WESTERN REGION

2008
HSC Preliminary Course
FINAL EXAMINATION

Mathematics - Extension 1

General Instructions

- Reading Time - 5 minutes.
- Working Time - 1½ hours.
- Write using a blue or black pen.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks (**60**)

- Attempt Questions 1- 5.
- All questions are of equal value.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n \neq -1;\quad x \neq 0,\text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a \neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a > 0,\quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Solve the inequality $\frac{3x}{3x-4} \leq 3$ **3**

(b) Six students are to be seated in a row on the stage for an assembly. How many ways can they be placed if: **3**

i) there are no restrictions on where they sit?

ii) two particular students insist on sitting next to each other?

iii) two particular students refuse to sit next to each other?

(c) A committee of 5 is to be chosen from 8 Labour Party members and 4 Liberal Party members. It is necessary that there is majority of Labour Party Members and there is at least 1 Liberal Party member on the committee. In how many ways can the committee be chosen? **2**

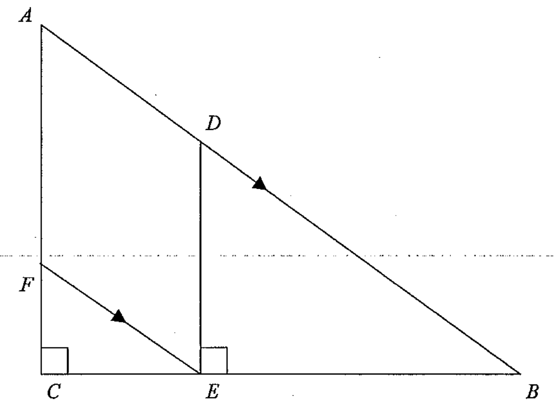
(d) Find the acute angle between the lines $x+2y=0$ and $x-3y=0$. **2**

(e) For the points $A(-3,-7)$ and $B(-1,-4)$. Find the co-ordinates of the point $P(x,y)$ which divides the interval AB externally in the ratio $4:3$. **2**

End of Question 1

Question 2 **(12 Marks)** Use a Separate Sheet of paper **Marks**

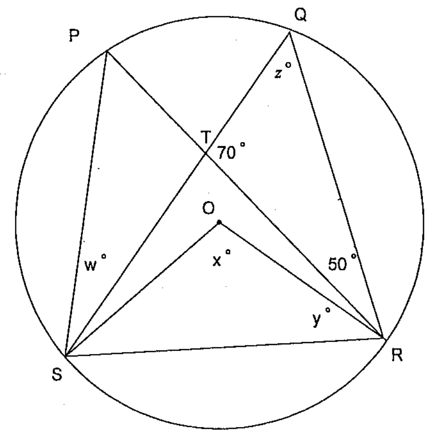
(a) In the triangle below, $AB \parallel FE$, and $\angle FCE = \angle DEB = 90°$ **4**



i) Prove that $\Delta ABC \,|||\, \Delta DBE$ and $\Delta ABC \,|||\, \Delta FCE$

ii) If $DE:FC = 5:2$, $FE = 3.2$ cm and $CE = 2.4$ cm find the length of AB

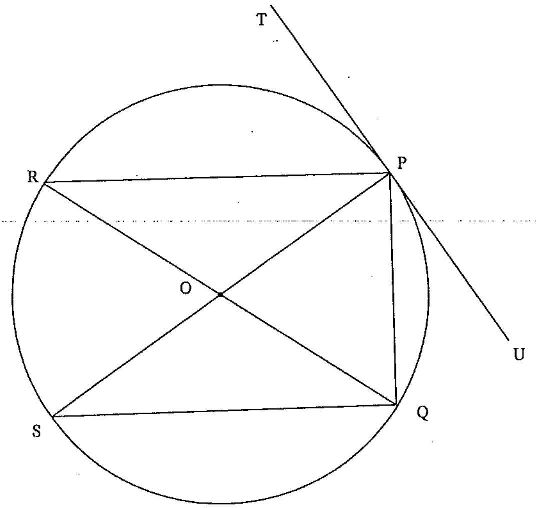
(b) Find the values of w, x, y and z, giving reasons. **4**



Question 2 continues on page 4

Question 2 continued. **Marks**

(c) The point O is the centre of the circle. TU is a tangent to the circle, contacting the circle at P. **4**



i) Show that $\angle ROP = 2\angle RPT$

ii) Show that $\angle RPT$ and $\angle QPU$ are complementary.

iii) Show that $RP \parallel SQ$.

End of Question 2

Question 3 (12 Marks) Use a Separate Sheet of paper **Marks**

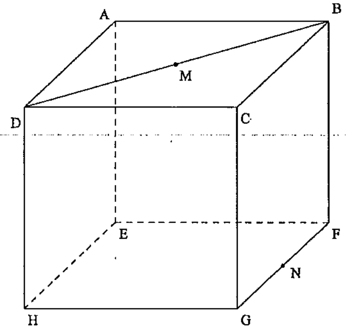
(a) Write $\sin\theta + \cos\theta$ in terms of t where $t = \tan\dfrac{\theta}{2}$. **1**

(b) The cube ABCDEFGH has sides of length 12 cm. M is the midpoint of the diagonal BD on the face ABCD and N is the midpoint of the edge GF. **4**

i) Calculate the length of the line MH.

ii) Find the size of $\angle MHF$ to the nearest minute.

iii) Find the size of $\angle MHN$ to the nearest minute.



(c) Show that $\sin\left(2x+30^\circ\right) = \sqrt{3}\sin x\cos x + \cos^2 x - \dfrac{1}{2}$. **2**

(d) Solve $\sin x\cos x = \dfrac{1}{2\sqrt{2}}$ for $0^\circ \le x \le 360^\circ$ **2**

(e) i) Express $\sqrt{2}\sin x + \cos x$ in the form $r\sin\left(x+\alpha\right)$. **3**

ii) Hence or otherwise, solve $\sqrt{2}\sin x + \cos x = 1$ for $0^\circ \le x \le 360^\circ$, giving your answer to the nearest minute.

End of Question 3

Question 4 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Find the centre and radius of the circle with equation $x^2+8x+y^2-12y+27=0$ **2**

(b) Sketch the parabola $x^2-12y+2x+25=0$, clearly showing the vertex P, the focus S and the directrix. **3**

(c) For the parabola defined by the equations $x=6t$, $y=3t^2$; **3**

i) Give the Cartesian Equation of the parabola.

ii) Find the equation of the focal chord which passes through the point where $t=3$.

iii) Find the equation of the tangent to the parabola at the point where $t=3$.

(d) Find the equation of the locus of a point P which moves such that it's distance from the point A(2, -3) is twice it's distance from the point B(-4, 12). **2**

(e) Find the Cartesian equation of the curve which has the parametric equations below. **2**

$x=3p-2$

$y=2-3p^3$

End of Question 4

Question 5 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Differentiate the following. (You do not need to simplify your answers after finding the derivative.) **3**

i) $\left(x^2-2x\right)\sqrt{x^3-2x^2}$

ii) $\sqrt[3]{\dfrac{x^4}{x^3-2x^2}}$

(b) The polynomial $P(x)=2x^4-ax^2+bx-1$ is exactly divisible by $(x+1)$, and has a remainder of -3 on division by $(x-2)$. Find the values of a and b. **3**

(c) i) Show that x-3 is a factor of $f(x)=x^3+3x^2-9x-27$. **3**

ii) Find the intercepts that the graph of $y=f(x)$ makes with the x axis.

(d) If α, β, and γ are the roots of the equation $x^3-3x+1=0$, find: **3**

i) $\alpha\beta+\beta\gamma+\gamma\alpha$

ii) $\alpha^2+\beta^2+\gamma^2$

iii) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$.

End of Examination

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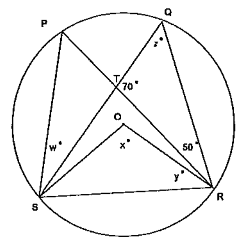
Mathematics Extension 1

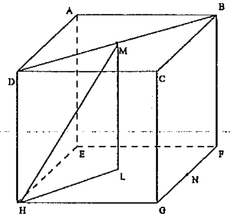
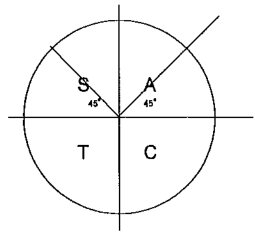
SOLUTIONS

Question 1	Preliminary Examination - Extension 1		2008
Part	Solution	Marks	Comment
(a)	$\frac{3x}{3x-4} \le 3$ $(3x-4)^2 \frac{3x}{3x-4} \le 3(3x-4)^2$ $9x^2 - 12x \le 27x^2 - 72x + 48$ $0 \le 18x^2 - 60x + 48$ $0 \le 3x^2 - 10x + 8$ $0 \le 3x^2 - 6x - 4x + 8$ $0 \le 3x(x-2) - 4(x-2)$ $0 \le (3x-4)(x-2)$ but $x \ne \frac{4}{3}$ $x < \frac{4}{3}$ $x \ge 2$	3	1 for quadratic inequation or equivalent method 1 for working 1 for solution
(b)	i) Ways of arranging 6 people = 6! = 720	1	
	ii) Ways of arranging 5 groups = 5! = 120 Ways of arranging a group of 2 = 2! = 2 Ways of arranging 6 groups with 2 people together = 5!×2! = 240	1	
	iii) Ways with two who refuse to be together = ways of arranging 6 – ways with two together = 720 – 240 = 480	1	
(c)	Possible arrangements 4 Labour & 1 Liberal = ${}^8C_4 \times {}^4C_1 = 70 \times 4 = 280$ 3 Labour and 2 Liberal = ${}^8C_3 \times {}^4C_2 = 56 \times 6 = 336$ Total arrangements = 280 + 336 = 616	2	1 for partial arrangements 2 for correct result

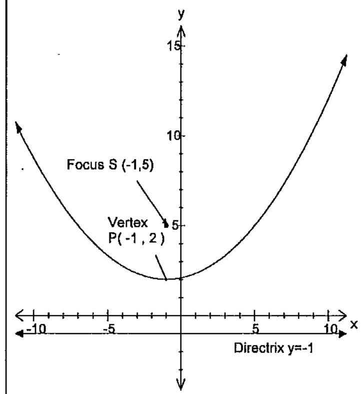
Question 1	Preliminary Examination - Extension 1	2008	
Part	**Solution**	**Marks**	**Comment**
(d)	$x+2y=0$ $x-3y=0$ $2y=-x$ $3y=x$ $y=-\frac{1}{2}x$ $y=\frac{1}{3}x$ $m_1=-\frac{1}{2}$ $m_2=\frac{1}{3}$ $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $=\left\|\frac{\left(\frac{-1}{2}\right)-\left(\frac{1}{3}\right)}{1+\left(\frac{-1}{2}\right)\left(\frac{1}{3}\right)}\right\|$ $\tan\theta=1$ $\theta=45^\circ$ $=\left\|\frac{\frac{-5}{6}}{\frac{5}{6}}\right\|=1$	2	1 mark for gradients 1 for angle
(e)	$x=\frac{mx_2-nx_1}{m-n}$ $y=\frac{my_2-ny_1}{m-n}$ $=\frac{4(-1)-3(-3)}{4-3}$ $=\frac{4(-4)-3(-7)}{4-3}$ $=5$ $=5$ The point is (5, 5)	2	1 for each coordinate

Question 2	Preliminary Examination - Extension 1	2008	
Part	**Solution**	**Marks**	**Comment**
(a)	i) In ΔABC and ΔDBE $\angle ACB=\angle DEB=90^\circ$ (given) $\angle ABC=\angle DBE$ (common angle) $\angle BAC=\angle BDE$ (angle sum Δ) $\therefore \Delta ABC \text{ \|\|\| } \Delta DBE$ (corresponding angles equal) In ΔABC and ΔFCE $\angle ACB=\angle FCE=90^\circ$ (given) $\angle ABC=\angle FEC$ (corresponding angles on \\ lines) $\angle BAC=\angle EFC$ (angle sum Δ) $\therefore \Delta ABC \text{ \|\|\| } \Delta FCE$ (corresponding angles equal)	2	1 mark for each similarity proof (requires at least 2 angles equal and reasons)
	ii) $DE:FC=5:2$ $\frac{DE}{FC}=\frac{5}{2}=\frac{EB}{2.4}$ $EB=\frac{5}{2}\times 2.4$ $EB=6$ $CB=EB+CE$ $=6+2.4$ $=8.4$ $\frac{CB}{CE}=\frac{8.4}{2.4}=\frac{7}{2}$ $\frac{AB}{FE}=\frac{7}{2}$ $\frac{AB}{3.2}=\frac{7}{2}$ $AB=\frac{7}{2}\times 3.2$ $=11.2$ cm	2	Different pairings of sides are possible to achieve this. Give 1 for a reasonable attempt involving similarity
(b)	 W = 50° Angles on same arc PQ $\angle STP=70^\circ$ (vert opp) $\angle SPT=60^\circ$ (angle sum ΔPST) z $=\angle SPT=60^\circ$ (angles on same arc SR) x = 2 × 60° =120° (angle at centre twice $\angle SPT$) y = (180-120)÷2 (isos Δ OSR) y = 30°	4	1 for each pronumeral
(c)	i) $\angle RPT=\angle RQP$ (angle between tang & chord) $\angle ROP=2\angle RQP$ (angle at centre twice angle at circ) $\therefore \angle ROP=2\angle RPT$	1	

Question 2	Preliminary Examination - Extension 1	2008	
Part	Solution	Marks	Comment
	ii) $\angle RPT = 90^\circ$ (Angle in a semicircle) $\angle RPT + \angle QPU + \angle RPT = 180^\circ$ (Straight line) $\angle RPT + \angle QPU + 90^\circ = 180^\circ$ $\angle RPT + \angle QPU = 90^\circ$ $\angle RPT$ and $\angle QPU$ are complementary.	1	
	iii) $\angle PRQ = \angle PSQ$ (Angles on same arc) $\angle PRQ = \angle RPS$ (Base angles in isos ΔROP) $\therefore \angle PSQ = \angle RPS$ (both equal $\angle PRQ$) $RP \parallel SQ$ (equal alternate angles)	2	

Question 3	Preliminary Examination - Extension 1	2008	
Part	Solution	Marks	Comment
(a)	$\sin\theta + \cos\theta = \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}$ $= \frac{1+2t-t^2}{1+t^2}$	1	
(b)	i) $BD^2 = 12^2 + 12^2$ $BD = \sqrt{288} = 12\sqrt{2}$ $MD = 6\sqrt{2}$ $MH^2 = \left(6\sqrt{2}\right)^2 + 12^2$ $MH = \sqrt{216} = 6\sqrt{6}$  	1	
	ii) Using triangle MLH $\tan \angle MHL = \frac{12}{6\sqrt{2}} = \frac{2}{\sqrt{2}}$ $\angle MHL = 54^\circ 44'$	1	
	iii) In ΔHGN and ΔMNL $HN^2 = MN^2 = 12^2 + 6^2$ $HN = MN = \sqrt{180} = 6\sqrt{5}$ In ΔMHN $\cos \angle MHN = \frac{\left(6\sqrt{6}\right)^2 + \left(6\sqrt{5}\right)^2 - \left(6\sqrt{5}\right)^2}{2\left(6\sqrt{6}\right)\left(6\sqrt{5}\right)}$ $= 0.5477$ $\angle MHN = 56^\circ 47'$	2	
(c)	$\sin x \cos x = \frac{1}{2\sqrt{2}}$ $2\sin x \cos x = \frac{1}{\sqrt{2}}$ $\sin 2x = \frac{1}{\sqrt{2}}$  Acute value $2x = 45^\circ$ For $0 \le 2x \le 720$ $2x = 45^\circ, 135^\circ, 405^\circ, 495^\circ$ $x = 22.5^\circ, 67.5^\circ, 202.5^\circ, 247.5^\circ,$	3	1 for $\sin 2x$ equation 1 for $2x = 45^\circ$ 1 for all 4 solutions.

Question 3	Preliminary Examination - Extension 1	2008	
Part	**Solution**	**Marks**	**Comment**
(d)	i) $\sqrt{2}\sin x+\cos x$ $\tan^{-1}\alpha=\dfrac{1}{\sqrt{2}}$ and $r=\sqrt{\sqrt{2}^2+1^2}$ $\alpha=35°16'$ $\quad r=\sqrt{3}$ $\sqrt{2}\sin x+\cos x=\sqrt{3}\sin\left(x+35°16'\right)$	2	1 for value of r 1 for α
	ii) $\sqrt{2}\sin x+\cos x=1$ $\sqrt{3}\sin\left(x+35°16'\right)=1$ $\sin\left(x+35°16'\right)=\dfrac{1}{\sqrt{3}}$ (In 1st and 2nd quadrants) Acute value for $\left(x+35°16'\right)=35°16'$ For $35°16'\le x+35°16'\le 395°16'$ $\left(x+35°16'\right)=35°16',\ 180°-35°16',\ 360°+35°16'$ $x=0°,\ 109°28',\ 360°$	2	1 for solution acute value only 2 for final set of solutions Or deduct 1 if missing a value.

Question 4	Preliminary Examination - Extension 1	2008	
Part	**Solution**	**Marks**	**Comment**
(a)	$x^2+8x+y^2-12y+27=0$ $x^2+8x+16+y^2-12y+36=-27+16+36$ $(x+4)^2+(y-6)^2=25$ Centre (-4, 6), radius 5	2	1 for rearranging equation 1 for radius and centre
(b)	$x^2-12y+2x+25=0$ $x^2+2x+1=12y-24$ $(x+1)^2=4.3.(y-2)$ Vertex at (-1, 2) focal length 3  	3	1 each for correct placement of focus, directrix and vertex. Deduct 1 if direction of parabola incorrect.
(c)	i) $x=6t$, $y=3t^2$ $\quad a=3$ Cartesian equation $x^2=4ay$ $x^2=12y$	1	
	ii) Where $t=3$ $x=18$ $\quad y=27$ Focus (0, 3) Gradient $=\dfrac{27-3}{18-0}=\dfrac{24}{18}=\dfrac{4}{3}$ Equation $y=\dfrac{4}{3}x+3$ $4x-3y+9=0$	1	Any form of equation okay.

Question 4	Preliminary Examination - Extension 1	2008	
Part	Solution	Marks	Comment
	iii) $x^2=12y$ $y=\frac{x^2}{12}$ $\frac{dy}{dx}=\frac{2x}{12}=\frac{x}{6}$ At (18, 27) $\frac{dy}{dx}=\frac{18}{6}=3$ Equation $y-27=3(x-18)$ $y-27=3x-54$ $y=3x-27$	1	
(d)	P (*x*, *y*) A(2, -3) B(-4, 12). AP = 2PB $\sqrt{(x-2)^2+(y+3)^2}=2\sqrt{(x+4)^2+(y-12)^2}$ $(x-2)^2+(y+3)^2=4\left[(x+4)^2+(y-12)^2\right]$ $x^2-4x+4+y^2+6y+9=4\left[x^2+8x+16+y^2-24y+144\right]$ $x^2-4x+4+y^2+6y+9=4x^2+32x+64+4y^2-96y+576$ $3x^2+36x+3y^2-102y+627=0$ $x^2+12x+y^2-34+209=0$	2	Accept any version of equation
(e)	$x=3p-2$ $x+2=3p$ $\frac{x+2}{3}=p$ $\left(\frac{x+2}{3}\right)^3=p^3$ $y=2-3p^3$ $3p^3=2-y$ $p^3=\frac{2-y}{3}$ $\frac{2-y}{3}=\left(\frac{x+2}{3}\right)^3$ $\frac{2-y}{3}=\frac{(x+2)^3}{27}$ $18-9y=(x+2)^3$	2	Other expanded forms of equation are okay, as are all those shown for 2 marks. 1 mark for any partial solution that shows the right idea

Question 5	Preliminary Examination - Extension 1	2008	
Part	Solution	Marks	Comment
(a)	i) $\frac{d}{dx}\left[(x^2-2x)(x^3-2x^2)^{\frac{1}{2}}\right]$ $=(x^2-2x).\frac{1}{2}(x^3-2x^2)^{-\frac{1}{2}}(3x^2-4x)+(x^3-2x^2)^{\frac{1}{2}}(2x-2)$ The following is not required but provided in case students give this answer. $=\frac{(x^2-2x)(3x^2-4x)}{2\sqrt{x^3-2x^2}}+(2x-2)\sqrt{x^3-2x^2}$	1	Mark for completing the differentiation, no extra for simplifying
	ii) $\frac{d}{dx}\left[\left(\frac{x^4}{x^3-2x^2}\right)^{\frac{1}{3}}\right]=\frac{d}{dx}\left[\frac{x^{\frac{4}{3}}}{(x^3-2x^2)^{\frac{1}{3}}}\right]$ $=\frac{(x^3-2x^2)^{\frac{1}{3}}.\frac{4}{3}x^{\frac{1}{3}}-x^{\frac{4}{3}}.\frac{1}{3}(x^3-2x^2)^{-\frac{2}{3}}(3x^2-4x)}{(x^3-2x^2)^{\frac{2}{3}}}$	2	2 for use of both quotient and chain rules correctly. Deduct 1 if simple errors made on the way.
(b)	$P(x)=2x^4-ax^2+bx-1$ $P(-1)=2(-1)^4-a(-1)^2+b(-1)-1=0$ $1-a-b=0$ $P(2)=2(2)^4-a(2)^2+b(2)-1=-3$ $34-4a+2b=0$ .... (1) $2-2a-2b=0$ .....(2) $36-6a=0$ .....(1) + (2) $6a=36$ $a=6$ $1-6-b=0$ $b=-5$ $a=6$ and $b=-5$	3	1 for substitution and 1 each for solving simultaneously to give each value.

(c)	i) $f(3)=(3)^3+3(3)^2-9(3)-27$ $=27+27-27-27=0$ $\therefore\ x-3$ is a factor of $f(x)$	1	
	ii) $x-3\,\overline{)\,x^3+3x^2-9x-27}$ quotient x^2+6x+9 x^3-3x^2 $6x^2-9x$ $6x^2-18x$ $9x-27$ $9x-27$ Or by using the factor theorem and testing other values. $x^2+6x+9=(x+3)^2$ $f(x)=(x-3)(x+3)^2$ Intercepts with x axis $x=3$ and double root at $x=-3$	2	2 for finding full set of intercepts. 1 if one root only found.
(d)	$x^3-3x+1=0$ i) $\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}=-3$	1	
	ii) $\alpha+\beta+\gamma=\frac{-b}{a}=0$ $\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$ $=0^2-2(-3)$ $=6$	1	
	iii) $\alpha\beta\gamma=\frac{-d}{a}=-1$ $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$ $=\frac{-3}{-1}$ $=3$	1	