2009

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- **Reading Time- 5 minutes**
- **Working Time – 3 hours**
- **Write using a black or blue pen**
- **Approved calculators may be used**
- **A table of standard integrals is provided at the back of this paper.**
- **All necessary working should be shown for every question.**
- **Begin each question on a fresh sheet of paper.**

Total marks (120)

- **Attempt Questions 1-8**
- **All questions are of equal value**

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$

Total Marks – 120
Attempt Questions 1-8
All Questions are of equal value

Begin each question on a NEW SHEET of paper, writing your name and question number at the top of the page. Extra paper is available.

Question 1 (15 marks) Begin a NEW sheet of paper. **Marks**

a) By using the method of partial fractions, show that **4**

$$\int \frac{dx}{x^2-1} = \ln\sqrt{\frac{x-1}{x+1}} + c$$

b) By making a suitable trigonometric substitution, evaluate **4**

$$\int_0^1 \sqrt{1-x^2}\ dx$$

c) If $I = \int e^x \sin x\ dx$ **4**

Find I using the method of integration by parts.

d) Evaluate **3**

$$\int_0^{\frac{\pi}{2}} \cos x \sin^3 x\ dx$$

End of Question 1

Question 2 (15 marks) Begin a NEW sheet of paper. **Marks**

a) Given $A = 3-4i$ and $B = \sqrt{3}+i$.

i) Find AB in $x+iy$ form **1**

ii) Find $\frac{A}{B}$ in $x+iy$ form **1**

iii) Find $\sqrt{A}$ in $x+iy$ form **3**

iv) Find B in modulus- argument form **2**

v) Hence find B^4 in $x+iy$ form **1**

b) On separate Argand diagrams sketch the following loci:

i) $2 \geq |z| \geq 1$ **1**

ii) $\frac{3\pi}{4} > \arg z > \frac{\pi}{4}$ **1**

iii) $3 \geq \text{Re}Z \geq 0$ and $3 \geq \text{Im}Z \geq 1$ **2**

c) On the Argand diagram shown OABC is a rectangle with the length OA being twice OC.

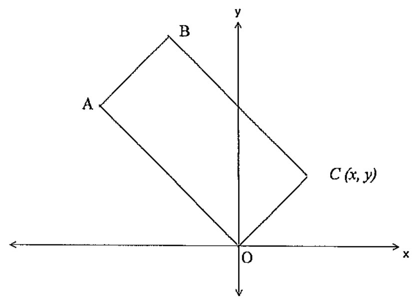
OC represents the complex number $x+iy$.

Find the complex number represented by

i) OA **1**

ii) OB **1**

iii) BC **1**



End of Question 2

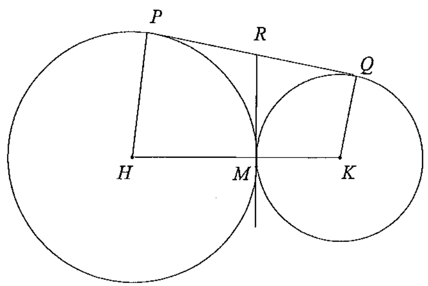
Question 3 (15 marks) Begin a NEW sheet of paper. **Marks**

a) A sequence of numbers u_n is given by $u_1 = 3, u_2 = 21$ and $u_n = 7u_{n-1} - 10u_{n-2}$ for $n \geq 3$. Use mathematical induction to show that $u_n = 5^n - 2^n$ for $n \geq 1$. **4**

b) i) Show that the area enclosed by a parabola of focal length a and its latus rectum is given by $A = \dfrac{8a^2}{3}$ units^2. **3**

ii) A solid is formed such that its base is a semicircle of radius 1 metre. Vertical sections parallel to the diameter are parabolas with each latus rectum being a chord of the semicircle parallel to the diameter. By using the result from i) and the technique of slicing, find the volume of this solid. **4**

c) Shown are two circles centres H and K which touch at M. PQ and RM are common tangents.



i) Show that quadrilaterals $HPRM$ and $MRQK$ are cyclic. **2**

ii) Prove that triangles PRM and MKQ are similar. **2**

End of Question 3

Question 4 (15 marks) Begin a NEW sheet of paper. **Marks**

a) Given that $a, b, c,$ and d represent positive integers and that $a + b + c = 3d$. Show that $100a + 10b + c$ is divisible by 3. **2**

b) The roots of $x^3 + 3px + q = 0$ are α, β and γ, (none of which are equal to 0).

i) Find the monic equation with roots $\dfrac{\beta\gamma}{\alpha}, \dfrac{\alpha\gamma}{\beta}$ and $\dfrac{\alpha\beta}{\gamma}$, giving the coefficients in terms of p and q. **4**

ii) Deduce that if $\gamma = \alpha\beta$ then $(3p - q)^2 + q = 0$ **2**

c) Determine the values of a and b given that $(x+1)^2$ is a factor of $P(x) = x^5 + 2x^2 + ax + b$. **3**

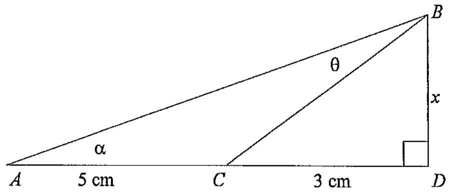
d) i) Solve $Z^5 = 1$ over the complex field giving your answers in modulus-argument form. **2**

ii) Hence write $Z^5 - 1$ as the product of linear and quadratic factors. **2**

End of Question 4

Question 5 (15 marks) Begin a NEW sheet of paper. **Marks**

a)

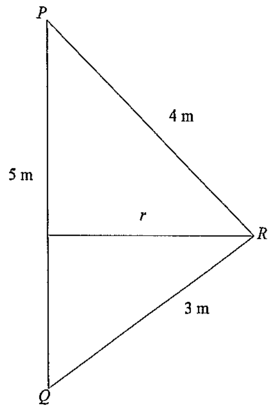


On the above sketch $\angle CDB = 90^o$, $AC = 5$ cm, $CD = 3$ cm and $BD = x$ cm. Also $\angle BAC = \alpha$ and $\angle ABC = \theta$

i) Determine the value x which will maximise θ. **4**

ii) Determine the maximum value of θ (nearest minute). **1**

b) The diagram shows a mass of 1 kg at R joined by two strings to a vertical rod at P and Q where $PR = 4$m and $QR = 3$ m. The mass is rotating horizontally with an angular velocity of 3π rad s^{-1} in horizontal circle ($g = 10$ ms^{-2})



i) Show that the radius of the rotation is $2{\cdot}4m$. **2**

ii) Calculate the tension in each string. **4**

iii) At what angular velocity would the tension in QR be zero? **2**

c) For the curve with equation $x^2 + 3xy - y^2 = 13$, determine the gradient of the tangent at the point (2, 3) on the curve. **2**

End of Question 5

Question 6 (15 marks) Begin a NEW sheet of paper. **Marks**

a) A mass of 1kg is falling under gravity (g) through a medium in which the resistance to the motion is proportional to the square of the velocity. (k = constant of proportionality)

i) Draw a sketch showing all forces acting. **1**

ii) Write an equation for the acceleration of this mass. **1**

iii) Show that the mass reaches a terminal velocity **1**

given by $v = \sqrt{\dfrac{g}{k}}$.

iv) Show that the distance it has fallen when it reaches **4**

a velocity v m/s is given by $x = \dfrac{1}{2k}\ln\left(\dfrac{g}{g - kv^2}\right)$

b) i) Show that the recurrence (reduction)formula for **4**

$I_n = \int \sec^n x dx$

is $I_n = \dfrac{1}{n-1}\tan x \sec^{n-2} x + \dfrac{n-2}{n-1} I_{n-2}$

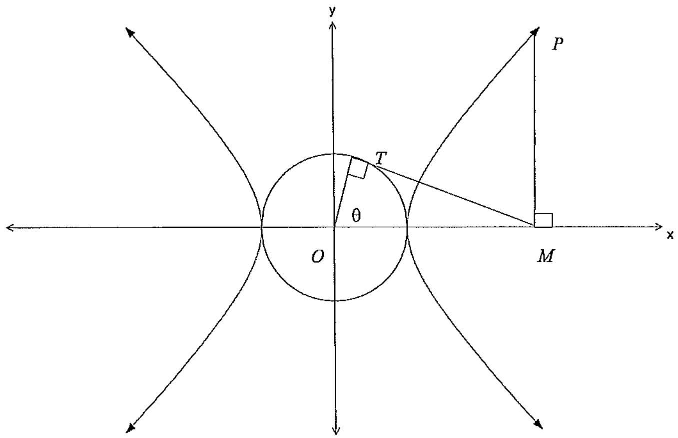
ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \sec^4 x dx$ **2**

c) A cubic equation in z has all real coefficients. If two of the roots are 3 and $2 + i$ determine the equation. **2**

End of Question 6

Question 7 (15 marks) Begin a NEW sheet of paper. **Marks**

a)



The sketch shows the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = a^2$ with $a, b \geq 0$. T lies on the circle where $\angle TOX = \theta$ and $0 \leq \theta \leq \frac{\pi}{2}$. The tangent at T meets OX at M and MP is perpendicular to OX with P on the hyperbola.

i) Find the equation of the tangent TM and hence the coordinates of M. **3**

ii) Hence show that the coordinates of P are $(a\sec\theta, b\tan\theta)$ **1**

iii) If $Q(a\sec\beta, b\tan\beta)$ is another point on the hyperbola, where $\theta + \beta = \frac{\pi}{2}$ and $\theta \neq \frac{\pi}{4}$, show that the equation of PQ is $ay = b(\cos\theta + \sin\theta)x - ab$. **3**

iv) Every such chord PQ passes through a fixed point, find its coordinates. **2**

v) Show that as θ approaches $\frac{\pi}{2}$, PQ approaches a line parallel to an asymptote. **2**

Question 7 continues on page 9.

Question 7 continued. **Marks**

b) i) By letting $Z = \cos\theta + i\sin\theta$ show that

$$Z^n + \frac{1}{Z_n} = 2\cos n\theta.$$

1

ii) Hence express $\cos^4\theta$ in terms of $\cos n\theta$ **3**

End of Question 7

Question 8 (15 marks) Begin a NEW sheet of paper. **Marks**

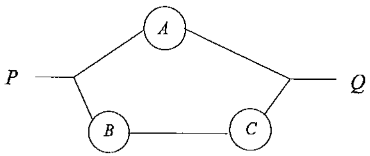
a) A particle is projected to just clear two poles of height h metres at distances of b and c metres from the point of projection. If v is the velocity of the projection at an angle θ to the horizontal:

i) Show that $v^2 = \dfrac{(b+c)g\sec^2\theta}{2\tan\theta}$ **3**

ii) Hence or otherwise show that $\tan\theta = \dfrac{h(b+c)}{bc}$ **2**

iii) Also find an expression in terms of h, b and c, for the greatest height the particle reaches. **2**

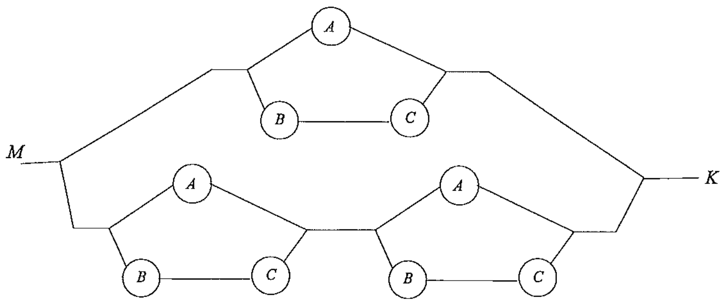
b)



The sketch shows a circuit with components A, B and C each, with a probability of being defective of p. If a component is defective current will not pass the component.

i) Show that the probability of current not flowing from P to Q is $p^2(2-p)$. **3**

ii) A more complex circuit is created using repetitions of the basic circuit in part i). Find the probability that current cannot flow from M to K. **2**



Question 8 continues on page 12.

Question 8 continued. **Marks**

c) Consider the word EXTENSION.

i) How many distinct arrangements can be made of all the letters? **1**

ii) In how many of these arrangements will the first and last letters be N? **1**

iii) In how many of these arrangements will the vowels be grouped together? **1**

End of Examination

WESTERN REGION

2009
TRIAL HSC
EXAMINATION

Mathematics

Extension 2

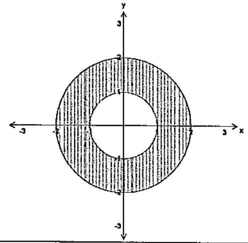
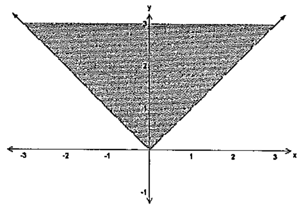
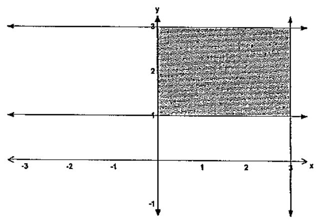
SOLUTIONS

Question 1	Trial HSC Examination- Mathematics Extension 2		2009
Part	**Solution**	**Marks**	**Comment**
(a)	Let $\frac{A}{x+1}+\frac{B}{x-1}=\frac{1}{x^2-1}$	1	
	$\therefore A(x-1)+B(x+1)=1$		
	If $x=1 \quad 2B=1$		
	$B=\frac{1}{2}$		Any method to find A and B
	If $x=-1 \quad 2A=-1$		
	$A=-\frac{1}{2}$	1	
	$\frac{dx}{x^2-1}=\frac{1}{2}\int\frac{1}{x-1}-\frac{1}{x+1}dx$	1	
	$\therefore \int \quad =\frac{1}{2}[\ln(x-1)-\ln(x+1)]+C$		No mark for C
	$=\frac{1}{2}\ln\left(\frac{x-1}{x+1}\right)+C$	1	
	$=\ln\sqrt{\frac{x-1}{x+1}}+c$		Total = 4

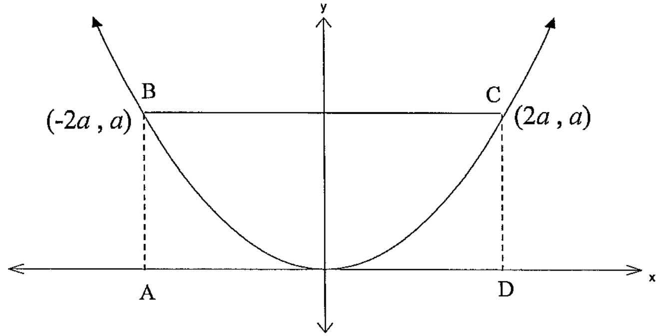
Question 1	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(b)	Let $x=\cos\theta$ When $x=1$ $\theta=0$ $dx=-\sin\theta d\theta$ $x=0$ $\theta=\frac{\pi}{2}$	1	Can use alternate methods ie $x=\sin\theta$
	$\therefore \int_0^1 \sqrt{1-x^2}\,dx = \int_{\frac{\pi}{2}}^0 \sqrt{1-\cos^2\theta}\,(-\sin\theta d\theta)$		
	$=\int_{\frac{\pi}{2}}^0 -\sin^2\theta d\theta$	1	
	$=\frac{1}{2}\int_{\frac{\pi}{2}}^0 (\cos 2\theta-1)d\theta$		
	$=\frac{1}{2}\left[\frac{1}{2}\sin 2\theta-\theta\right]_{\frac{\pi}{2}}^0$	1	
	$=\frac{1}{2}\left[(0)-\left(0-\frac{\pi}{2}\right)\right]$		
	$=\frac{\pi}{4}$	1	
			Total = 4
(c)	$I=\int e^x \sin x dx$		
	$=e^x\sin x-\int e^x\cos x dx$	1	
	$=e^x\sin x-(e^x\cos x-\int e^x[-\sin x]dx)$	1	
	$=e^x\sin x-e^x\cos x-I$		
	$2I=e^x(\sin x-\cos x)+C$	1	
	$\therefore I=\frac{e^x}{2}(\sin x-\cos x)+C$	1	Total = 4

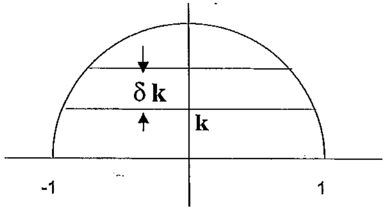
Question 1	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(d)	$\int_0^{\frac{\pi}{2}} \cos x\sin^3 x\, dx=\left[\frac{1}{4}\sin^4 x\right]_0^{\frac{\pi}{2}}$	1	May be done by a substitution of $u=\sin x$
	$=\frac{1}{4}\left[\sin^4\left(\frac{\pi}{2}\right)-\sin^4(0)\right]$	1	$du=\cos x\, dx$
	$=\frac{1}{4}[1-0]$		
	$=\frac{1}{4}$	1	Total = 3

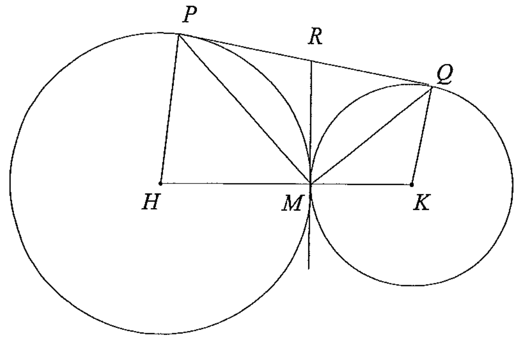
Question 2	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(a) (i)	$AB=(3-4i)(\sqrt{3}+i)$ $=3\sqrt{3}+4+(3-4\sqrt{3})i$	1	
(ii)	$\frac{A}{B}=\frac{(3-4i)}{(\sqrt{3}+i)}\times\frac{(\sqrt{3}-i)}{(\sqrt{3}-i)}$ $=\frac{3\sqrt{3}-4}{4}+\frac{(-4\sqrt{3}-3)i}{4}$	1	
(iii)	$x+iy=\sqrt{3-4i}\,(x+y\,real)$ $\therefore x^2-y^2+2xyi=3-4i$ $\therefore x^2-y^2=3.........(\alpha)$ $2xy=-4...........(\beta)$ Squaring $x^4-2x^2y^2+y^4=9, 4x^2y^2=16$ $x^4+2x^2y^2+y^4=25$ $(x^2+y^2)^2=25$ $x^2+y^2=5..........(\gamma)$ Adding $(\alpha+\gamma)2x^2=8$ $x=\pm 2$ $\therefore y=\mp 1$ $\therefore \sqrt{A}=\pm(2-i)$	1 1 1	Other methods okay Total = 3
(iv)	$B=\sqrt{3}+i$ $=2(\frac{\sqrt{3}}{2}+\frac{1}{2}i)$ $\|B\|=2 \quad ArgB=\frac{\pi}{6}$ $=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$	1 for mod 1 for arg	Total = 2

Question 2	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(v)	$B^4=2^4(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6})$ $=16(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$ $=-8+8\sqrt{3}i$	1 1	Total = 2
(b) (i)		1	
(ii)		1	
(iii)		1	
(c) (i)	$OA=2(-y+ix)$	1	
(ii)	$OB=OA+AB$ $=-2y+2ix+x+iy$ $=(x-2y)+(2x+y)i$	1	
(iii)	$BC=-OA$ $=2y-2xi$	1	

Question 3	Trial HSC Examination- Mathematics Extension 2		2009
Part	**Solution**	**Marks**	**Comment**
(a)	$u_n = 5^n - 2^n$ $u_1 = 5 - 2 = 3$ $\therefore$ True for $n = 1$	1	
	Assume this is true for $n = k$ i.e. $u_k = 5^k - 2^k$ For $u_{k+1} = 7u_k - 10u_{k-1}$ $= 7(5^k - 2^k) - 10(5^{k-1} - 2^{k-1})$ $= 7.5^k - 7.2^k - 2.5^k + 5.2^k$ $= 5.5^k - 2.2^k$ $= 5^{k+1} - 2^{k+1} \quad k \geq 2$	2	
	$\therefore$ If true for $n = k$ also true for $n = k + 1$ Hence by induction true for positive integers n $\therefore u_n = 5^n - 2^n$ *for* $n \geq 1$	1	Total = 4

Part	Solution	Marks	Comment
(b) (i)			
	Consider parabola $x^2 = 4ay$ Length of latus rectum $= 4a$ *units* Required area is area of rectangle *ABCD* minus area under curve between $x = -2a$ and $x = 2a$	1	
	Area $= 4a.a - \int_{-2a}^{2a} \frac{x^2}{4a} dx$	1	
	$= 4a^2 - 2\left[\frac{x^3}{12a}\right]_0^{2a}$ $= 4a^2 - 2\left[\frac{8a^3}{12a} - 0\right]$ $= 4a^2 - \frac{4a^2}{3}$	1	Total = 3
	Area $= \frac{8a^2}{3}$ unit^2		

(ii)	Let semicircle be as shown with equation $x^2+y^2=1 \quad ; y\geq 0$  Let slice be made at $y=k$ with thickness δk When $y=k$, $x=\pm\sqrt{1-k^2}$ $\therefore$ Length of latus rectum $=2\sqrt{1-k^2}$ $4a=2\sqrt{1-k^2}$ $a=\dfrac{\sqrt{1-k^2}}{2}$ From (i) Area of Section $=\dfrac{8}{3}\left(\dfrac{\sqrt{1-k^2}}{2}\right)^2$ $=\dfrac{8(1-k^2)}{12}$ $=\dfrac{2(1-k^2)}{3}$ Volume of slice $=\dfrac{2(1-k^2)}{3}\delta \text{k}$ Volume of solid $=\dfrac{2}{3}\int_0^1(1-k^2)\,dk$ $=\dfrac{2}{3}\left[k-\dfrac{k^3}{3}\right]_0^1$ $=\dfrac{2}{3}\left[\left(1-\dfrac{1}{3}\right)-(0)\right]$ $=\dfrac{4}{9}m^3$	1 (at a) 1 (at area $\frac{2(1-k^2)}{3}$) 1 (at integration) 1 (at substitution)	Students do not need to use k Total = 4

(c) (i)	 In $HPRM$ $\angle HPR=\angle RMH=90^O$ (Angle between radius and tangent) $\therefore HPRM$ is a cyclic quadrilateral (Opposite angles supplementary) Similarly for $MRQK$	1 1	Total = 2
(ii)	Join PM and QM. in ΔPRM and MQK $\angle PRM=\angle MKQ$(Exterior angle of cyclic quad $MRQK$) $PR=RM$ (Tangent from external point) $KM=KQ$(radii) $\therefore$ Triangle isoseles. $\therefore \angle RPM=\angle RMP=\angle KMQ$ $\therefore \Delta PRM\ /// \ \Delta MQK$(Equiangular)	1 1	Total = 2

Question 4	Trial HSC Examination- Mathematics Extension 2	2009	
Part	Solution	Marks	Comment
(a)	100a+10b+c=99a+9b+a+b+c	1	
	=9(11a+b)+3d		
	=3(3[11a+b]+d)		
	Which is divisible by 3.	1	Total = 2
(b) (i)	From $x^3+3px+q=0$		
	$\alpha+\beta+\gamma=0,\ \alpha\beta+\alpha+\beta\gamma+=3p\ \ \alpha\beta\gamma=-q$		
	$\therefore \frac{\beta\gamma}{\alpha}+\frac{\alpha\gamma}{\beta}+\frac{\alpha\beta}{\gamma}=\frac{(\beta\gamma)^2+(\alpha\gamma)^2+(\alpha\beta)^2}{\alpha\beta\gamma}$		
	$=\frac{(\beta\gamma+\alpha\gamma+\alpha\beta)^2-2(\alpha\beta\gamma^2+\alpha\beta^2\gamma+\alpha^2\beta\gamma)}{\alpha\beta\gamma}$	1	
	$=\frac{(\beta\gamma+\alpha\gamma+\alpha\beta)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{\alpha\beta\gamma}$		
	$=\frac{(3p)^2+2q(0)}{-q}=-\frac{9p^2}{q}$		
	$\frac{\beta\gamma}{\alpha}.\frac{\alpha\gamma}{\beta}+\frac{\alpha\gamma}{\beta}.\frac{\alpha\beta}{\gamma}+\frac{\beta\gamma}{\alpha}.\frac{\alpha\beta}{\gamma}=\gamma^2+\alpha^2+\beta^2$		
	$=(\gamma+\alpha+\beta)^2-2(\alpha\beta+\alpha\gamma+\beta\gamma)$	1	
	$=0-2.3p$		
	$=-6p$		
	$\frac{\beta\gamma}{\alpha}.\frac{\alpha\gamma}{\beta}.\frac{\alpha\beta}{\gamma}=\alpha\beta\gamma$	1	
	$=-q$	1	
	$\therefore$ Required equation is $x^3+\frac{9p^2}{q}x^2-6px+q=0$		Total = 4

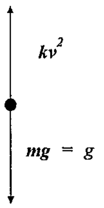
Question 4	Trial HSC Examination- Mathematics Extension 2	2009	
Part	Solution	Marks	Comment
(b) (ii)	For $\gamma=\alpha\beta$		
	$\frac{\alpha\beta}{\gamma}=1$ is a root	1	
	$\therefore 1+\frac{9p^2}{q}-6p+q=0$		
	$q+9p^2-6pq+q^2=0$		
	$\therefore (3p-q)^2+q=0$	1	Total = 2
(c)	Let $P(x)=x^5+2x^2+ax+b$		
	If $(x+1)^2$ is a factor $P(-1)=P'(-1)=0$		
	$\therefore P(-1)=-1+2-a+b=0$	1	
	$\therefore -a+b=-1$		
	$P'(x)=5x^4+4x+a$	1	
	$P'(-1)=5-4+a=0$		
	$a=-1$		
	$\therefore b=-2$	1	Total = 3
(d)	Let $Z=cis\theta$		
	$Z^5=cis5\theta$		
	$\therefore \cos5\theta+i\sin5\theta=0$		
	$\theta=0,\frac{2\pi}{5},\frac{4\pi}{5},\frac{6\pi}{5},\frac{8\pi}{5}$		
	i.e. $=0,\frac{2\pi}{5},\frac{4\pi}{5}\frac{-4\pi}{5},\frac{-2\pi}{5}$	1	
	$\therefore$ Roots are $Z_1=1,\quad Z_2=cis\frac{2\pi}{5},\quad Z_3=cis\frac{4\pi}{5}$		
	$Z_4=cis(-\frac{4\pi}{5})\quad Z_5=cis(-\frac{2\pi}{5})$		Total = 2
	$=\overline{Z}_3\quad =\overline{Z}_2$	1	

Question 4	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(e) ii)	z^5-1 $=(z-z_1)(z-z_2)(z-z_5)(z-z_3)(z-z_4)$ $=(z-z_1)(z^2-(z_2+z_2z_5)(z^2-(z_3+z_4)z+z_3z_4)$ $=(z-1)(z^2-2z\cos\frac{2\pi}{5}+1)(z^2-2z\cos\frac{4\pi}{5}+1)$	 1 1	Total = 2

Question 5	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(a) (i)	$\angle BCD=\alpha+\theta$(Exterior $\angle$ of $\Delta\, ABC$) $\tan\alpha=\frac{x}{8}$ $\tan(\alpha+\theta)=\frac{x}{3}$ $\therefore\frac{x}{3}=\frac{\tan\alpha+\tan\theta}{1-\tan\alpha\tan\theta}$ $x-x\tan\alpha\tan\theta=3\tan\alpha+3\tan\theta$ $x-x.\frac{x}{8}\tan\theta=\frac{3x}{8}+3\tan\theta$ $\therefore(\frac{x^2}{8}+3)\tan\theta=\frac{5x}{8}$ $\therefore\tan\theta=\frac{5x}{8}\times\frac{8}{x^2+24}$ $\tan\theta=\frac{5x}{x^2+24}$ Differentiate both sides $\sec^2\theta d\theta=\frac{(x^2+24)5-5x.2x}{(x^2+24)^2}dx$ $\frac{d\theta}{dx}=\frac{120-5x^2}{(x^2+24)^2}.\cos^2$ For maximum θ, $\frac{d\theta}{dx}=0$ $\theta\pm90^0\ \therefore120-5x^2=0$ $x^2=24\quad x=\sqrt{24}=2\sqrt{6}$ $\therefore x=2\sqrt{6}$ cm gives the maximumvalue of θ *Test* $\quad x \quad \sqrt{6} \quad 2\sqrt{6} \quad 3\sqrt{6}$ $\quad\frac{d\theta}{dx}\quad + \quad 0 \quad -$ $\therefore$ max	 1 1 1 1	Any method Total = 4

Question 5	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(a) (ii)	$\tan\theta = \dfrac{5\times\sqrt{24}}{24+24}$ $\theta = 27°2'$ is maximum value.	1	
(b) (i)	Let centre for rotation be S Let $PS = x$ $\therefore SQ = 5 - x$ $\therefore x^2 + r^2 = 16$ $(5-x)^2 + r^2 = 9$ $x^2 - 25 + 10x - x^2 = 7$ $10x = 32$ $x = 3.2$ $\therefore (3.2)^2 + r^2 = 16$ $\therefore r = 2.4$	1 1	Total = 2
(b) (ii)	Let tension in $PR = T_1(N)$ and in $QR = T_2(N)$ Let $\angle QPR = \alpha$ and $\angle PQR = \beta$ Resolving forces at R Vertically $T_1\cos\alpha = T_2\cos\beta + g$ $\therefore T_1\cos\alpha - T_2\cos\beta = g$ i.e. $\frac{4}{5}T_1 - \frac{3}{5}T_2 = 10$ ________________(1) Horizontally $T_1\sin\alpha + T\sin\beta = mrw^2$ i.e. $\frac{3}{5}T_1 + \frac{4}{5}T_2 = 2.4(3\pi)^2$ ____________(2) Solving simultaneously gives $T_1 = 135.9$ Newtons(1dp) and $T_2 = 164.5$ Newtons(1dp)	1 1 1 1	Total = 4

Question 5	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(b) iii)	In (1) above when $T_2 = 0$ $\frac{4}{5}T_1 = 10$ $T_1 = 12.5$ $\therefore$ Substitute in (2) $\frac{3}{5}\times 12.5 = 2.4w^2$ $\therefore w^2 = 3.125$ $w = 1.77\ (2dp)$ $\therefore T_2 = 0$ when angular velocity 1.77 rad s^{-1}	1 1	Total = 2
(c)	$x^2 + 3xy - y^2 = 13$ $2x + 3y + 3x.\frac{dy}{dx} - 2y.\frac{dy}{dx} = 0$ $\frac{dy}{dx}(3x-2y) = -(2x+3y)$ $\frac{dy}{dx} = \frac{-(2x+3y)}{(3x-2y)}$ $\therefore$ at $(2,3)$ $\quad \frac{dy}{dx} = -\frac{(4+9)}{6-6}$ $\therefore$ Gradient infinite $\therefore$ Tangent is vertical	1 1	Total = 2

Question 6	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(a) (i)		1	
(a) (ii)	$\ddot{x} = g - kv^2$	1	
(a) (iii)	$When\ \ddot{x} = 0$ $g = kv^2$ $\therefore v = \sqrt{\frac{g}{k}}$	1	
(a) (iv)	$\ddot{x}\frac{d}{dx}(\frac{1}{2}v^2) = g - kv^2$ $= \frac{d}{dv}(\frac{1}{2}v^2).\frac{dv}{dx} = g - kv^2$ $= v.\frac{dv}{dx} = g - kv^2$ $\therefore \frac{dv}{dx} = \frac{g - kv^2}{v}$ $\frac{dx}{dv} = \frac{v}{g - kv^2}$ $\therefore x = -\frac{1}{2k}\ln(g - kv^2) + c$ $When\ x = 0\ v = 0$ $\therefore \quad c = \frac{1}{2k}\ln g$ $\therefore \quad x = -\frac{1}{2k}\ln(g - kv^2) + \frac{1}{2k}\ln g$ $= \frac{1}{2k}\ln(\frac{g}{g - kv^2})$	1 1 1 1	Total = 4

Question 6	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(b) (i)	$I_n = \int \sec^n x dx$ $= \int \sec^{n-2} x.\sec^2 x dx$ $= \tan x.\sec^{n-2} x - \int (n-2)\sec^{n-3} x.\tan x.\sec x \tan x dx$ $= \tan x \sec^{n-2} x - (n-2)\int \sec^{n-2} x.\tan^2 x dx$ $= \tan x \sec^{n-2} x - (n-2)\int \sec^{n-2} x.(\sec^2 x - 1)dx$ $= \tan x \sec^{n-2} x - (n-2)\int (\sec^n x - \sec^{n-2} x)dx$ $= \tan x \sec^{n-2} x - (n-2)I_n + (n-2)I_{n-2}$ $I_n + (n-2)I_n = (n-1)I_n = \tan x \sec^{n-2} x + (n-2)I_{n-2}$ $I_n = \frac{1}{n-1}\tan x \sec^{n-2} x + \frac{n-2}{n-1}I_{n-2}$	1 1 1 1	Total = 4
(b) (ii)	$\int_0^{\frac{\pi}{4}} \sec^4 x dx = I_4 = \left[\frac{1}{3}\tan x \sec^2 x + \frac{2}{3}\int \sec^2 x\right]_0^{\frac{\pi}{4}}$ $= \left[\frac{1}{3}\tan x \sec^2 x + \frac{2}{3}\tan x\right]_0^{\frac{\pi}{4}}$ $= \left(\frac{1}{3}\tan\frac{\pi}{4}\sec^2\frac{\pi}{4} + \frac{2}{3}\tan\frac{\pi}{4}\right) - \left(\frac{1}{3}\tan 0 \sec^2 0 + \frac{2}{3}\tan 0\right)$ $= \left(\frac{1}{3}\times 1\times 2 + \frac{2}{3}\times 1\right) - 0$ $= \frac{4}{3}$	1 1	Total = 2

Question 6	Trial HSC Examination- Mathematics Extension 2	2009	
Part	Solution	Marks	Comment
(c)	If one root is $2+i$, another is $2-i$ $\therefore (z-(2+i))(z-(2-i))(z-3)=0$ $(z-2-i)(z-2+i)(z-3)=0$ $(z^2-4z+5)(z-3)=0$ $z^3-7z^2+17z-15=0$	 1 1	Total = 2

Question 7	Trial HSC Examination- Mathematics Extension 2	2009	
Part	Solution	Marks	Comment
(a) (i)	Coordinates of T are $(a\cos\theta, a\sin\theta)$ $x^2+y^2=a^2$ $\therefore 2x+2y\dfrac{dy}{dx}=0$ $\therefore \dfrac{dy}{dx}=-\dfrac{x}{y}$ at T $\dfrac{dy}{dx}=-\dfrac{\cos\theta}{\sin\theta}$ $\therefore$ Equation TM $y-a\sin\theta=-\dfrac{\cos\theta}{\sin\theta}(x-a\cos\theta)$ $x\cos\theta+y\sin\theta=a$ *When* $y=0$ $x=a\sec\theta$ $\therefore$ Coordinates M are $(a\sec\theta, 0)$	 1 1 1	Total = 3
(a) (ii)	On $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ when $x=a\sec\theta$ $a^2\dfrac{\sec^2\theta}{a^2}-\dfrac{y^2}{b^2}=1$ $\dfrac{y^2}{b^2}=\sec^2\theta-1$ $\dfrac{y^2}{b^2}=\tan^2\theta$ $\therefore y=b\tan\theta$ Coordinates of P are $(a\sec\theta, b\tan\theta)$	 1	

(a) iii)	$a\sec\beta = a\sec(\frac{\pi}{2}-\theta)$ $= a\cos ec$ $b\tan\beta = b\tan(\frac{\pi}{2}-\theta)$ $= b\cot\theta$	1	
	Gradient $PQ = \frac{b\cot\theta - b\tan\theta}{a\cos ec\theta - a\sec\theta}$ $= \frac{b}{a}\left\{\frac{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{1}{\sin\theta}-\frac{1}{\cos\theta}}\right\}$ $= \frac{b}{a}\left\{\frac{\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}}{\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}}\right\}$ $= \frac{b}{a}\left(\frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}\right)$ $= \frac{b}{a}(\cos\theta+\sin\theta)$	1	
	$\therefore$ Equation PQ is $y - b\tan\theta = \frac{b}{a}(\cos\theta+\sin\theta)(x - a\sec\theta)$ $y - b\tan\theta = \frac{b}{a}(\cos\theta+\sin\theta)x - b\cos\theta\sec\theta - b\sin\theta\sec\theta$ $y - b\tan\theta = \frac{b}{a}(\cos\theta+\sin\theta)x - b - b\tan\theta$ $y = \frac{b}{a}(\cos\theta+\sin\theta)x - b$ $ay = b(\cos\theta+\sin\theta)x - ab$	1	Total = 3
(a) (iv)	All of the lines have the same y intercept. i.e. $y = -b$	1	
	$\therefore$ The fixed point is the intercept $(0,-b)$	1	Total = 2

(a) (v)	Equations of Asymptotes are $y = \pm\frac{b}{a}x$ Gradients of asymptotes are $m = \pm\frac{b}{a}$	1	
	As $\theta \to \frac{\pi}{2}$ the equation of PQ $\to ay = b(0+1)x - ab$ $\therefore$ Gradient of PQ $\to \frac{b}{a}$ $\therefore PQ$ approaches a line which is parallel to an asymptote.	1	Total = 2
(b) i)	$z = \cos\theta + i\sin\theta$ $\frac{1}{z} = z^{-1} = \cos\theta - i\sin\theta$ By De Moivres Theorem $z^n = \cos n\theta + i\sin n\theta$ $z^{-n} = \cos n\theta - i\sin n\theta$ $z^n + z^{-n} = \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$ $z^n + \frac{1}{z^n} = 2\cos n\theta$	1	
ii)	$\left(z+\frac{1}{z}\right)^4 = z^4 + 4z^3\frac{1}{z} + 6z^2\frac{1}{z^2} + 4z\frac{1}{z^3} + \frac{1}{z^4}$ $= \left(z^4+\frac{1}{z^4}\right) + 4\left(z^2+\frac{1}{z^2}\right) + 6$	1	
	$(2\cos\theta)^4 = 2\cos 4\theta + 4(2\cos 2\theta) + 6$ $2^4\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$	1	
	$\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$	1	Total = 3

Question 8	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(a) (i)	Equation of trajectory is $y = x\tan\theta - \frac{gx^2}{2v^2}.\sec^2\theta$ Passes through (b,h) and (c,h) $\therefore \quad h = b\tan\theta - \frac{gb^2}{2v^2}\sec^2\theta$ ________(1) and $h = c\tan\theta - \frac{gc^2}{2v^2}\sec^2\theta$ ________(2) $\therefore b\tan\theta - \frac{gb^2}{2v^2}\sec^2\theta = c\tan\theta - \frac{gc^2}{2v^2}\sec^2\theta$ $(b-c)\tan\theta = (b^2-c^2)\frac{g}{2v^2}\sec^2\theta$ $\tan\theta = (b+c)\frac{g}{2v^2}\sec^2\theta$ $v^2 = \frac{(b+c)g\sec^2\theta}{2\tan\theta}$	1 1 1	Total = 3
(a) (ii)	Substitute result into (1) $h = b\tan\theta - \frac{gb^2\sec^2\theta}{2}\frac{2\tan\theta}{(b+c)g\sec^2\theta}$ $h = b\tan\theta - \frac{b^2\tan\theta}{b+c}$ $h(b+c) = (b^2+bc)\tan\theta - b^2\tan\theta$ $= bc\tan\theta$ $\therefore \tan\theta = \frac{h(b+c)}{bc}$	1 1	Total = 2

Question 8	Trial HSC Examination- Mathematics Extension 2	2009	
Part	**Solution**	**Marks**	**Comment**
(a) (iii)	Greatest height at $x = \frac{b+c}{2}$ $\therefore y = \left(\frac{b+c}{2}\right)\tan\theta - \left(\frac{b+c}{2}\right)^2 . \frac{g\sec^2\theta.2\tan\theta}{2(b+c)g\sec^2\theta}$ $y = \left(\frac{b+c}{2}\right)\tan\theta - \left(\frac{b+c}{4}\right)\tan\theta$ $y = \left(\frac{b+c}{4}\right)\tan\theta$ $y = \left(\frac{b+c}{4}\right)\frac{h(b+c)}{bc}$ $y = \frac{h(b+c)^2}{4bc}$	1 1	Total = 2
(b) (i)	No current if failure of A and (B or C or B and C) $\therefore \text{P}(Failure) = p\left[p(1-p)+(1-p)p+p.p\right]$ $= p\left(p-p^2+p-p^2+p^2\right)$ $= p\left(2p-p^2\right)$ $= p^2(2-p)$	1 1 1	Total = 3
(b) (ii)	Let $q = p^2(2-p)$ $P(Failure) = q^2(2-q)$ $= \left[p^2(2-p)\right]^2\left[2-p^2(2-p)\right]$ $= p^4(2-p)^2\left(p^3-2p^2+2\right)$	1 1	Total = 2
(c) (i)	$\frac{9!}{2!\times 2!} = 90\,720$	1	
(c) (ii)	$\frac{7!}{2!} = 2\,520$	1	
(c) (iii)	$\frac{6!}{2!}\times\frac{4!}{2!} = 4\,320$	1	