WESTERN REGION

2009
TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes.
- Working Time - 3 hours.
- Write using a blue or black pen.
- Approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**120**)

- Attempt Questions 1-10.
- All questions are of equal value.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Express $3\cdot5\dot{3}\dot{1}$ as a fraction in simplest form. **2**

(b) If $\tan\theta = \frac{7}{8}$ and $\cos\theta < 0$, find the exact value of $\operatorname{cosec}\theta$ **1**

(c) Evaluate $\dfrac{3.24^2 - 2.1^2}{\sqrt{36 + 2.1}}$ correct to 3 significant figures. **1**

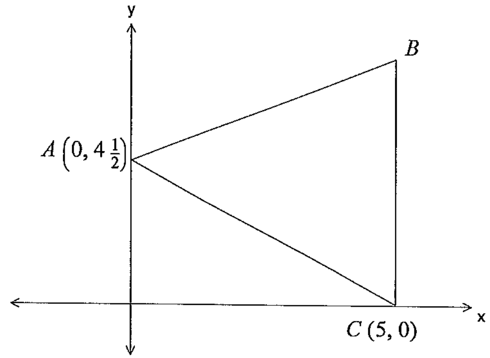
(d) Solve $|15 - 4x| \le 3$ **2**

(e) If $k = \frac{1}{3}m\left(v^2 - u^2\right)$ find the value of m when $k = 724$, $v = 14\cdot2$ and $u = 7\cdot4$. **2**

(f) Find the period and amplitude for the graph of $3y = \sin\left(2x - \frac{\pi}{4}\right)$. **2**

(g) Paint at the local hardware store is sold at a profit of 30% on the cost price. If a drum of paint is sold for \$67·50, find the cost price. **2**

Question 2 **(12 Marks)** Use a Separate Sheet of paper **Marks**



The lines AB and CB have equations $x - 2y + 9 = 0$ and $4x - y - 20 = 0$ respectively.

(a) Find the coordinates of the point B. **2**

(b) Show that the equation of the line AC is $9x + 10y - 45 = 0$. **2**

(c) Calculate the distance AC in exact form. **2**

(d) Find the equation of the line perpendicular to BC which passes passes through A. **2**

(e) Calculate the shortest distance between the point B and the line AC. Hence find the area of the triangle ABC. **2**

(f) State the inequalities that together define the area bounded by the triangle ABC. **2**

Question 3 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Differentiate with respect to x.

i. $3x\sqrt[3]{x}$ **2**

ii. $\dfrac{\sin 2x}{e^{2x}}$ **2**

(b) Find:

i. $\displaystyle\int \frac{dx}{e^{3x}}$ **2**

ii. $\displaystyle\int_0^{\pi} \sec^2 \frac{x}{4}\, dx$. **2**

(c) If α and β are the roots of the equation $3x^2 - 4x - 7 = 0$
Find:

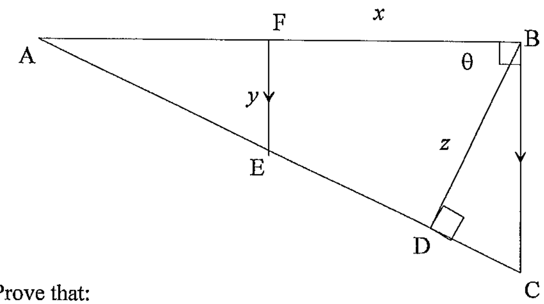
i. $\alpha + \beta$. **1**

ii. $2\alpha^2 + 2\beta^2$. **1**

iii. the equation with roots $2\alpha^2$ and $2\beta^2$. **2**

Question 4 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) The right triangle ABC is shown below. BC || FE, BD ⊥ AC, ∠FBD = θ, BF = x, EF = y and BD = z.



Prove that:

i. ∠FEA = θ **2**

ii. AF = $y \tan\theta$ **1**

iii. $z = (x + y\tan\theta)\cos\theta$ **1**

iv. $z = x\cos\theta + y\sin\theta$ **1**

(b) The federal government distributes $500 million in order to stimulate the economy. Each recipient spends 80% of the money that he or she receives. In turn, the secondary recipient spends 80% of the money that they receive, and so on. What was the total spending that results from the original $500 million into the economy? **2**

(c) A ship sails from port A, 60 nautical miles due west, to a port B. It then proceeds a distance of 50 nautical miles on a bearing of 210° to a port C.

i. Draw a diagram to illustrate the information given. **1**

ii. Find the distance (nearest nautical mile) and bearing of C from A. **4**

Question 5 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) In a raffle in which 1000 tickets are sold, there is a first prize of \$1000, a second prize of \$500 and a third prize of \$200. The prize winning tickets are drawn consecutively without replacement, with the first ticket winning first prize.

Find the probability that:

i. a person buying one ticket in the raffle wins:

α. first prize. **1**

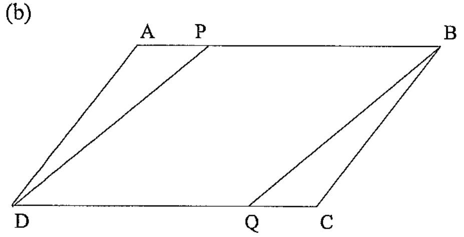
β. at least \$500 **1**

γ. no prizes. **1**

ii. a person buying two tickets in the raffle wins:

α. at least \$500 **1**

(b) **3**



ABCD is a parallelogram, BP = DQ.

Prove DP = BQ

(c) i. Is the series $\log 3 + \log 9 + \log 27 + \ldots\ldots$ arithmetic or geometric? Give reasons for your answer. **2**

iii. Find the sum of the first 10 terms of the series. **1**

(d) Find the radius and centre of the circle with equation **2**

$$4x^2 - 4x + 4y^2 + 24y + 21 = 0$$

Question 6 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) A curve has a gradient function with equation $\frac{dy}{dx} = 6(x-1)(x-2)$.

i. If the curve passes through the point (1, 2), what is the equation of the curve? **2**

ii. Find the coordinates of the stationary points and determine their nature. **2**

iii. Find any points of inflexion. **2**

iv. Graph the function showing all the main features. **2**

(b) Show that $\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta} = \tan\theta$ **3**

(c) Evaluate $\lim_{\theta \to 0} \frac{\sin 2\theta}{3\theta}$ **1**

Question 7 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) The parabola $y = x^2$ and the line $y = x + 2$ intersect at points A and B respectively. Find the coordinates of the points A and B. Hence find the area bounded by the parabola and the line. **4**

(b) The minute hand on a clock face is 12 centimetres long.
In 40 minutes

i. Through what angle does the hand move (in radians)? **1**

ii. How far does the tip of the hand move? **1**

iii. What area does the hand sweep through in this time? **1**

(c) Use Simpson's rule to evaluate $\int_1^{2.5} f(x)\,dx$, to 1 decimal place using the 7 function values in the table below. **2**

x	1·00	1·25	1·50	1·75	2·00	2·25	2·50
$f(x)$	3·43	2·17	0·38	1·87	2·65	2·31	1·97

(d) A function is defined by the following features: **3**

$$\frac{d^2y}{dx^2} > 0 \text{ for } x < -1 \text{ and } 1 < x < 3.$$

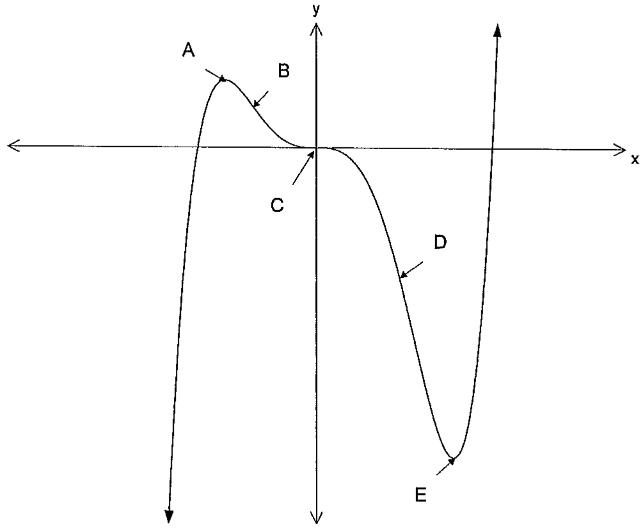
$$\frac{dy}{dx} = 0 \text{ when } x = -3, 1 \text{ and } 5.$$

$$y = 0 \text{ when } x = 1.$$

Sketch a possible graph of the function.

Question 8 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) The graph of the curve $y = f(x)$ is drawn below.



i. Name the points of inflexion. **1**

ii. When is the graph decreasing? **1**

iii. Sketch the gradient function. **1**

(b) Steve borrows \$15 000 for a new car. He decides to repay the loan plus interest at 6% pa compounded monthly. He repays the loan in monthly installments of \$P.

i. Show that after three months the amount that Steve owes is \$[15226·13 – P(3·015025)]. **2**

ii. After two years of repaying his loan, Steve still owes \$10 000 on the loan. What was the monthly repayment? **3**

(c) Sketch the graph of the parabola $2x = y^2 - 8y + 4$, showing the vertex, focus and the directrix. **4**

Question 9 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) A particle moves in a straight line so that its displacement (in m) from a fixed point O at time t seconds is given by $x = 2\sin 2t,\ 0 \le t \le 2\pi$.

Find:

i. The initial velocity **1**

ii. The acceleration after $\frac{\pi}{12}$ seconds. **1**

iii. When the particle is at rest. **2**

iv. The displacement of the particle when it is at rest. **2**

(b) The area bounded by the curve $y = \sqrt{\frac{2x}{3x^2 - 1}}$ between the lines $x = 1$ and $x = 3$ is rotated about the x-axis. Find the volume of the solid of revolution formed. **3**

(c) The rate at which Carbon Dioxide will be produced when conducting an experiment is given by $\frac{dV}{dt} = \frac{1}{100}\left(30t - t^2\right)$ where V cm^3 is the volume of gas produced after t minutes.

i. At what rate is the gas being produced 15 minutes after the experiment begins. **1**

ii. How much Carbon Dioxide has been produced during this time? **2**

Question 10 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) An open cylindrical can is made from a sheet of metal with an area of 300cm^2.

i. Show that the volume of the can is given by $V = 150r - \frac{1}{2}\pi r^3$. **2**

ii. Find the radius of the cylinder that gives the maximum volume and calculate this volume. **4**

(b) The population of a certain town grows at a rate proportional to the population. If the population grows from 20 000 to 25 000 in two years, find:

i. The population of the town, to the nearest hundred, after a further 8 years. **3**

ii. Calculate the rate of change at this time. **1**

(c) If $\log_a 2 + 2\log_a x - \log_a 6 = \log_a 3$ find the value of x. **2**

End of Examination.

Western Region

2009

TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

Mathematics

Solutions

Solutions	Marks/Comments

Question 1

(a) Let $x = 0\cdot53131..$

$10x = 5\cdot3131$

$100x = 53\cdot1313$

$1000x = 531\cdot3131$

$990x = 526$

$x = \frac{526}{990} = \frac{263}{495}$

$\therefore 3\cdot5\dot{3}\dot{1} = 3\frac{263}{495}$

or $3\cdot5\dot{3}\dot{1} = 5 + \frac{5}{10} + \frac{31}{1000} + \frac{31}{100000}....$

Limiting Sum $\frac{31}{1000} + \frac{31}{100000} + \frac{31}{10000000}....$

$a = \frac{31}{1000}$ $\quad S_\infty = \frac{a}{1-r}$

$r = \frac{1}{100}$ $\quad = \frac{\frac{31}{1000}}{1 - \frac{1}{100}}$

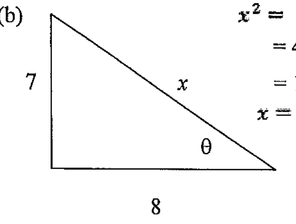
$= \frac{31}{1000} \div \frac{99}{100}$

$= \frac{31}{990}$

$\therefore 3\cdot5\dot{3}\dot{1} = 3 + \frac{5}{10} + \frac{31}{990} = 3\frac{263}{495}$

2 marks – 1 for correct method
1 correct answer

(b)



$x^2 = 7^2 + 8^2$

$= 49 + 64$

$= 113$

$x = \sqrt{113}$

Since $\tan\theta = \frac{7}{8}$ and $\cos\theta < 0$

3^{rd} Quadrant $\therefore \operatorname{cosec}\theta < 0$

$\therefore \operatorname{cosec}\theta = -\frac{\sqrt{113}}{7}$

1 Mark – Correct Answer

(c) $\frac{3\cdot24^2 - 2\cdot1^2}{\sqrt{36 + 2\cdot1}} = 0\cdot986242288$

$= 0\cdot986$

1 Mark – Correct rounded answer

(d) $|15 - 4x| \le 3$

$15 - 4x \le 3$ or $15 - 4x \ge -3$

$-4x \le -12$ $\quad -4x \ge -18$

$x \ge 3$ or $\quad x \le 4\frac{1}{2}$

2 Marks – 1 for each solution

(e) $k = \frac{1}{3}m(v^2 - u^2)$

$724 = \frac{1}{3}m(14\cdot2^2 - 7.4^2)$

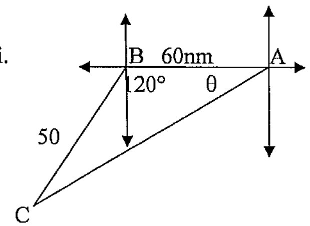
2 Marks – 1 for substitution
- 1 for answer

$2172 = m(146\cdot88)$ $m = \frac{2172}{146\cdot88} = 14\cdot7875817$ $= 14\cdot8$ (3sf)	
(f) $3y = \sin\left(2x - \frac{\pi}{4}\right)$ $y = \frac{1}{3}\sin\left(2x - \frac{\pi}{4}\right)$ $\therefore$ amplitude $= \frac{1}{3}$ period $= \frac{2\pi}{2} = \pi$	**2 marks - 1 for period** **1 for amplitude**
(g) $130\% = \$67\cdot50$ $1\% = \frac{67\cdot50}{130} = 0\cdot51923\ldots\ldots$ Cost Price $= \frac{67\cdot50}{130} \times 100 = \$51\cdot92$	**2 Marks - 1 for 1%** **1 for Cost price**

Solutions	Marks/Comments
Question 2 (a) $x - 2y + 9 = 0$ -----(1) $4x - y - 20 = 0$ -----(2) From (2) $y = 4x - 20$ ------(3) Sub (3) into (1) B is the point (7, 8) $x - 2(4x - 20) + 9 = 0$ $x - 8x + 40 + 9 = 0$ $-7x = -49$ $x = 7$ Hence $y = 8$	**2 marks – 1 for method** **1 for correct answer**
(b) $m(AC) = \frac{4\frac{1}{2} - 0}{0 - 5}$ $= -\frac{9}{10}$ $y - y_1 = m(x - x_1)$ $y - 0 = -\frac{9}{10}(x - 5)$ $10y = -9x + 45$ $9x + 10y - 45 = 0$	**2 marks – 1 for gradient** **1 for equation**
(c) $AC = \sqrt{(0-5)^2 + \left(4\frac{1}{2} - 0\right)^2}$ $= \sqrt{(-5)^2 + \left(4\frac{1}{2}\right)^2}$ $= \sqrt{25 + \frac{81}{4}}$ $= \frac{\sqrt{181}}{2}$	**2 marks – 1 for substitution** **1 for answer**
(d) $m(BC) = \frac{8-0}{7-5} = \frac{8}{2} = 4$ $\therefore m(line) = -\frac{1}{4}$ $y - y_1 = m(x - x_1)$ $y - 4\frac{1}{2} = -\frac{1}{4}(x - 0)$ $4y - 18 = -x$ $x + 4y - 18 = 0$	**2 marks – 1 for gradient of line** **1 for equation**
(e) $d = \frac{\lvert 9(7) + 10(8) - 45\rvert}{\sqrt{9^2 + 10^2}} = \frac{\lvert 63 + 80 - 45\rvert}{\sqrt{81 + 100}} = \frac{98}{\sqrt{181}}$ Area $= \frac{1}{2}bh = \frac{1}{2} \times \frac{\sqrt{181}}{2} \times \frac{98}{\sqrt{181}} = 24\frac{1}{2}$ square units.	**2 marks – 1 for substitution** **1 for answer**
(f) $x - 2y + 9 \geq 0$ $4x - y - 20 \leq 0$ $9x + 10y - 45 \geq 0$	**2 marks - lose 1 mark for each incorrect**

Solutions	Marks/Comments
Question 3	
(a) i. $\frac{d}{dx}\left[3x\sqrt[3]{x}\right] = vu' + uv'$ $= 3 \times x^{\frac{1}{3}} + 3x \times \frac{1}{3}x^{-\frac{2}{3}}$ $= 4x^{\frac{1}{3}}$ $= 4\sqrt[3]{x}$ **OR** By using indices $3x\sqrt[3]{x} = 3x \times x^{\frac{1}{3}}$ $= 3x^{\frac{4}{3}}$ Derivative $= 4\sqrt[3]{x}$	**2 marks – 1 for method** **1 for answer**
ii. $\frac{d}{dx}\left[\frac{\sin 2x}{e^{2x}}\right] = \frac{(e^{2x})(2\cos 2x) - (\sin 2x)(2e^{2x})}{(e^{2x})^2}$ $= \frac{2e^{2x}\left[\cos 2x - \sin 2x\right]}{(e^{2x})^2}$ $= \frac{2\left[\cos 2x - \sin 2x\right]}{e^{2x}}$	**2 marks – 1 for method** **1 for answer**
(b) i. $\int \frac{dx}{e^{3x}} = \int e^{-3x}dx = -\frac{1}{3}e^{-3x} + C$	**2 marks – 1 for method** **1 for answer**
ii. $\int_0^{\pi} \sec^2 \frac{x}{4}dx = 4\left[\tan\frac{x}{4}\right]_0^{\pi}$ $= 4\left[\tan\frac{\pi}{4} - \tan 0\right] = 4$	**2 marks – 1 for integral** **1 for answer**
(c) i. $\alpha + \beta = -\frac{b}{a} = -\frac{-4}{3} = \frac{4}{3}$ $2\alpha^2 + 2\beta^2 = 2\left(\alpha^2 + \beta^2\right)$ $= 2\left[\left(\alpha+\beta\right)^2 - 2\alpha\beta\right]$	**1 mark**
ii. $= 2\left[\left(\frac{4}{3}\right)^2 - 2\left(\frac{-7}{3}\right)\right]$ $= 2\left[\left(\frac{16}{9}\right) + \left(\frac{14}{3}\right)\right]$ $= \frac{116}{9}$	**1 mark**
iii. Equation with roots $2\alpha^2$ *and* $2\beta^2$ has equation $x^2 - (2\alpha^2 + 2\beta^2)x + (2\alpha^2 \times 2\beta^2) = 0$ i.e. $x^2 - 2[(\alpha+\beta)^2 - 2\alpha\beta]x + 4(\alpha\beta)^2 = 0$	**2 marks – 1 for method** **1 for answer**

$x^2 - 2\left[\left(\frac{4}{3}\right)^2 - 2\left(\frac{-7}{3}\right)\right]x + 4\left(\frac{-7}{3}\right)^2 = 0$ $x^2 - 2\left[\frac{16}{9} + \frac{14}{3}\right]x + \frac{196}{9} = 0$ $x^2 - 2\left[\frac{58}{9}\right]x + \frac{196}{9} = 0$ $x^2 - \frac{116}{9}x + \frac{196}{9} = 0$ $9x^2 - 116x + 196 = 0$	

Solutions	Marks/Comments
Question 4	
(a) i. $\angle FBD = \theta$ (Given) $\angle DBC = 90° - \theta$ $\angle BCD = 180° - 90° - (90 - \theta)$ (angle sum of ΔBCD) $= \theta$ $\therefore \angle FEA = \theta$ (Corresponding angles to $\angle BCD$, $FE \parallel BC$)	**2 marks – 1 for proof** **1 for reasons**
ii. $\angle AFE = 90°$ (Corresponding angles $FE \parallel BC$) $\tan\theta = \dfrac{AF}{y}$ $\therefore AF = y\tan\theta$	**1 mark**
iii. In ΔABD, $\cos\theta = \frac{z}{AF+FE}$ $z = (AF + FB)\cos\theta$ $= (x + y\tan\theta)\cos\theta$	**1 mark**
iv. $z = (x + y\tan\theta)\cos\theta$ $= x\cos\theta + y\tan\theta\cos\theta$ $= x\cos\theta + y\frac{\sin\theta}{\cos\theta}\cos\theta$ $= x\cos\theta + y\sin\theta$	**1 mark**
(b) $a = 500\ 000000 \times 0.8$ $r = \frac{4}{5}$ $S_\infty = \frac{a}{1-r}$ $= \frac{500\ 000\ 000 \times 0.8}{1-\frac{4}{5}}$ $= \frac{400\ 000\ 000}{\frac{1}{5}}$ \$2 000 000 000	**2 marks – 1 for substation into formula** **- 1 for answer**
(c) i. 	**1 for correct diagram**
ii. $AC^2 = 50^2 + 60^2 - 2 \times 50 \times 60\cos 120°$ AC = 95·3939......... = 95 nm	**2 marks – 1 for substitution** **- 1 for answer**
$\frac{\sin\theta}{50} = \frac{\sin 120}{95.3939..}$ $\sin\theta = \frac{50\sin 120}{95.3939...}$ $\theta = 27°$ (nearest degree) Bearing $= 270 - 27$ $= 243°$	**2 marks – 1 for 27°** **- 1 for bearing**

Question 5	
(a) i. α. P(Wins first Prize) $= \frac{1}{1000}$	**1 mark**
β. P(At least \$500) $= \frac{2}{1000}$ $or\ 0\cdot002$	**1 mark**
γ. P(no prizes) $= 1 - \frac{3}{1000} = \frac{997}{1000}$ $or\ 0\cdot997$	**1 mark**
ii. P(At least \$500) $= 1 - (\frac{998}{1000} \times \frac{997}{999})$ $= 0\cdot003997997$	**1 mark**
(b) ABCD is a parallelogram and BP = DQ Then AP = AB – PB = DC – DQ = QC In Δ□s APD, BCQ AP = QC (proven above) AD = BC (opposite sides of parallelogram) $\angle PAD = \angle QCB$ (opposites angles of a parallelogram) $\therefore \Delta APD \equiv \Delta BCQ$ (SAS) $\therefore$ DP = BQ (corresponding side in congruent triangles)	**3 marks – 1 for showing AP = QC** **1 for proving triangles congruent** **1 for DP = BQ**
(c) i. $\log 3 + \log 9 + \log 27 + \ldots\ldots$ $\log 3 + \log 3^2 + \log 3^3 + \ldots\ldots$ $\log 3 + 2\log 3 + 3\log 3 + \ldots\ldots$ Series is Arithmetic with a common difference of $\log 3$	**2 marks – 1 for type of series** **1 for reason** **i.e the value of d**
ii. $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{10} = \frac{10}{2}[2(\log_a 3) + (10-1)\log_a 3]$ $= 5[2\log_a 3 + 9\log_a 3]$ $= 5[11\log_a 3]$ $= 55\log_a 3$ $= \log_a 3^{55}$	**1 for S_{10} in either form**
(d) $4x^2 - 4x + 4y^2 + 24y + 21 = 0$ $x^2 - x + y^2 + 6y = -\frac{21}{4}$ $\left(x^2 - x + \frac{1}{4}\right) + (y^2 + 6y + 9) = -\frac{21}{4} + \frac{1}{4} + 9$ $\left(x - \frac{1}{2}\right)^2 + (y+3)^2 = 4$ Centre $\left(\frac{1}{2}, -3\right)$, Radius = 2	**2 marks – 1 for centre** **1 for radius**

Question 6

(a) i. $\frac{dy}{dx} = 6(x-1)(x-2)$ $\qquad \frac{d^2y}{dx^2} = 12x - 18$

$= 6(x^2 - 3x + 2)$

$= 6x^2 - 18x + 12$

$y = \int (6x^2 - 18x + 12)\, dx$

$= 2x^3 - 9x^2 + 12x + C$

When $x = 1$, $y = 2$

$2 = 2(1)^3 - 9(1)^2 + 12(1) + C$

$2 = 2 - 9 + 12 + C$

$0 = 3 + C$

$C = -3$

Equation of curve is $y = 2x^3 - 9x^2 + 12x - 3$

2 marks – 1 for integration
1 for equation with correct value of "c"

ii. $\frac{dy}{dx} = 6(x-1)(x-2)$ but $\frac{dy}{dx} = 0$ for Stationary Points

i.e. $6(x-1)(x-2) = 0$

$x = 1$ or $x = 2$

i.e. (1, 2), (2,1)

At (1, 2), $\frac{d^2y}{dx^2} < 0$ Maximum at (1, 2)

At (2, 1), $\frac{d^2y}{dx^2} > 0$ Mimimum at (2,1)

2 marks – 1 for points
1 for testing points

iii. $\frac{d^2y}{dx^2} = 12x - 18 = 0$ for inflexion point

$12x = 18$

$x = 1\frac{1}{2}$ $\qquad$ i.e. (1½, 1½)

2 marks – 1 for inflexion point
1 for testing

At (1½, 1½)

x	1	1½	2
$\frac{d^2y}{dx^2}$	−	0	+

Concavity changes at $x = 1\frac{1}{2}$

Point of inflexion at (1½, 1½)

At $x = -1$, $y = 2(-1)^3 - 9(-1)^2 + 12(-1) - 3$

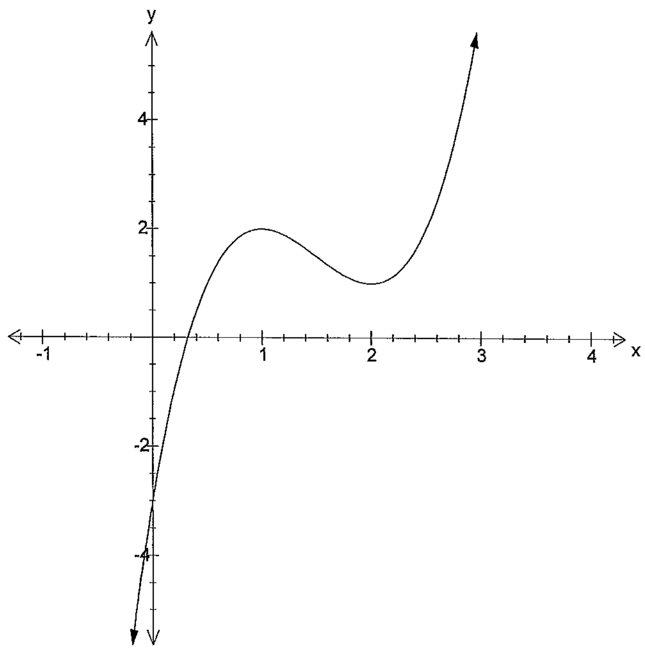
$= -26$

At $x = 3$, $y = 2(3)^3 - 9(3)^2 + 12(3) - 3$

$= 6$

At $x = 0$, $y = -3$

Solutions	Marks/Comments



2 marks – 1 for graph
1 for labels

(b) $\frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta} = \tan\theta$ $\qquad$ **(3 marks)**

LHS $= \frac{(1+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta} = \frac{\sec^2\theta \cdot \cot\theta}{\text{cosec}^2\theta}$ $\qquad$ 1 mark

$= \frac{1}{\cos^2\theta} \cdot \frac{\cos\theta}{\sin\theta} \div \frac{1}{\sin^2\theta}$ $\qquad$ 1 mark

$= \frac{1}{\cos\theta \sin\theta} \times \frac{\sin^2\theta}{1}$

$= \frac{\sin\theta}{\cos\theta}$ $\qquad$ 1 mark

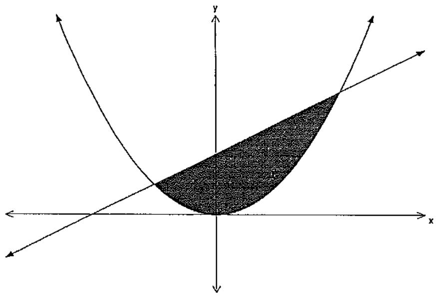
$= \tan\theta \quad =$ RHS

(c)

$\lim_{\theta \to 0} \frac{\sin 2\theta}{3\theta} = \frac{2}{3} \lim_{\theta \to 0} \frac{\sin 2\theta}{2\theta}$ $\qquad$ **1 mark**

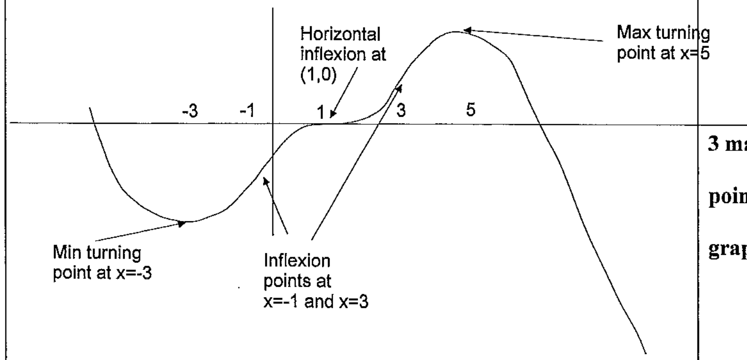
$= \frac{2}{3} \cdot 1$

$= \frac{2}{3}$

Solutions	Marks/Comments
Question 7	
(a) $y = x^2$ - - - (1) $y = x + 2$ - - - (2) (1) In (2) $x^2 = x + 2$ $x^2 - x - 2 = 0$ $(x + 1)(x - 2) = 0$ $x = -1$ or $x = 2$ i.e. $A(-1, 1)$ $B(2, 4)$	**4 marks – 1 for each point (2)** **1 for integration** **1 for answer**
	
$A = \left\lvert\int_a^b (f(x) - g(x))\,dx\right\rvert$ $= \left\lvert\int_{-1}^{2} (x + 2 - x^2)\,dx\right\rvert$ $= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$ $= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$ $= 3\frac{1}{3} - -1\frac{1}{6}$ $= 4\frac{1}{2}$ *sq units*	
(b) i. Angle $= \frac{40}{60} \times 360$ $= 240 \times \frac{\pi}{180}$ $= \frac{4\pi}{3}$	**1 mark**
ii. $l = r\theta$ $= 12\left(\frac{4\pi}{3}\right)$ $= 16\pi$ cm.	**1 mark**
iii. $A = \frac{1}{2}r^2\theta$ $= \frac{1}{2}(12)^2\left(\frac{4\pi}{3}\right)$ $= 96\pi$ cm.	**1 mark**

(c) $A = \frac{h}{3}[ends + 2odds + 4\,evens]$ $= \frac{0.25}{3}[(3\cdot43 + 1\cdot97) + 2(0\cdot38 + 2.65) + 4(2\cdot17 + 1\cdot87 + 2\cdot31)]$ $= 3\cdot071\dot{6}$ $= 3\cdot1$ (1dp)	**2 marks – 1 for use of formula** **1 for answer**

(d)



3 marks – 1 for inflexions
1 for stationary points
1 for point on graph (1, 0)

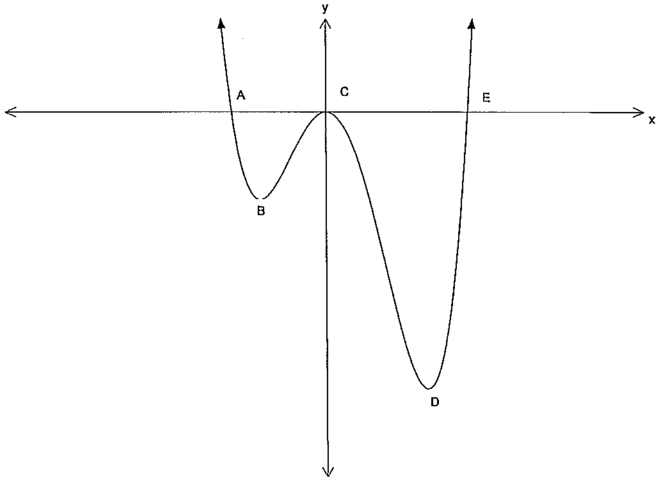
Solutions	Marks/Comments

Question 8

(a) i. B, C, D — **1 mark**

ii. From A to C and then from C to E — **1 mark**

iii.



1 mark

(b) i. Let \$P be the amount repaid each month

\$$A_n$ - Amount owing after n repayments

2 marks - 1 for working
1 for proof

$A_1 = 15\,000 \times 1{\cdot}005 - P$

$A_2 = A_1 \times 1{\cdot}005 - P$

$= (15\,000 \times 1{\cdot}005 - P) \times 1{\cdot}005 - P$

$= 15\,000 \times 1{\cdot}005^2 - P(1 + 1{\cdot}005)$

$A_3 = A_2 \times 1{\cdot}005 - P$

$= [15\,000 \times 1{\cdot}005^2 - P(1 + 1{\cdot}005)] \times 1.005 - P$

$= 15\,000 \times 1{\cdot}005^3 - P(1{\cdot}005 + 1{\cdot}005^2) - P$

$= 15\,000 \times 1{\cdot}005^3 - P(1 + 1{\cdot}005 + 1{\cdot}005^2)$

$= 15226{\cdot}13 - P(3{\cdot}015025)$

ii. $A_{24} = 15\,000 \times 1{\cdot}005^{24} - P(1 + 1{\cdot}005 + \ldots\ldots\ldots + 1{\cdot}005^{23})$

but $A_{24} = 10\,000$

3 marks - 1 for working
- 1 for sum of GS
- 1 for answer

$\therefore 10\,000 = 15\,000 \times 1{\cdot}005^{24} - P(1 + 1{\cdot}005 \ldots\ldots + 1{\cdot}005^{23})$

$P(1 + 1{\cdot}005 \ldots\ldots + 1{\cdot}005^{23}) = 15\,000 \times 1{\cdot}005^{24} - 10\,000$

$P = \frac{15\,000 \times 1{\cdot}005^{24} - 10\,000}{(1 + 1{\cdot}005 \ldots\ldots + 1{\cdot}005^{23})}$

GS with a = 1, r = 1·005, n = 24

$= 15\,000 \times 1{\cdot}005^{24} - 10\,000 \div \frac{1{\cdot}005^{24} - 1}{0{\cdot}005}$

$S = \frac{1(1{\cdot}005^{24} - 1)}{1{\cdot}005 - 1}$

$= \frac{(15\,000 \times 1{\cdot}005^{24} - 10\,000) \times 0{\cdot}005}{1{\cdot}005^{24} - 1}$

$= \frac{1{\cdot}005^{24} - 1}{0{\cdot}005}$

= \$271·60

(c) $2x = y^2 - 8y + 4$

$y^2 - 8y = 2x - 4$

$y^2 - 8y + 16 = 2x - 4 + 16$

$(y-4)^2 = 2x + 12$

$(y-4)^2 = 2(x+6)$

4 marks – 1 for sketch
1 for vertex
1 for focus
1 for directrix

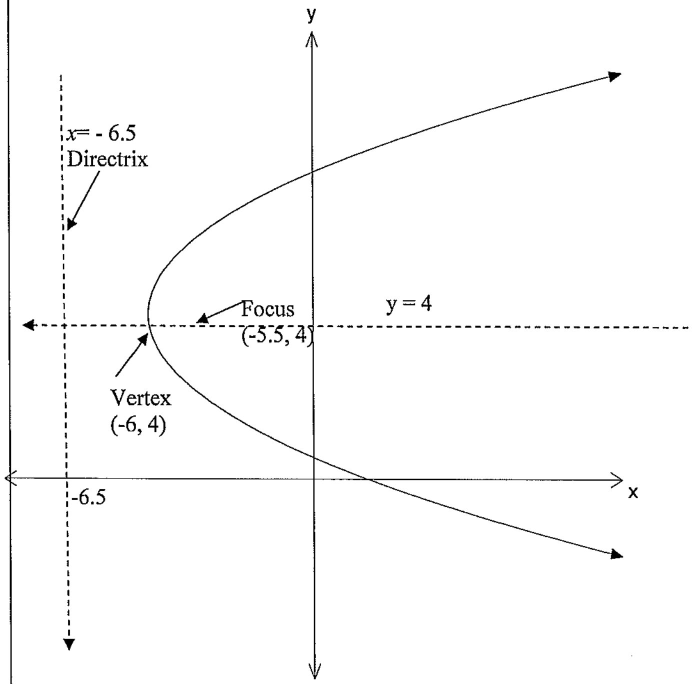
Vertex = (−6, 4)

$4a = 2$

$a = \frac{1}{2}$

Focus = $\left(-5\frac{1}{2}, 4\right)$

Directrix : $x = -6\frac{1}{2}$



Solutions	Marks/Comments
Question 9	
(a) $x = 2\sin 2t$ $\dot{x} = 4\cos 2t$ $\ddot{x} = -8\sin 2t$	
i. $t = 0 \quad \dot{x} = 4\cos 2(0)$ $= 4 \times 1$ $= 4\ m/s$	**1 mark**
ii. $t = \frac{\pi}{12} \quad \ddot{x} = -8\sin 2\left(\frac{\pi}{12}\right)$ $= -8\sin\left(\frac{\pi}{6}\right)$ $= -8 \times \frac{1}{2}$ $= -4\ m/s^2$	**1 mark**
iii. $\dot{x} = 0 \quad then\ 4\cos 2t = 0$ i.e. $\cos 2t = 0$ $2t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots\ldots$ $t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots\ldots$	**2 marks – 1 for working 1 for answer**
iv. $x = 2\sin 2t$ $= 2\sin 2\left(\frac{\pi}{4}\right)$ $= 2$ $\therefore x = \pm 2\,\text{m}$	**2 marks - 1 for working 1 for answer**
(b) $V = \int_a^b [f(x)]^2\, dx$ $= \int_1^3 \left(\sqrt{\frac{2x}{3x^2-1}}\right)^2 dx$ $= \int_1^3 \frac{2x\, dx}{3x^2-1}$ $= \frac{1}{3}[\ln(3x^2-1)]_1^3$ $= \frac{1}{3}[\ln(3 \times 3^2 - 1) - \ln(3 \times 1^2 - 1)]$ $= \frac{1}{3}(\ln 26 - \ln 2)$ $= \frac{1}{3}\left(\ln \frac{26}{2}\right)$ $= \frac{1}{3}(\ln 13)$	**3 marks – 1 use of formula 1 for Integral 1 for answer**

(c) i. $\frac{dV}{dt} = \frac{1}{100}(30t - t^2)$ when $t = 15$ $\frac{dV}{dt} = \frac{1}{100}[30(15) - (15)^2]$ $= \frac{225}{100}$ $= 2\frac{1}{4}\ cm^3/min$	**1 mark**
ii. $V = \int_0^{15} \frac{1}{100}(30t - t^2)$ $= \frac{1}{100}\left[15t^2 - \frac{t^3}{3}\right]_0^{15}$ $= \frac{1}{100}\left\{\left[15(15)^2 - \frac{15^3}{3}\right] - [0]\right\}$ $= \frac{1}{100}[3375 - 1125]$ $= \frac{1}{100}(2250)$ $= 22{\cdot}5\ cm^3$	**2 marks – 1 for integral 1 for answer**

Solutions	Marks/Comments

Question 10

(a) i. $SA = \pi r^2 + 2\pi rh = 300$

$2\pi rh = 300 - \pi r^2$

$h = \frac{300 - \pi r^2}{2\pi r}$

$V = \pi r^2 h$

$= \pi r^2 \left(\frac{300 - \pi r^2}{2\pi r}\right)$

$= 150r - \frac{\pi r^3}{2}$

2 marks – 1 for "h"
1 for "V"

ii. $V = 150r - \frac{1}{2}\pi r^3$

$\dot{V} = 150 - \frac{3}{2}\pi r^2$

$\ddot{V} = -3\pi r$ which is less than 0 for positive r

4 marks – 1 for differentials
1 for value of 'r'
1 for test
1 for max volume

Stat Pts when $\dot{V} = 0$

i.e. $150 - \frac{3}{2}\pi r^2 = 0$

$150 = \frac{3}{2}\pi r^2$

$100 = \pi r^2$

$r^2 = \frac{100}{\pi}$

$r^2 = \pm\sqrt{\frac{100}{\pi}}$

Now max Volume when $r > 0$ i.e. $r = \sqrt{\frac{100}{\pi}} = 5{\cdot}641895835$

$V = 150\sqrt{\frac{100}{\pi}} - \frac{\pi}{2}\left(\sqrt{\frac{100}{\pi}}\right)^3 = 564{\cdot}1895835$

$= 564\ m^3$ (nearest m^3)

(b) i. $\frac{dP}{dt} = kP \quad \therefore P = P_0 e^{kt}$

When $t = 0$, $P = 20\ 000$, $\therefore P_0 = 20\ 000$

So $P = 20\ 000e^{kt}$

When $t = 2$, $P = 25\ 000$

$25\ 000 = 20\ 000e^{2k}$

$\frac{5}{4} = e^{2k}$

$\ln\left(\frac{5}{4}\right) = 2k$

$k = \ln\left(\frac{5}{4}\right) \div 2$

$k = 0{\cdot}111571775$

$\therefore P = 20\ 000e^{0{\cdot}111571775\,t}$

3 marks – 1 for value of 'k'
1 for equation
1 population

When $t = 10$ $P = 20\ 000e^{0{\cdot}111571775\,(10)}$

$= 61\ 000$ people (nearest 100)

ii. $\frac{dp}{dt} = 61\ 000 \times 0{\cdot}111571775$

$= 6\ 806$ people / year

1 for rate of change

(c) $\log_a 2 + 2\log_a x - \log_a 6 = \log_a 3$

$\log_a 2 + \log_a x^2 - \log_3 6 = \log_a 3$

$\text{Log}_a \frac{2x^2}{6} = \log_a 3$

$\therefore \frac{2x^2}{6} = 3$

$2x^2 = 18$

$x^2 = 9$

$x = \pm 3$

going back to original equation, cannot have log (-3) so

$x = 3$

2 marks – 1 manipulation of logs
1 for answer