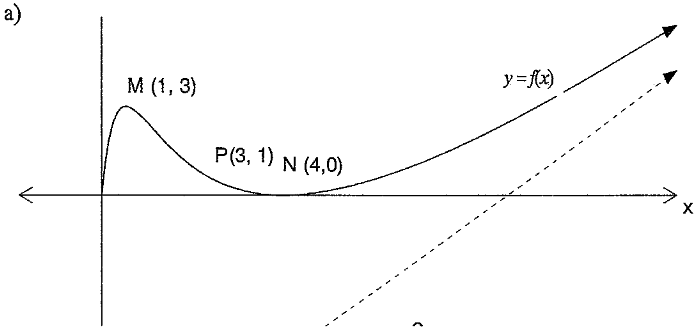
iii) BC **1**

iv) AC **2**

End of Question 2

Question 3 (15 marks) Use a SEPARATE sheet of paper. **Marks**

a)



The diagram shows the graph of $y = f(x)$ for $x \geq 0$.
M(1, 3) and N(4, 0) are stationary points of $y = f(x)$ and P(3, 1) is a point of inflexion of $y = f(x)$. The line $y = x - 9$ is an asymptote as $x \to \infty$. Draw separate one third page sketches showing any special features for the following:

i) $f'(x)$ **2**

ii) $\dfrac{1}{f(x)}$ **2**

iii) $-(f(x))^2$ **2**

b) Determine the gradient of the tangent to the curve $x^2 + 2xy - y^2 = 17$ at the point $(3,\ 2)$ **2**

c) The zeros of $x^3 - 3x^2 - 2x + 4$ are α, β and γ.

i) Find a cubic polynomial whose zeros are α^2, β^2 and γ^2 **2**

ii) Hence or otherwise find the value of $\alpha^2 + \beta^2 + \gamma^2$ **1**

iii) Determine the value of $\alpha^3 + \beta^3 + \gamma^3$ **2**

d) The equation $P(x) = x^3 + 3x^2 - 24x + k = 0$ has a double root. Find the possible values of k. **2**

End of Question 3

Question 4 (15 marks) Use a SEPARATE sheet of paper. **Marks**

a) i) Show that a reduction formula for $I_n = \int (ln\,x)^n dx$ is $I_n = x(ln\,x)^n - nI_{n-1}$ **3**

ii) Hence evaluate $\int_1^{e^4} (ln\,x)^3\, dx$ **4**

b) A solid shape has as its base the parabola $y = x^2$ in the XY plane. Sections taken perpendicular to the axis of the parabola are equilateral triangles.

i) Show that at $y = a$ the area of the triangle is $\sqrt{3}a$ units^2 **2**

ii) Hence, using the method of slicing, determine the volume of the solid, if the length of the axis of the parabola is 16cm. **2**

c) The arc of the curve $y = 6x - x^2 - 8$ where $y \geq 0$ is rotated about the line $x = 1$. By applying the technique of cylindrical shells determine the exact volume of the solid formed. **4**

End of Question 4

Question 5 (15 marks) Use a SEPARATE sheet of paper. **Marks**

a) The line $y = mx + a$ intersects the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at two points which have x coordinates x_1 and x_2.

i) Express x_2 in terms of m, a, b and x_1. **3**

ii) Hence or otherwise show that the line is a tangent to the ellipse at the point where $x=\frac{-a^2m}{b^2+a^2m^2}$. **1**

b) A parabola has parametric equations $x = 2at$ and $y = at^2$.

i) Find the equation of the normal to the parabola at the point where $t = p$. **2**

ii) Hence show that, through the point (x_1, y_1), it is possible to draw up to three normals to the parabola. **2**

c) Given the complex number $z = \cos\theta + i\sin\theta$

i) Use DeMoivres Theorem and the binomial expansion find an expression for $\cos 4\theta$ in terms of $\cos\theta$. **3**

ii) Also, using $z^n+\frac{1}{z^n}=2\cos n\theta$ determine an expansion for $\cos^4\theta$ in terms of $\cos n\theta$. **2**

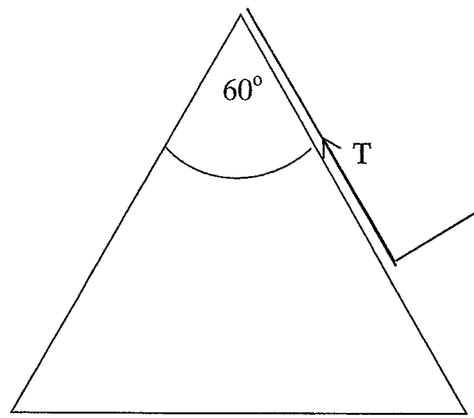
iii) Hence evaluate $\int_0^{\frac{\pi}{2}} \cos^4\theta \, d\theta$ **2**

End of Question 5

Question 6 (15 marks) Use a SEPARATE sheet of paper. **Marks**

a) A mass of 3kg is attached to the vertex of a cone of vertical angle 60^0 by an inelastic string of length 1 metre. The mass is moving in a horizontal circle on the curved, frictionless surface of the cone.

T = Tension in string
N = Normal reaction of the cone surface on the mass.



i) If the mass is moving at a speed of 1ms^{-1}, by resolving forces vertically and horizontally find the values of T and N. **4**

ii) What is the maximum speed of the particle for it to just remain on the cone`s surface and what will be the string's tension at this time? **3**

b) A mass is allowed to fall under gravity from rest at the surface of a medium in which the retardation on the mass is proportional to the distance fallen (x).

i) Write the equation for this motion. **1**

ii) How far does it fall before it becomes stationary? **2**

iii) Show that the displacement in terms of t is **5**

$$x=\frac{g}{k}\left(\sin\left(\frac{2\sqrt{k}t-\pi}{2}\right)+1\right)$$

End of Question 6

Question 7 (15 marks) Use a SEPARATE sheet of paper. **Marks**

a) If $y = f(x).g(x)$ **3**
By taking the logarithm of each side and differentiating implicitly, verify the rule for differentiating a product.

b) Given that $z^5 - 1 = 0$

i) Solve for Z over the complex field (£) in the form $\cos\theta + i\sin\theta$. **3**

ii) Hence express $z^5 - 1$ as the product of linear and quadratic factors. **2**

iii) Write down the complex roots of $z^4 + z^3 + z^2 + z + 1 = 0$. **1**

iv) Without evaluating, show that $\cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = -\frac{1}{2}$. **2**

c) Two different circles touch externally at K. A line ABCD cuts one circle at A and B and the other at C and D. The line CK cuts the first circle at P and DK cuts it at Q.

i) Draw a sketch to show this information. **1**

ii) Prove that PQ is parallel to the line AD. **3**

End of Question 7

Question 8 (15 marks) Use a SEPARATE sheet of paper. **Marks**

a) i) Show that for all values of x and y **1**
$\sin(x+y) - \sin(x-y) = 2\cos x \sin y$

ii) Use mathematical induction to show that for all positive integers **4**

$$n\ ,\ \cos x + \cos 2x + \cos 3x + \text{L}\ \cos nx = \frac{\sin\left(n+\frac{1}{2}\right)x - \sin\frac{1}{2}x}{2\sin\frac{1}{2}x}$$

iii) Hence show that : **4**
$\cos 2x + \cos 4x + \cos 6x + \text{L}\ + \cos 16x\ =\ 8.\cos 9x.\cos 4x.\cos 2x.\cos x$

b) Find a relationship between the coefficients of **4**
$p(x) = x^3 + ax^2 + bx + c = 0$
if the roots are three consecutive terms of an arithmetic series.

c) Solve the differential equation $\frac{dy}{dx} = 2y$ for y given that when $x = 1$, **2**
$y = 1$.

End of Examination

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Mathematics
Extension 2

SOLUTIONS

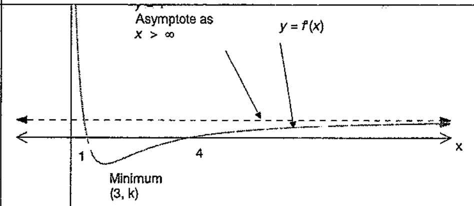
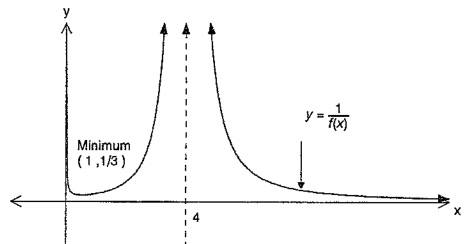
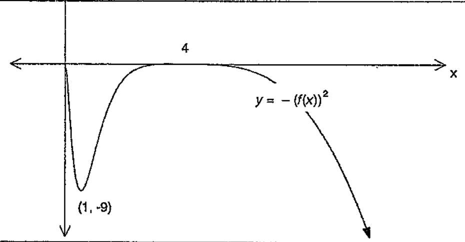
Question 1	Trial HSC Examination- Mathematics Extension 2		2008
Part	**Solution**	**Marks**	**Comment**
(a)	$\int \frac{dx}{\sqrt{16-9x^2}} = \frac{1}{3}\int \frac{dx}{\sqrt{\frac{16}{9}-x^2}}$ $= \frac{1}{3}\sin^{-1}\frac{x}{4/3}+c$ $= \frac{1}{3}\sin^{-1}\frac{3x}{4}+c$	2	1 for rearranging 1 for inv trig integral
(b)	$\int 5\cos x\ \sin^2 x\ dx = \frac{5}{3}\sin^3 x + c$	2	2 for solution 1 if simple error made
(c)	$\int_1^e x\ln x\ dx = \left[\frac{x^2}{2}\ln x\right]_1^e - \int_1^e \frac{x^2}{2}\frac{1}{x}dx$ $= \left[\frac{x^2}{2}\ln x\right]_1^e - \int_1^e \frac{x}{2}dx$ $= \left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^e$ $= \left[\frac{x^2}{4}(2\ln x - 1)\right]_1^e$ $= \frac{e^2}{4}(2-1) - \frac{1}{4}(-1)$ $= \frac{e^2}{4}+\frac{1}{4}$	3	1 for breakup into parts 1 for integral 1 for final answer

Question 1	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
(d)	Let $\frac{A}{x+1}+\frac{B}{x-1}=\frac{1}{x^2-1}$ $A(x-1)+B(x+1)=1$ When $x=1$ $2B=1$ $\rightarrow B=\frac{1}{2}$ When $x=-1$ $-2A=1$ $\rightarrow A=-\frac{1}{2}$ $\int_2^3 \frac{dx}{x^2-1}=\frac{1}{2}\int_2^3\left(\frac{-1}{x+1}+\frac{1}{x-1}\right)dx$ $=\frac{1}{2}\left[ln(x-1)-ln(x+1)\right]_2^3$ $=\frac{1}{2}\left[ln\left(\frac{x-1}{x+1}\right)\right]_2^3$ $=\frac{1}{2}\left[ln\frac{1}{2}-ln\frac{1}{3}\right]_2^3$ $=\frac{1}{2}ln\frac{3}{2}$	4	1 value of B 1 value of A 1 integral 1 for answer

Question 1	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
(e)	If $t=tan\frac{\theta}{2}$ $\frac{dt}{d\theta}=\frac{1}{2}sec^2\frac{\theta}{2}$ $2cos^2\frac{\theta}{2}dt=d\theta$ $cos^2\frac{\theta}{2}=\frac{1}{1+t^2}$ $d\theta=\frac{2}{1+t^2}dt$ Limits $\theta=\frac{\pi}{2}\longrightarrow t=1$ $\theta=0\longrightarrow t=0$ $\int_0^{\frac{\pi}{2}}\frac{d\theta}{2+\cos\theta}=\int_0^1\frac{1}{2+\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}dt$ $=\int_0^1\frac{1+t^2}{(2+2t^2+1-t^2)}\cdot\frac{2}{1+t^2}dt$ $=\int_0^1\frac{2}{(3+t^2)}dt$ $=2\left[\frac{1}{\sqrt{3}}tan^{-1}\frac{t}{\sqrt{3}}\right]_0^1$ $=\frac{2}{\sqrt{3}}\left(tan^{-1}\frac{1}{\sqrt{3}}-tan^{-1}0\right)$ $=\frac{2}{\sqrt{3}}\cdot\frac{\pi}{6}$ $=\frac{\pi}{3\sqrt{3}}$	4	1 for dθ 1 for correct statement of integral including limits 1 for completing integral 1 for result

Question 2	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
(a) i)	$\frac{a}{b}=\frac{3+4i}{2-2i}\times\frac{2+2i}{2+2i}$ $=\frac{6+6i+8i-8}{4+4}$ $=\frac{-2+4i}{8}$ $=-\frac{1}{4}+\frac{7}{4}i$	2	1 for multiplying by conjugate. 1 for correct answer
ii)	Let $\sqrt{A}=x+iy$ $\therefore A=x^2-y^2+2xyi$ $\therefore 3+4i=x^2-y^2+2xyi$ $\therefore 3=x^2-y^2$ ....(1) $\therefore 4=2xy$ ....(2) $(1)^2+(2)^2$ $x^4+2x^2y^2+y^4=25$ $(x^2+y^2)^2=25$ $x^2+y^2=5$ ....(3) (1) + (3) $2x^2=8$ $x=\pm2$ $y=\pm1$ $\sqrt{A}=\pm(2+i)$	3	1 for squaring and equating real and imaginary 1 for eliminating y (or x) 1 for final solution
iii)	$A-\overline{B}=3+4i-(2+2i)$ $=1+2i$	1	1 for answer
(b) i)	$1+\sqrt{3}i=2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$ $\cos\theta=\frac{1}{2}$ and $\sin\theta=\frac{\sqrt{3}}{2}$ $\therefore\theta=\frac{\pi}{3}$ $\therefore 1+\sqrt{3}i=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$	2	1 value of θ 1 for result

Question 2	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
ii)	$(1+\sqrt{3}i)^6=2^6\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^6$ $=64(\cos2\pi+i\sin2\pi)$ by De Moivres Theorem $=64(1+0i)$ $=64$ Which is totally real.	2	1 Use of De Moivre Thm 1 for answer
(c) i)	$OA=2iOC$ $=2i(p+iq)$ $=-2q+2pi$	1	1 for answer
ii)	$OB=OC+OA$ $=(p+iq)+(-2q+2pi)$ $=(p-2q)+(2p+q)i$	1	1 for answer
iii)	$BC=-OA$ $=2q-2pi$	1	1 for answer
iv)	$AC=AB+BC$ $=OC-OA$ $=(p+iq)-(-2q+2pi)$ $=(p+2q)+(q-2p)i$	2	1 for sum of vectors 1 for answer

Question 3	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
(a) i)		2	1 for basic shape 1 for asymptote
ii)		2	1 for basic shape 1 for discontinuity
iii)		2	1 for shape 1 for below axis
(b)	$x^2+2xy-y^2=17$ $2x+2y+2x\frac{dy}{dx}-2y\frac{dy}{dx}=0$ $\frac{dy}{dx}(2x-2y)=-(2x+2y)$ $\frac{dy}{dx}=\frac{-(x+y)}{(x-y)}$ $=\frac{x+y}{y-x}$ At (3,2) Gradient of tangent = $\frac{3+2}{2-3}=-5$	2	1 for implicit differentiation 1 for derivative

Question 3	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
(c) i)	For $x^3-3x^2-2x+4=0$ $x=\alpha,\ \beta$ and γ Let $X=x^2$ $\sqrt{X}=x$ $X\sqrt{X}-3X-2\sqrt{X}+4=0$ $\sqrt{X}(X-2)=3X-4$ Squaring $X(X^2-4X+4)=9X^2-24X+16$ $X^3-4X^2+4X=9X^2-24X+16$ $\therefore$ Required polynomial is $x^3-13x^2+28x-16=0$	2	Any method okay. 1 mark for partial solution or complete solution with simple error.
ii)	As above has roots $\alpha^2,\ \beta^2$ and γ^2 $\alpha^2+\beta^2+\gamma^2=\frac{-b}{a}=13$	1	1 for answer
iii)	As $\alpha,\ \beta$ and γ are roots of $x^3-3x^2-2x+4=0$ Then $\alpha^3-3\alpha^2-2\alpha+4=0$ $\beta^3-3\beta^2-2\beta+4=0$ $\gamma^3-3\gamma^2-2\gamma+4=0$ Adding $(\alpha^3+\beta^3+\gamma^3)-3(\alpha^2+\beta^2+\gamma^2)-2(\alpha+\beta+\gamma)+12=0$ $(\alpha^3+\beta^3+\gamma^3)-3(13)-2(3)+12=0$ $(\alpha^3+\beta^3+\gamma^3)=-33$	2	Any method okay. 1 mark for partial solution or complete solution with simple error.
(d)	$P(x)=x^3+3x^2-24x+k$ $P'(x)=3x^2+6x-24$ $=3(x-2)(x+4)$ If $P'(x)=0$ $x=2,$ $x=-4$ If $x=2$ is a double zero, $P(2)=(2)^3+3(2)^2-24(2)+k=0$ $k=28$ $P(-4)=(-4)^3+3(-4)^2-24(-4)+k=0$ $k=-80$	2	1 possible zeros 1 values of k

Question 4	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
(a) i)	$I_n = \int (\ln x)^n\,dx = x(\ln x)^n - \int x.n(\ln x)^{n-1}.\frac{1}{x}$ $= x(\ln x)^n - n\int (\ln x)^{n-1}\,dx$ $= x(\ln x)^n - nI_{n-1}$	3	1 for use of Int by parts 1 for simplifying 1 for result in terms of I_n
ii)	Consider $\int (\ln x)^3\,dx = I_3$ $\therefore I_3 = x(\ln x)^3 - 3I_2$ Now $I_2 = x(\ln x)^2 - 2I_1$ and $I_1 = x(\ln x) - 1I_0$ $= x(\ln x) - x$ $\therefore I_3 = x(\ln x)^3 - 3\left(x(\ln x)^2 - 2\left(x(\ln x) - x\right)\right)$ $= x(\ln x)^3 - 3x(\ln x)^2 + 6x(\ln x) - 6x$ $\therefore \int_1^{e^4} (\ln x)^3 dx = \left(e^4.64 - 3e^4.16 + 6e^4.4 - 6e^4\right) - (-6)$ $= 34e^4 + 6$	4	1 for I_3 1 for I_2 1 full expression including I_1 1 sub and evaluate
(b) i)	$y = x^2$ at $y = a \quad x = \pm\sqrt{a}$ $\therefore$ Length of Δ side $= 2\sqrt{a}$ $\therefore$ Area of $\Delta = \frac{1}{2}.2\sqrt{a}.2\sqrt{a}\sin 60^\circ$ $= 2a.\frac{\sqrt{3}}{2}$ $\sqrt{3}a$ unit^2	2	1 for side 1 for area

Question 4	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
ii)	Let thickness of slice at $y = a$ be δ and the volume be δV Then $\delta V = \sqrt{3}a.\delta a$ units^3 For whole solid, $V = \lim_{\delta a \to 0} \sum_{a=0}^{16} \sqrt{3}a.\delta a$ $= \int_0^{16} \sqrt{3}a\,da$ $= \left[\frac{\sqrt{3}a^2}{2}\right]_0^{16}$ $= 128\sqrt{3}$ units^3	2	1 expression for integral 1 for solution
(c)	$y = 6x - x^2 - 8$ $y = 0 \quad x = 2, 4$ For shells about Y axis $V = 2\pi\int_a^b xy\,dx$ About $x = 1$ $V = 2\pi\int_a^b (x-1)y\,dx$ $= 2\pi\int_2^4 (x-1)(6x - x^2 - 8)\,dx$ $= 2\pi\int_2^4 (7x^2 - x^3 - 14x + 8)\,dx$ $= 2\pi\left[\frac{7x^3}{3} - \frac{x^4}{4} - 7x^2 + 8x\right]_2^4$ $= 2\pi\left[\left(\frac{448}{3} - 64 - 112 + 32\right) - \left(\frac{56}{3} - 4 - 28 + 16\right)\right]$ $= \frac{16\pi}{3}$ units^3	4	4 marks for full solution 3 marks if simple error made 2 marks if major error or 2 simple errors 1 mark if start made using correct formula or from scratch with correct method.

Question 5	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
(a) i)	Solving simultaneously $y = mx + a$ ....(1) $b^2x^2 + a^2y^2 = a^2b^2$ .....(2) sub (1) into (2) $b^2x^2 + a^2(mx+a)^2 = a^2b^2$ $b^2x^2 + a^2m^2x^2 + 2a^3mx + a^4 = a^2b^2$ $(b^2 + a^2m^2)x^2 + 2a^3mx + a^4 - a^2b^2 = 0$ If $x = x_1$ and x_2 are the roots then $x_1 + x_2 = \frac{-2a^3m}{b^2 + a^2m^2}$ $\therefore x_2 = \frac{-2a^3m}{b^2 + a^2m^2} - x_1$	3	1 for sub into equation 1 for simplify 1 for expression for x_2
ii)	For a tangent $x_1 = x_2 = x$ $x_1 + x_2 = 2x = \frac{-2a^3m}{b^2 + a^2m^2}$ $\therefore x = \frac{-a^3m}{b^2 + a^2m^2}$	1	1 for answer
(b) i)	$x = 2at$ and $y = at^2$ Grad of tangent $= \frac{dy}{dx} = t$ Grad of normal $= = \frac{-1}{t}$ At $t = p$ $m = \frac{-1}{p}$ [point $(2ap, ap^2)$] Equation of normal $y - ap^2 = \frac{-1}{p}(x - 2ap)$ $py - ap^2 = -x + 2ap$ $x + py - ap^3 - 2ap = 0$	2	1 for gradient of normal 1 for equation of normal

Question 5	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
ii)	Normal passes through (x_1, y_1) then $x_1 + py_1 - ap^3 - 2ap = 0$ To find intersection with the parabola, this equation must be solved for p. As the equation is a cubic in p, there can be from 1 to 3 values for p. $\therefore$ Up to three normals can be drawn	2	2 for any reasonable explanation
(c) i)	$z = \cos\theta + i\sin\theta = c + is$ $z^4 = (c + is)^4$ $= c^4 + 4c^3(is) + 6c^2(-s^2) + 4c(-is^3) + s^4$ $= c^4 - 6c^2s^2 + s^4 + i(4c^3s - 4cs^3)$ By De Moivres Thm $z^4 = \cos 4\theta + i\sin 4\theta$ Equating real parts $\cos 4\theta = c^4 - 6c^2(1 - c^2) + (1 - c^2)^2$ $= c^4 - 6c^2 + 6c^4 + 1 - 2c^2 + c^4$ $= 8c^4 - 8c^2 + 1$ $= 8\cos^4\theta - 8\cos^2 + 1$	3	1 for expanding 1 for De Moivre 1 for solution
(ii)	$\left(z + \frac{1}{z}\right)^4 = (2\cos\theta)^4$ $= 16\cos^4\theta$ and $\left(z + \frac{1}{z}\right)^4 = z^4 + 4z^3\frac{1}{z} + 6z^2\frac{1}{z^2} + 4z\frac{1}{z^3} + \frac{1}{z^4}$ $= z^4 + \frac{1}{z^4} + 4\left(z^2 + \frac{1}{z^2}\right) + 6$ $16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$ $\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$	2	1 for expansion 1 for expression

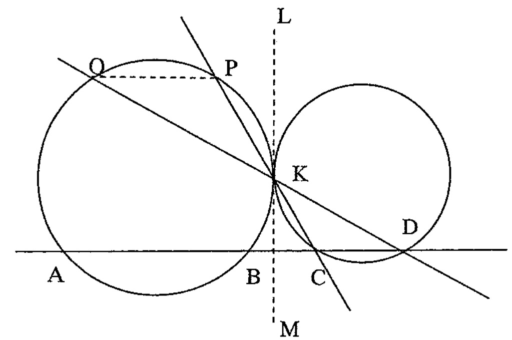
Question 5	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
iii)	$\int_0^{\frac{\pi}{2}} \cos^4\theta \, d\theta = \int_0^{\frac{\pi}{2}} \left(\frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}\right) d\theta$ $= \left[\frac{1}{32}\sin 4\theta + \frac{1}{4}\sin 2\theta + \frac{3}{8}\theta\right]_0^{\frac{\pi}{2}}$ $= \left[\left(\frac{3}{8}.\frac{\pi}{2}\right) - (0)\right]$ $= \frac{3\pi}{16}$	2	1 for integral 1 for evaluating

Question 6	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
(a) i)	Vertically $T\sin 60° + N\cos 60° = 30$ Horizontally $T\cos 60° - N\sin 60° = \frac{mv^2}{r} = \frac{3\times 1}{0.5} = 6$ $\frac{\sqrt{3}}{2}T + \frac{1}{2}N = 30$ $\frac{1}{2}T - \frac{\sqrt{3}}{2}N = 6$ Solving simultaneously, $T = 15\sqrt{3} + 3$ $N = 15 - 3\sqrt{3}$ Tension is $15\sqrt{3} + 3$ Newtons and Normal force is $15 - 3\sqrt{3}$ Newtons.	4	1 for Vertical component 1 for Horizontal Comp 1 for tension 1 for normal
ii)	For mass to stay on surface $N = 0$ $\therefore \frac{\sqrt{3}}{2}T = 30 \qquad T = 20\sqrt{3}$ And $\frac{1}{2}T = \frac{mv^2}{r}$ And $10\sqrt{3} = \frac{3v^2}{0.5}$ $v^2 = \frac{5\sqrt{3}}{3}$ Velocity is $\sqrt{\frac{5\sqrt{3}}{3}} ms^{-1}$ with tension $20\sqrt{3}$ Newtons.	3	1 for value of T 1 for equation 1 for Velocity
b) i)	$\ddot{x} = g - kx$ (k is a constant)	1	1 for equation

Question 6	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
ii)	$\frac{d}{dx}\left(\frac{1}{2}v^2\right) = g - kx$ $\frac{1}{2}v^2 = gx - \frac{1}{2}kx^2 + c$ When $x = 0,\ \ v = 0\ \ \therefore c = 0$ $\frac{1}{2}v^2 = gx - \frac{1}{2}kx^2$ When $v = 0\ \ gx - \frac{1}{2}kx^2 = 0$ $x\left(g - \frac{1}{2}kx\right) = 0$ $\therefore x = 0$ or $\frac{2g}{k}$ $\therefore$ Distance fallen is $\frac{2g}{k}$ metres	2	1 for equation linking v and x 1 for distance

Question 6	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
iii)	$\frac{1}{2}v^2 = gx - \frac{1}{2}kx^2$ $\therefore v = \frac{dx}{dt} = \sqrt{2gx - kx^2}$ $\frac{dt}{dx} = \frac{1}{\sqrt{2gx - kx^2}}$ $= \frac{1}{\sqrt{k}\sqrt{\frac{2g}{k}x - x^2}}$ Completing the square $= \frac{1}{\sqrt{k}\sqrt{\frac{g^2}{k^2} - \frac{g^2}{k^2} + \frac{2g}{k}x - x^2}}$ $\frac{dt}{dx} = \frac{1}{\sqrt{k}\sqrt{\frac{g^2}{k^2} - \left(x - \frac{g}{k}\right)^2}}$ $t = \frac{1}{\sqrt{k}}\sin^{-1}\left(\frac{kx - g}{g}\right) + c$ When $t = 0\ \ x = 0\ \therefore c = \frac{-1}{\sqrt{k}}\sin^{-1}(-1) = \frac{\pi}{2\sqrt{k}}$ $\therefore t = \frac{1}{\sqrt{k}}\sin^{-1}\left(\frac{kx - g}{g}\right) + \frac{\pi}{2\sqrt{k}}$ $t - \frac{\pi}{2\sqrt{k}} = \frac{1}{\sqrt{k}}\sin^{-1}\left(\frac{kx - g}{g}\right)$ $\frac{2\sqrt{k}t - \pi}{2} = \sin^{-1}\left(\frac{kx - g}{g}\right)$ $\frac{kx - g}{g} = \sin\left(\frac{2\sqrt{k}t - \pi}{2}\right)$ $kx - g = g\sin\left(\frac{2\sqrt{k}t - \pi}{2}\right)$ $kx = g\sin\left(\frac{2\sqrt{k}t - \pi}{2}\right) + g$ $x = \frac{g}{k}\left(\sin\left(\frac{2\sqrt{k}t - \pi}{2}\right) + 1\right)$	5	1 for $\frac{dt}{dx}$ 2 for expression for t 2 for expression for x

Question 7	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
(a)	$y=f(x).g(x)$ $\ln y=\ln[f(x)]+\ln[g(x)]$ $\frac{1}{y}.\frac{dy}{dx}=\frac{f'(x)}{f(x)}+\frac{g'(x)}{g(x)}$ $=\frac{g(x)f'(x)+f(x)g'(x)}{f(x).g(x)}$ $\frac{1}{y}.\frac{dy}{dx}=\frac{g(x)f'(x)+f(x)g'(x)}{y}$ $\frac{dy}{dx}=g(x)f'(x)+f(x)g'(x)$	3	1 1 1
(b) i)	$z^5=1$ By De Moivres Thm $\cos 5\theta+i\sin 5\theta=1$ Equating real and imaginary $\cos 5\theta=1 \quad \sin 5\theta=0$ $\therefore\ 5\theta=0,\ 2\pi,\ 4\pi,\ 6\pi,\ 8\pi$ $\theta=0,\ \frac{2\pi}{5},\ \frac{4\pi}{5},\ \frac{6\pi}{5},\ \frac{8\pi}{5}$ $z_1=cis0=1$ $z_2=cis\frac{2\pi}{5}$ $z_3=cis\frac{4\pi}{5}$ $z_4=cis\frac{6\pi}{5}=cis\frac{-4\pi}{5}=\overline{z}_3$ $z_5=cis\frac{8\pi}{5}=cis\frac{-2\pi}{5}=\overline{z}_2$	3	1 values of θ 2 for values of z_1 - z_5
ii)	$z^5-1=(z-z_1)(z-z_2)(z-z_5)(z-z_3)(z-z_4)$ $=(z-z_1)(z^2-(z_2+z_5)z+z_2z_5)(z^2-(z_3+z_4)z+z_3z_4)=$ $=(z-z_1)\left(z^2-2\cos\frac{2\pi}{5}z+1\right)\left(z^2-2\cos\frac{4\pi}{5}z+1\right)$	2	1 factors 1 in quadratics
iii)	$z^5-1=0$ $(z-1)(z^4+z^3+z^2+z+1)=0$ Roots are $z_2,\ z_3,\ z_4,\ z_5$ from above.	1	

Question 7	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
iv)	Sum of roots of $z^5-1=0$ is zero. $\therefore z_1+z_2+z_3+z_4+z_5=0$ $z_1+z_2+\overline{z}_2+z_3+\overline{z}_3=0$ $1+2\cos\frac{2\pi}{5}+2\cos\frac{4\pi}{5}=0$ $2\cos\frac{2\pi}{5}+2\cos\frac{4\pi}{5}=-1$ $\cos\frac{2\pi}{5}+\cos\frac{4\pi}{5}=\frac{-1}{2}$	2	1 for conjugates 1 for answer
(c) i)			
ii)	Draw common tangent through LKM Join PQ $\angle PQK=\angle PKL$ (angle in the alternate segment) $\angle KDC=\angle MKC$ (angle in the alternate segment) $\angle PKL=\angle MKC$ (vertically opposite angle) $\therefore \angle PQK=\angle KDC$ (from above) $\therefore PQ=AD$ (alternate angles equal)	3	1 1 1

Question 8	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
(a)	$\sin(x+y)-\sin(x-y)=\sin x\cos y+\cos x\sin y-(\sin x\cos y-\cos x\sin y)$ $=2\cos x\sin y$	1	1 for answer

Question 8	Trial HSC Examination- Mathematics Extension 2	2008	
Part	**Solution**	**Marks**	**Comment**
a) ii)	If $n=1$ LHS $=\cos x$ RHS $=\dfrac{\sin\left(\frac{3x}{2}\right)-\sin\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)}$ Using i) above RHS $=\dfrac{2\cos x\sin\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)}$ $=\cos x=$ LHS ∴ true for $n=1$ Assume true for $n=k$ i.e. $\cos x+\cos 2x+\cos 3x\ldots\ldots+\cos kx=\dfrac{\sin\left(k+\frac{1}{2}\right)x-\sin\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)}$ When $n=k+1$ $\cos x+\cos 2x+\cos 3x\ldots+\cos kx+\cos(k+1)x=\dfrac{\sin\left(k+\frac{1}{2}\right)x-\sin\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)}+\cos(k+1)x$ $=\dfrac{\sin\left(k+\frac{1}{2}\right)x-\sin\left(\frac{x}{2}\right)+\cos(k+1)x.2\sin\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)}$ $=\dfrac{\sin\left(k+\frac{1}{2}\right)x-\sin\left(\frac{x}{2}\right)+\sin\left((k+1)x+\frac{x}{2}\right)-\sin\left((k+1)x-\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)}$ using i) above $=\dfrac{\sin\left(k+\frac{1}{2}\right)x-\sin\left(\frac{x}{2}\right)+\sin\left(k+\frac{3}{2}\right)x-\sin\left(k+\frac{1}{2}\right)x}{2\sin\left(\frac{x}{2}\right)}$ $=\dfrac{\sin\left((k+1)+\frac{1}{2}\right)x-\sin\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)}$ ∴ True for $n=k+1$ ∴ Since true for $n=1$, by induction is true for all positive integral values of $k\geq 1$	4	1 for n =1 case 1 for using i) 1 for simplifying 1 for stating k+1 case

Question 8	**Trial HSC Examination- Mathematics Extension 2**	**2008**	
Part	**Solution**	**Marks**	**Comment**
(a) iii)	$\cos 2x+\cos 4x+\cos 6x......+\cos 16x=\cos(2x)+\cos 2(2x)+\cos 3(2x)......+\cos 8(2x)$ $=\dfrac{\sin\left(8+\frac{1}{2}\right)2x-\sin\left(\frac{2x}{2}\right)}{2\sin\left(\frac{2x}{2}\right)}$ $=\dfrac{\sin 17x-\sin x}{2\sin x}$ $=\dfrac{\sin(9+8)x-\sin(9-8)x}{2\sin x}$ $=\dfrac{2\cos 9x\sin 8x}{2\sin x}$ Using i) above $=\dfrac{2\cos 9x.2\sin 4x\cos 4x}{2\sin x}$ Using double angle on $\sin 8x$ $=\dfrac{4\cos 9x.2\sin 2x\cos 2x\cos 4x}{2\sin x}$ Using double angle on $\sin 4x$ $=\dfrac{8\cos 9x.2\sin x\cos x\cos 2x\cos 4x}{2\sin x}$ Using double angle on $\sin 2x$ $=\dfrac{8\cos 9x.\cancel{2\sin x}\cos x\cos 2x\cos 4x}{\cancel{2\sin x}}$ $=8\cos 9x\cos 4x\cos 2x\cos x$	4	1 for sub into expression 1 for breaking up 17x 2 for completing simplification

Question 8	**Trial HSC Examination- Mathematics Extension 2**	**2008**	
Part	**Solution**	**Marks**	**Comment**
(b)	Let roots be $\alpha-d$, α and $\alpha+d$ $\therefore$ Sum of the roots $=(\alpha-d)+\alpha+(\alpha+d)=-a$ $\therefore 3\alpha=-a$ $\alpha=\dfrac{-a}{3}$ $\therefore$ Sum of the roots 2 at a time $=(\alpha-d)\alpha+(\alpha-d)(\alpha+d)+(\alpha+d)\alpha=b$ $\alpha^2-\alpha d+\alpha^2-d^2+\alpha^2+\alpha d=b$ $3\alpha^2-d^2=b$ $d^2=3\alpha^2-b$ $d^2=3\left(\dfrac{-a}{3}\right)^2-b=\dfrac{a^2}{3}-b$ $\therefore$ Product of the roots $=\alpha(\alpha-d)(\alpha+d)=c$ $\alpha^3-\alpha d^2=c$ $\left(\dfrac{-a}{3}\right)^3-\left(\dfrac{-a}{3}\right)\left(\dfrac{a^2}{3}-b\right)=c$ $\dfrac{-a^3}{27}+\dfrac{a^3}{9}-\dfrac{ab}{3}=c$ $\dfrac{2a^3}{27}-\dfrac{ab}{3}=c$ $\therefore 2a^3-9ab-27c=0$	4	1 each for expressions for sums & products = 3 marks 1 for substitution and simplifying
(c)	$\dfrac{dy}{dx}=2y$ $\therefore \dfrac{dx}{dy}=\dfrac{1}{2y}$ $\therefore x=\dfrac{1}{2}\ln y+c$ When $x=1, y=1$ $\therefore c=1$ $\therefore x=\dfrac{1}{2}\ln y+1$ $\dfrac{1}{2}\ln y=x-1$ $\ln y=2x-2$ $y=e^{2x-2}$	2	1 for expression for x 1 for result