WESTERN REGION

2008
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions
- Reading Time – 5 minutes
- Working Time – 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120
- Attempt Questions 1 – 10
- All questions are of equal value

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Total Marks – 120
Attempt Questions 1 – 10
All questions are of equal value

Marks

Begin each question on a SEPARATE sheet of paper. Extra paper is available.

Question 1 (12 marks) Use a SEPARATE sheet of paper or booklet.

a) Evaluate $\left(\frac{1}{e^{2.5}}-1\right)^2$ correct to 3 significant figures. **2**

b) Solve $|2x-4| \le 2$ **2**

c) If $\frac{4}{2-\sqrt{3}} = a+b\sqrt{3}$ find the values of a and b. **2**

d) Find the sum of the first ten terms of the series $4\frac{1}{2} + 3 + 1\frac{1}{2} + \ldots$ **2**

e) Factorise $2z^2 + 6zy + xz + 3xy$ **2**

f) Find the perpendicular distance from the point (1, 3) to the line $6x-8y+5=0$ **2**

End of Question 1

Question 2 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) Differentiate with respect to x

(i) $2x^3 + x^{-3}$ **2**

(ii) $\frac{1}{e^{2x}} - \sin x$ **2**

b) (i) Find $\int \sec^2 x - e^{4x} \, dx$ **2**

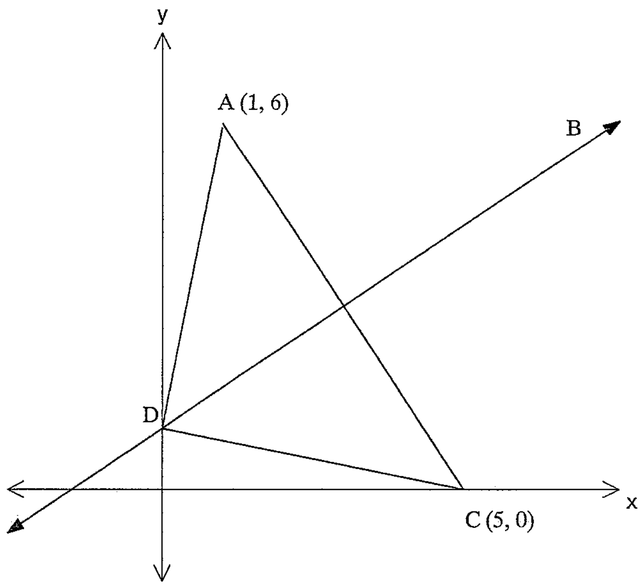
(ii) Evaluate $\int_1^e x^2 + \frac{2}{x} dx$ **3**

c) Find the area enclosed by the curve $y = 2\cos 3x$, the line $x = \frac{\pi}{12}$ and the x and y axes. **3**

End of Question 2

Question 3 (12 marks) Use a SEPARATE writing booklet. **Marks**

a)



The points A and C have coordinates (1, 6) and (5, 0) respectively.
The line BD has an equation of $2x-3y+3=0$ and meets the y axis in D.

i) The point M is the midpoint of AC. Show that M has coordinates (3, 3). **1**

ii) Show that M lies on BD. **1**

iii) Find the gradient of the line AC. **1**

iv) Show that BD is perpendicular to AC. **2**

v) Find the distance AC. **1**

vi) Explain why the quadrilateral ABCD is a kite regardless of the position of B. **1**

Question 3 continues on page 5

Question 3 continued **Marks**

b) Michael is training for a local marathon. He has trained by completing practice runs over the marathon course. So far he has completed three practice runs with times shown below.

Week 1	Week 2	Week 3
3 hours	2 hours 51 minutes	2 hours 42 minutes 27 seconds

i) Show that these times form a geometric series with a common ratio $r = 0.95$. **1**

ii) If this series continues, what would be his expected time in Week 5, to the nearest second? **1**

iii) How many hours, minutes and seconds (to the nearest second) will he have run in total in his practice runs in these 5 weeks? **1**

iv) If the previous winning time for the marathon was 2 hours and 6 minutes, how many weeks must he keep practicing to be able to run the marathon in less that the previous winning time? **2**

End of Question 3

Question 4 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) Show that: **2**

$$\sqrt{\frac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = \tan x$$

b) Two dice are painted so that the first has four blue and two red faces and the second has one blue and five red faces. The two dice are rolled together.

i) What is the probability that the dice both show blue faces uppermost? **1**

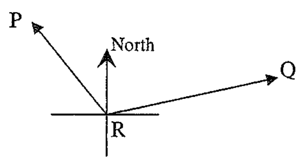
ii) What is the probability that different colours show uppermost? **2**

c) Ally and Bella are standing on level ground on opposite sides of a tower which is 142 metres high. Ally is due west and measures the angle of elevation of the top of the tower as 16°. Bella is due east and measures the angle of elevation of the top of the tower as 20°.

i) Draw a diagram to illustrate this information. **1**

ii) Calculate the distance between Ally and Bella. **2**

d) Peta and Quentin are pilots of two light planes which leave Resthaven station at the same time. Peta flies on a bearing of 330° at a speed of 180 km/h and Quentin flies on a bearing of 080° at a speed of 240 km/h. Copy the diagram below onto your answer page and mark the information on the diagram.



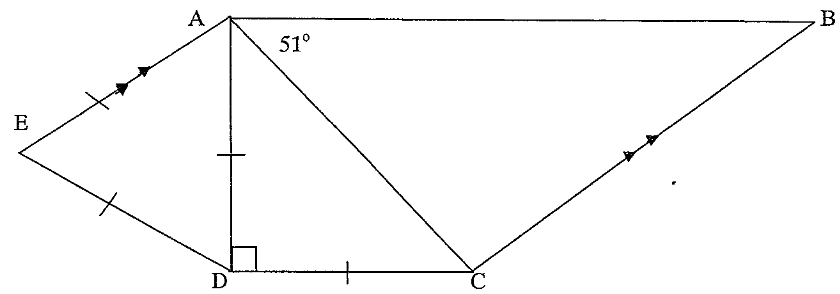
i) How far apart are Peta and Quentin after 2 hours? **2**

ii) What is the bearing of Quentin from Peta after 2 hours. **2**

End of Question 4

Question 5 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) In the diagram below AE = ED = AD = DC, $\angle ADC = 90°$ and AE || BC. $\angle BAC = 51°$



i) Find the size of $\angle EAB$. Give reasons for your answer. **3**

ii) Find the size of $\angle ABC$. Give reasons for your answer. **1**

b) A particle moves in a straight line so that its displacement, in metres, is given by

$x = \dfrac{4t^2 + t + 8}{4t + 1}$ where t is measured in seconds.

i) Find the initial position of the particle. **1**

ii) Find an expression for the velocity of the particle. **1**

iii) Show that the particle is stationary when $t = \dfrac{-1 + 4\sqrt{2}}{4}$ seconds. **2**

iv) Describe the motion of the particle in the first two seconds. **2**

c) Solve the pair of simultaneous equations **2**

$$3x - y = 10$$
$$x = y + 2$$

End of Question 5

Question 6 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) For the function $y = x^6 - 6x^4$

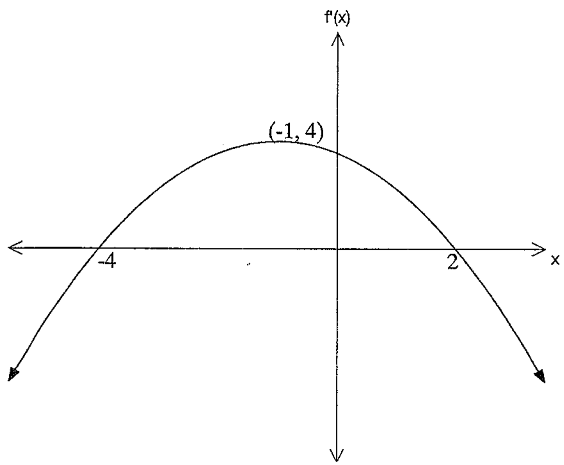
i) Find the x coordinates of the points where the curve crosses the axes. **2**

ii) Find the coordinates of the stationary points and determine their nature. **4**

iii) Find the coordinates of the points of inflexion. **2**

iv) Sketch the graph of $y = x^6 - 6x^4$ indicating clearly the intercepts, stationary points and points of inflexion. **2**

b) For a certain function $y = f(x)$, the sketch of $y = f'(x)$ is shown.



Give the x coordinates of the stationary points on $y = f(x)$ and indicate if they are maxima or minima. **2**

End of Question 6

Question 7 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) For the parabola with equation $x^2 = -8y$.

i) Find the coordinates of the focus (S) of the parabola. **1**

ii) Find the equation of the directrix of the parabola. **1**

iii) Show that the point A(-8, -8) lies on the parabola. **1**

iv) Find the equation of the focal chord of the parabola which passes through A. **2**

v) Find the equation of the tangent to the parabola at A. **2**

b) i) Show that the curves $y = x^2 - 3x$ and $y = 5x - x^2$ intersect at the points (0, 0) and (4, 4). **2**

ii) Find the area enclosed between the two curves. **3**

End of Question 7

Question 8 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) A city has a population which is growing at a rate that is proportional to the current population. The population at time t years is given by
$P = Ae^{kt}$

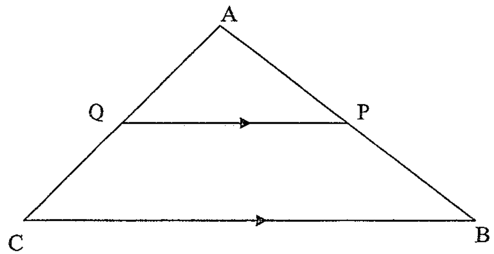
i) Show that $P = Ae^{kt}$ satisfies the equation $\frac{dP}{dt} = kP$. **1**

ii) If the population at the start of 2006 when $t = 1$ was 147 200 and at the start of 2007 when $t = 2$ was 154 800, find the values of A and k. **2**

iii) Find the population at the start of 2009. **1**

iv) Find during which year the population will first exceed 200 000. **1**

b) In the diagram below, P is the midpoint of the side AB of the ΔABC. PQ is drawn parallel to BC.



i) Prove that ΔABC ||| ΔAPQ. **2**

ii) Explain why Q is the midpoint of AC. **2**

c) Find an approximation for $\int_1^3 g(x)\,dx$ by using Simpson's Rule with the values in the table below. **2**

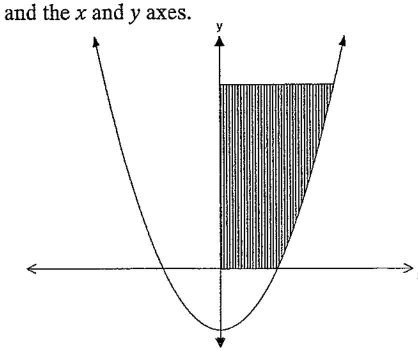
x	1	1.5	2	2.5	3
$g(x)$	12	8	0	3	5

d) Evaluate $\sum_{n=2}^{5} n^2 - 1$ **1**

End of Question 8

Question 9 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) The diagram shows the region bounded by the curve $y = 2x^2 - 2$ the line $y = 6$ and the x and y axes. **3**



Find the volume of the solid of revolution formed when the region is rotated about the y axis.

b) Paul plays computer games competitively. From past experience, Paul has a 0.8 chance of winning a game of *Beastie* and a 0.6 chance of winning a game of *Dragonfire*. In one afternoon of competition he plays two games of *Beastie* and one of *Dragonfire.*

i) What is the probability that he will win all three games? **1**

ii) What is the probability that he will win no games? **1**

iii) What is the probability that he will win at least one game? **1**

c) A car dealership has a car for sale for a cash price of \$20 000. It can also be bought on terms over three years. The first six months are interest free and after that interest is charged at the rate of 1% per month on that months balance. Repayments are to be made in equal monthly instalments beginning at the end of the first month.

A customer buys the car on these terms and agrees to monthly repayments of \$$M$. Let \$$A_n$ be the amount owing at the end of the nth month.

i) Find an expression for A_6. **1**

ii) Show that $A_8 = (20\,000 - 6M)1{\cdot}01^2 - M(1 + 1{\cdot}01)$ **1**

iii) Find an expression for A_{36}. **2**

iv) Find the value of M. **2**

End of Question 9

Question 10 (12 marks) Use a SEPARATE writing booklet. **Marks**

a) A plant nursery has a watering system which repeatedly fills a storage tank then empties it's contents to water different sections of the nursery.
The volume of water (in cubic metres) in the tank at a time t is given by the equation

$$V = 2 - \sqrt{3}\cos t - \sin t \text{ where t is measured in minutes.}$$

i) Give an equation for $\dfrac{dV}{dt}$, the rate of change of the volume at a time t. **1**

ii) Is the tank initially filling or emptying? **1**

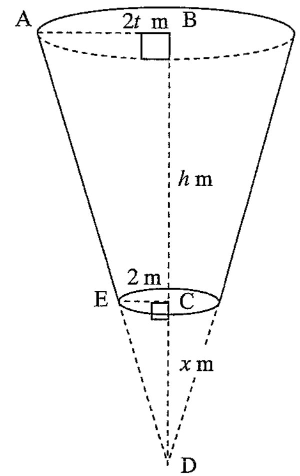
iii) At what time does the tank first become completely full and what is it's capacity when full? **3**

Question 10 continues on page 13

Question 10 continued

Marks

b) A truncated cone is to be used as a part of a hopper for a grain harvester. It has a total height of h metres. The top radius is to be t times greater than the bottom radius which is 2 metres.



AB = $2t$ metres
BC = h metres
EC = 2 metres
CD = x metres

i) If x is the height of the removed section of the original cone, show using similar triangles that $x = \dfrac{h}{t-1}$ **2**

ii) Show that the volume of the truncated cone is given by **2**
$$V = \left(\frac{4\pi h}{3}\right)\left(t^2 + t + 1\right)$$

iii) If the upper radius plus the lower radius plus the height of the truncated cone must total 12 metres, calculate the maximum volume of the hopper. **3**

End of Examination

WESTERN REGION

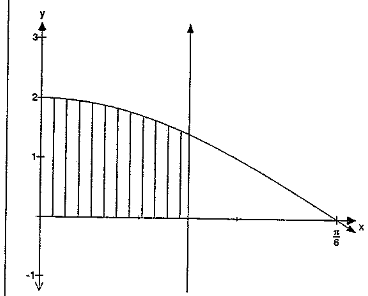
2008
TRIAL HSC
EXAMINATION

Mathematics

SOLUTIONS

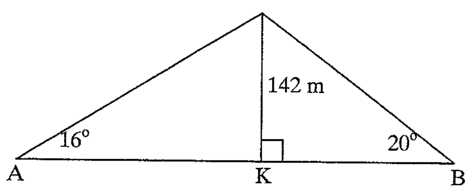
Question 1	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(a)	$\left(\frac{1}{e^{2.5}}-1\right)^2=0.84256=0.843$ (3 sig fig)	2	1 for answer 1 for rounding
(b)	$\lvert 2x-4\rvert\le 2$ $-2\le 2x-4\le 2$ $2\le 2x\le 6$ $1\le x\le 3$	2	1 for each part of the inequality
(c)	$\frac{4}{2-\sqrt{3}}=a+b\sqrt{3}$ $\frac{4}{2-\sqrt{3}}=\frac{4}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$ $=\frac{8+4\sqrt{3}}{4-3}$ $a+b\sqrt{3}=8+4\sqrt{3}$ $a=8$ and $b=4$	2	1 for correct method 1 for values of a and b
(d)	$4\frac{1}{2}+3+1\frac{1}{2}+\ldots$ Series is arithmetic with $a=4\frac{1}{2}$ and $d=-1\frac{1}{2}$ $S_n=\frac{n}{2}(2a+(n-1)d)$ $S_{10}=\frac{10}{2}(9+(9)\left(1\frac{1}{2}\right))$ $=5\left(22\frac{1}{2}\right)$ $=112\frac{1}{2}$	2	1 for formula 1 for answer
(e)	$2z^2+6zy+xz+3xy=2z(z+3y)+x(z+3y)$ $=(2z+x)(z+3y)$	2	1 for partial factorisation or simple mistake leading to incorrect factorisation 2 for correct factorisation.

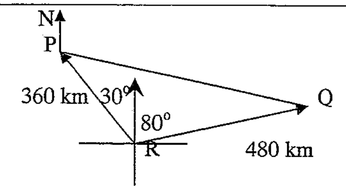
Question 1	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(f)	$d=\left\lvert\frac{6(1)-8(3)+5}{\sqrt{6^2+(-8)^2}}\right\rvert$ $=\left\lvert\frac{-13}{\sqrt{100}}\right\rvert$ $=1.3$	2	1 for use of formula 1 for answer

Question 2	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(a) i)	$\frac{d}{dx}\left(2x^3+x^{-3}\right)=6x^2-3x^{-4}$	2	1 for each part of derivative.
ii)	$\frac{d}{dx}\left(\frac{1}{e^{2x}}-\sin x\right)=\frac{d}{dx}\left(e^{-2x}-\sin x\right)$ $=-2e^{-2x}-\cos x$	2	1 for each part of derivative.
(b) i)	$\int \sec^2 x-e^{4x}\ dx=\tan x-\frac{e^{4x}}{4}+c$	2	1 for each part of integral.
ii)	$\int_1^e x^2+\frac{2}{x}dx=\left[\frac{x^3}{3}+2\ln x\right]_1^e$ $=\frac{e^3}{3}+2\ln e-\frac{1}{3}-2\ln 1$ $=\frac{e^3}{3}+2-\frac{1}{3}-0$ $=\frac{e^3-1}{3}+2$	3	1 for each part of the integral. 1 for substitution.
(c)	$y=2\cos 3x$  $Area=\int_0^{\frac{\pi}{12}}2\cos 3x\ dx$ $=\left[\frac{2\sin 3x}{3}\right]_0^{\frac{\pi}{12}}$ $=\frac{2\sin\frac{\pi}{4}}{3}-\frac{2\sin 0}{3}$ $=\frac{2}{3\sqrt{2}}-0$ $=\frac{\sqrt{2}}{3}$ square units.	3	1 for using correct integral including units. 1 for integration 1 for evaluation

Question 3	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(a) i)	Midpoint of (1, 6) and (5, 0). $MP=\left(\frac{1+5}{2},\frac{6+0}{2}\right)=\left(\frac{6}{2},\frac{6}{2}\right)=(3,3)$	1	1 for answer
ii)	Show that (3,3) lies on $2x-3y+3=0$ $LHS=2(3)-3(3)+3$ $=6-9+3$ $=0=RHS$ So M lies on BD.	1	1 for answer
iii)	Gradient $AC=m_1=\frac{6-0}{1-5}=\frac{6}{-4}=-\frac{3}{2}$	1	1 for answer
iv)	Find gradient m_2 of BD $2x-3y+3=0$ $2x-3y+3=0$ $3y=2x+3$ $y=\frac{2}{3}x+1$ $\therefore m_2=\frac{2}{3}$ $m_1.m_2=-\frac{3}{2}.\frac{2}{3}=-1$ $\therefore$ BD is perpendicular to AC	2	1 for gradient of BD 1 for showing product of gradients = -1
v)	$AC=\sqrt{(5-1)^2+(0-6)^2}$ $=\sqrt{16+36}$ $=\sqrt{52}$ $=2\sqrt{13}$	1	1 for answer
(vi	The lines AC and BD would form the diagonals of the quadrilateral ABCD. BD is the perpendicular bisector of AC from ii and iv above.. The diagonals of a kite meet at right angles and one diagonal bisects the other, so ABCD meets the criteria for a kite.	1	Need to mention perpendicular (or meet at right angles) and midpoint (or bisect) for point for mark.

Question 3	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(b) i)	$\frac{\text{2 hours 51 minutes}}{\text{3 hours}}=\frac{2.85}{3}=0.95$ $\frac{\text{2 hours 42 minutes 27 seconds}}{\text{2 hours 51 minutes}}=\frac{2.7075}{2.85}=0.95$ $\therefore$ forms a geometric series with $r=0.95$	1	1 for answer
ii)	$a=3, n=5$ and $r=0.95$ $u_5=ar^{n-1}$ $=3\times0.95^4$ $=2.4435$ $=2$ hours 26 min 37 sec	1	1 for answer
iii)	$S_n=\frac{a(1-r^n)}{(1-r)}$ $=\frac{3(1-0.95^5)}{(1-0.95)}$ $=13.573$ $=13$ hours 34 min 23 sec	1	1 for answer
iv)	$u_n=ar^{n-1}$ 2 hours 6 min $=3\times0.95^{n-1}$ $3\times0.95^{n-1}=2.1$ $0.95^{n-1}=2.1\div3=0.7$ $\ln(0.95^{n-1})=\ln(0.7)$ $(n-1)\ln(0.95)=\ln(0.7)$ $n-1=\frac{\ln(0.7)}{\ln(0.95)}$ $n-1=6.953$ $n=7.953$ Would need to continue for 8 weeks to better the time.	2	1 for equation to be solved 1 for answer either using logs as shown or by guess and check.

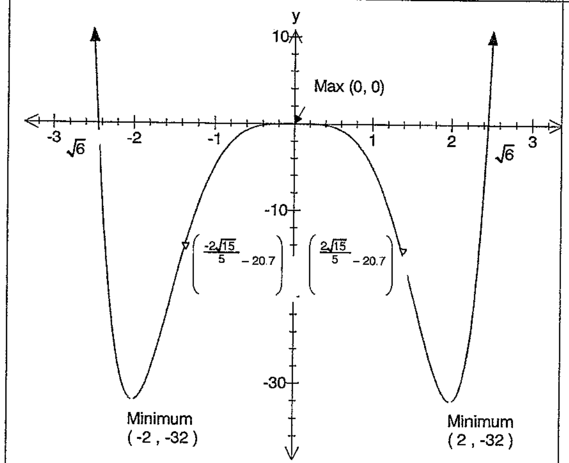
Question 4	Trial HSC Examination- Mathematics	2008	
Part	**Solution**	**Marks**	**Comment**
(a)	$\sqrt{\dfrac{\operatorname{cosec}^2 x - \cot^2 x - \cos^2 x}{\cos^2 x}} = \sqrt{\dfrac{1-\cos^2 x}{\cos^2 x}}$ $= \sqrt{\dfrac{\sin^2 x}{\cos^2 x}}$ $= \dfrac{\sin x}{\cos x}$ $= \tan x$	2	2 marks for required result. 1 mark for partial work toward required result.
(b) i)	P(Blue on first die) $= \frac{4}{6} = \frac{2}{3}$ P(Blue on 2nd die) $= \frac{1}{6}$ P(2 Blue showing) $= \frac{2}{3} \times \frac{1}{6} = \frac{1}{9}$	1	1 mark for answer.
(b) ii)	P(Different)= P(BR)+ P(RB) $= \frac{2}{3} \times \frac{5}{6} + \frac{1}{3} \times \frac{1}{6}$ $= \frac{5}{9} + \frac{1}{18}$ $= \frac{11}{18}$ OR =1-P(same) =1-(P(RR)+PBB)) $=1-\left(\frac{5}{6} \times \frac{1}{3} + \frac{2}{3} \times \frac{1}{6}\right)$ $=1-\frac{7}{18}$ $= \frac{11}{18}$	2	2 for correct answer. 1 mark if correct strategy used, but mistake made in the process.
(c) i)		1	1 mark if diagram shows angles and distances correctly.
(c) ii)	$\tan 16° = \frac{142}{AK}$ $\tan 20° = \frac{142}{KB}$ $AK = \frac{142}{\tan 16°}$ $KB = \frac{142}{\tan 20°}$ $= 495.2$ $= 390.1$ Distance AB = 495.2 + 390.1 = 885 m (nearest m)	2	1 mark for answer to each part of the required distance.

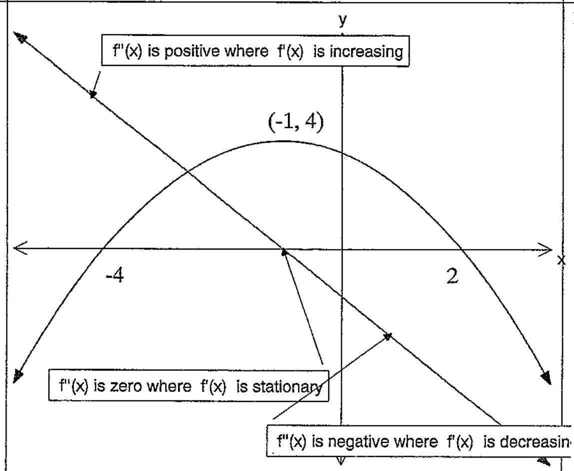
Question 4	Trial HSC Examination- Mathematics	2008	
Part	**Solution**	**Marks**	**Comment**
(d) i)	 $PQ^2 = 360^2 + 480^2 - 2 \times 360 \times 480 \cos 110°$ $PQ^2 = 478202$ $PQ = 692$ km (nearest km)	2	2 for complete answer . 1 if started using the cos rule correctly but simple mistake made.
(d) ii)	First find $\angle QPR$ $\frac{\sin \angle QPR}{480} = \frac{\sin 110°}{692}$ $\sin \angle QPR = \frac{480 \times \sin 110°}{692}$ $\sin \angle QPR = 0.652$ $\angle QPR = 41°$ $\angle NPR = 150°$ $Bearing(\angle NPQ) = 150° - 41°$ $= 109°$ Can also be found using cos rule using the 3 sides..	2	2 for complete answer . 1 if started using the sin or cos rule correctly but simple mistake made.

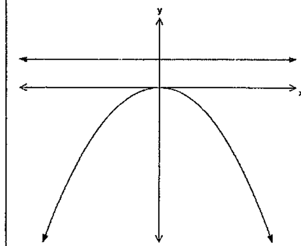
Question 5	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(a) i)	$\angle EAD = 60^\circ$ (equilateral Δ) $\angle DAC = 45^\circ$ (isosceles right Δ) $\therefore \angle EAB = \angle EAD + \angle DAC + \angle CAB$ $= 60^\circ + 45^\circ + 51^\circ$ $= 156^\circ$	3	1 for equilateral ? 1 for isosceles ? 1 for answer
ii)	$\angle ABC = 180^\circ - 156^\circ$ (cointerior $\angle$ on $\parallel$ lines AE and BC) $= 24^\circ$	1	1 for answer
(b) i)	$x = \dfrac{4t^2+t+8}{4t+1}$ $x = \dfrac{4(0)^2+(0)+8}{4(0)+1}$ when t = 0 $= 8$	2	1 for answer 1 for working
(b) ii)	$x = \dfrac{4t^2+t+8}{4t+1}$ $\dot{x} = \dfrac{(4t+1)(8t+1)-(4t^2+t+8)(4)}{(4t+1)^2}$ $= \dfrac{32t^2+12t+1-16t^2-4t-32}{(4t+1)^2}$ $= \dfrac{16t^2+8t-31}{(4t+1)^2}$	1	1 for answer

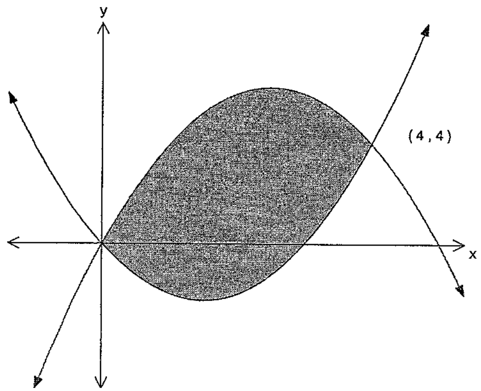
Question 5	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(b) iii)	$\dot{x} = 0$ $\dfrac{16t^2+8t-31}{(4t+1)^2} = 0$ $16t^2+8t-31 = 0$ $t = \dfrac{-8 \pm \sqrt{8^2-4(16)(-31)}}{2(16)}$ $= \dfrac{-8 \pm \sqrt{2048}}{32}$ $= \dfrac{-8 \pm 32\sqrt{2}}{32}$ $= \dfrac{-1 \pm 4\sqrt{2}}{4}$ So it is stationary when t $= \dfrac{-1 \pm 4\sqrt{2}}{4}$ Only us positive value so t $= \dfrac{-1+4\sqrt{2}}{4}$	2	1 for quadratic 1 for solution
(b) iv)	$t = \dfrac{-1+4\sqrt{2}}{4} \approx 1.2$sec $v = 0, x \approx 2.6$ When $t = 0$ $x = 8$ and $v = -31$ When $t = 2$ $x = 2.9$ and $v = 0.6$ Particle starts 8 units to the right of the origin, moving toward the origin, it decelerates and stops after 1.2 sec at 2.6 m to right of origin, then begins to move away from the origin, being 2.9 units to the right of the origin after 2 sec.	2	1 for mention of direction of travel before and after turning. 1 for mention of position at at least one point.
(c)	$3x - y = 10$ (1) $x = y + 2$ (2) $3(y+2) - y = 10$ (3) sub (2) in (1) $3y + 6 - y = 10$ $2y = 4$ $y = 2$ $x = (2) + 2$ sub y in (2) $x = 4$ solution (4, 2)	2	1 for substitution (or elimination) 1 for answer

Question 6	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(a) (i)	$y=x^6-6x^4$ Crosses axis where $x^6-6x^4=0$ $x^4\left(x^2-6\right)=0$ $x^4\left(x-\sqrt{6}\right)\left(x+\sqrt{6}\right)=0$ Crosses axis where $x=0$ and $x=\pm\sqrt{6}$	2	2 if all given. 1 if only one or two given
(a) (ii)	$y=x^6-6x^4$ $y'=6x^5-24x^3$ $=6x^3\left(x^2-4\right)$ $=6x^3(x-2)(x+2)$ $y''=30x^4-72x^2$ Stationary points where $x=0, y=0, y''=0$ $x=2, y=-32, y''=192$ $x=-2, y=-32, y''=192$ Stationary points $(-2,-32),(0,0)(2,-32)$ $y''=30x^4-72x^2$ At $x=0$ $y''=0$ so test either side At $x=1$ $y''=-42$ $\therefore$ concave down. At $x=-1$ $y''=-42$ $\therefore$ concave down $\therefore$ maximum at (0, 0) At $x=2$ $y''=192$ $\therefore$ minimum at (2, -32). At $x=-2$ $y''=192$ $\therefore$ minimum at (−2 , -32).	4	1 for derivative 1 for solving for x values 1 for y values 1 for nature

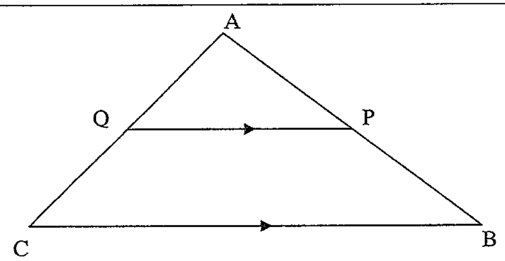
Question 6	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(a) (iii)	$y''=30x^4-72x^2$ $=6x^2\left(5x^2-12\right)$ $=6x^2\left(\sqrt{5}x-2\sqrt{3}\right)\left(\sqrt{5}x+2\sqrt{3}\right)$ $x=0 \quad y=0$ $x=\frac{2\sqrt{3}}{\sqrt{5}}=\frac{2\sqrt{15}}{5} \quad y=-20.736$ $x=-\frac{2\sqrt{3}}{\sqrt{5}}=-\frac{2\sqrt{15}}{5} \quad y=-20.736$ Check for changes of concavity From above, no change at (0, 0) but there is a change at $\left(\pm\frac{2\sqrt{15}}{5}, -20.736\right)$ Inflexions at $\left(\pm\frac{2\sqrt{15}}{5}, -20.736\right)$	2	1 for Coordinates 1 for testing to determine only 2 inflexions
		2	1 for shape of curve. 1 for points shown correctly.

Question 6	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(b)	 Stationary points on y occur where $f'(x)=0$ i.e. at $x=-4$ here $f''(x)$ is positive $\therefore$ min turning point at $x=-4$ and $x=2$ here $f''(x)$ is negative $\therefore$ max turning point at $x=2$	2	1 mark for x coordinates and 1 mark for nature of both.

Question 7	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(a) i)	$x^2=-8y$ $x^2=-4(2)y$ $a=2$  Focus is (0, -2)	1	1 for answer
ii)	Directrix is $y=2$	1	1 for answer
iii)	$x^2=-8y$ $x^2=(-8)^2=64$ $-8y=-8(-8)=64$ $\therefore(-8,-8)$ lies on the parabola.	1	1 for sub
iv)	Chord through (0, -2) and (-8, -8) $m=\frac{-8+2}{-8-0}=\frac{-6}{-8}=\frac{3}{4}$ $y-y_1=m(x-x_1)$ $y-(-2)=\frac{3}{4}(x-0)$ $y=\frac{3}{4}x-2$ $3x-4y-8=0$	2	1 for correct gradient 1 for equation
v)	$x^2=-8y$ $y=-\frac{x^2}{8}$ $y'=-\frac{x}{4}$ At A $y'=-\frac{-8}{4}=2$ $y-(-8)=2(x--8)$ $y+8=2x+16$ $y=2x+8$	2	1 for derivative 1 for equation

(b) i)	Substitute $y=x^2-3x$ into $y=5x-x^2$ $5x-x^2=x^2-3x$ $2x^2-8x=0$ $2x(x-4)=0$ $x=0,\quad y=0$ $x=4,\quad y=4$ Intersect at (0, 0) and (4, 4).	2	1 for x substitution 1 for points
ii)	 $Area=\int_0^4 5x-x^2dx-\int_0^4 x^2-3xdx$ $=\int_0^4 8x-2x^2\ dx$ $=\left[4x^2-\frac{2x^3}{3}\right]_0^4$ $=\left(64-\frac{128}{3}\right)-0$ $=\frac{64}{3}=21\frac{1}{3}u^2$	3	1 for correct integral stated 1 for integration 1 for sub and correct answer

Question 8	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
(a) i)	$P=Ae^{kt}$ $\frac{dP}{dt}=Ae^{kt}.k$ $=kAe^{kt}$ $=kP$	1	1 for answer
ii)	$t=1$ was 147 200 $P=Ae^{kt}$ $147200=Ae^{k}$ (i) $t=2$ was 154 800 $154800=Ae^{2k}$ (ii) $\frac{154800}{147200}=\frac{Ae^{2k}}{Ae^{k}}$ (ii) ÷ (i) $1.0516=e^{k}$ $k=\ln(1.0516)$ $k\approx 0.05$ $147200=Ae^{0.05(1)}$ $A=\frac{147200}{e^{0.05}}=139973$	2	1 for k 1 for A
iii)	When $t=4$ $P=Ae^{kt}$ $P=139973e^{0.05(4)}$ $=171197$	1	1 for answer
iv)	$P=Ae^{kt}$ $200000=139973e^{0.05t}$ $\frac{200000}{139973}=e^{0.05t}$ $1.429=e^{0.05t}$ $\ln(1.429)=\ln\left(e^{0.05t}\right)$ $0.05t=\ln(1.429)$ $t=\frac{\ln(1.429)}{0.05}$ $=7.1$ $t=7$ is start of 2012 Population will reach 200 000 in 2012	1	1 for answer

Question 8	Trial HSC Examination- Mathematics		2008
Part	Solution	Marks	Comment
(b) i)	 In ΔAPQ and ΔABC ∠A is common ∠AQP=∠ACB (Corresp ∠ on ‖ lines) ∠APQ= ∠ABC (Corresp ∠ on ‖ lines) ∴ ΔAPQ ||| ΔABC (Corresponding angles equal)	2	1 each for 2 pairs of equal angles and reasons
ii)	$\frac{AP}{AB}=\frac{1}{2}$ (P is midpoint of AB) $\frac{AP}{AB}=\frac{AQ}{AC}$ (sides of similar triangle in same ratio) $\frac{AQ}{AC}=\frac{1}{2}$ (from above) ∴ Q is midpoint of AC.	2	2 for any reasonable explanation using ratio of correspond sides 1 for partial explanation
(c)	$\int_1^3 g(x)\,dx \approx \frac{1}{6}\{12+4(8)+2(0)+4(3)+5\}$ $\approx \frac{61}{6}$ $\approx 10\frac{1}{6}$	2	1 for sub in formula correctly 1 evaluate correctly
(d)	$\sum_{n=2}^{5} n^2-1=(2^2-1)+(3^2-1)+(4^2-1)+(5^2-1)$ $=3+8+15+24$ $=50$	1	1 for answer

Question 9	Trial HSC Examination- Mathematics		2008
Part	Solution	Marks	Comment
(a)	$y=2x^2-2$ $V=\pi\int_0^6 x^2\,dy$ $=\pi\int_0^6 \frac{y+2}{2}\,dy$ $=\pi\left[\frac{y^2}{4}+y\right]_0^6$ $=\pi\left[\left(\frac{36}{4}+6\right)-(0)\right]$ $=15\pi\ \text{u}^3$	3	1 for formula used correctly 1 for integration 1 for evaluation
(b) i)	$P(WWW)=0.8\times0.8\times0.6=0.384$	1	1 answer
ii)	$P(LLL)=0.2\times0.2\times0.4=0.016$	1	1 answer
iii)	$P(\text{at least 1 win})=1-P(LLL)$ $=1-0.016$ $=0.984$	1	1 answer
(c) i)	$A_6=20000-6M$	1	1 answer
ii)	$A_7=(20000-6M)1.01-M$ $A_8=[(20000-6M)1.01-M]1.01-M$ $=(20000-6M)1.01^2-1.01M-M$ $=(20000-6M)1.01^2-M(1+1.01)$	1	1 answer
iii)	$A_9=(20000-6M)1.01^3-M(1+1.01+1.01^2)$ $A_n=(20000-6M)1.01^{n-6}-M(1+1.01+1.01^2+.......1.01^{n-7})$ $A_{36}=(20000-6M)1.01^{30}-M(1+1.01+1.01^2+.......1.01^{29})$	2	1 for developing series further 1 for result for A_{36}

Question 9	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
iv)	Since repaid after 36 months $A_{36}=0$ $(20000-6M)1.01^{30}-M(1+1.01+1.01^2+.......1.01^{29})=0$ $M(1+1.01+1.01^2+.......1.01^{29})=(20000-6M)1.01^{30}$ Need to evaluate $1+1.01+1.01^2+.......1.01^{29}$ Geometric series with $a=1,\ r=1.01,\ n=30$ $S_{30}=\frac{a(r^n-1)}{r-1}$ $=\frac{1(1.01^{30}-1)}{1.01-1}$ $=34.785$ $34.785M=(20000-6M)1.01^{30}$ $\frac{34.785M}{1.01^{30}}=20000-6M$ $6M+\frac{34.785M}{1.01^{30}}=20000$ $M\left(6+\frac{34.785}{1.01^{30}}\right)=20000$ $31.8M=20000$ $M=\frac{20000}{31.8}$ $=\$629$ (nearest dollar)	2	

Question 10	Trial HSC Examination- Mathematics	2008	
Part	Solution	Marks	Comment
a) i)	$V=2-\sqrt{3}\cos t-\sin t$ $\frac{dV}{dt}=\sqrt{3}\sin t-\cos t$	1	1 for answer
ii)	When $t=0$ $\frac{dV}{dt}=\sqrt{3}\sin 0-\cos 0$ $=-1$ $\therefore$ the tank is emptying at this time.	1	1 for answer or for correct value of derivative.
iii)	Full (or empty) when $\frac{dV}{dt}=0$ $\frac{dV}{dt}=0$ $\sqrt{3}\sin t-\cos t=0$ $\sqrt{3}\sin t=\cos t$ $\frac{\sin t}{\cos t}=\frac{1}{\sqrt{3}}$ $\tan t=\frac{1}{\sqrt{3}}$ $=\frac{\pi}{6},\frac{7\pi}{6},\frac{13\pi}{6}...$ As tank is initially emptying, second value corresponds to when full. Tank is first full when $t=\frac{7\pi}{6}$ $V=2-\sqrt{3}\cos t-\sin t$ $=2-\sqrt{3}\cos\frac{7\pi}{6}-\sin\frac{7\pi}{6}$ $=4$	3	2 if final value of *t* is given. 1 if equation formed correctly but not solved correctly 1 if an incorrect equation is formed and is solved correctly. 1 for finding volume from value of t