WESTERN REGION

2012

HIGHER SCHOOL CERTIFICATE

TRIAL EXAMINATION

Mathematics

General Instructions

- Reading Time - 5 minutes
- Working Time - 3 hours
- Write using a blue or black pen. Black pen is preferred
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11-16

Total marks (100)

Section I

Total marks (10)

- Attempt Questions 1-10
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II

Total marks (90)

- Attempt questions 11 – 16
- Answer on the blank paper provided, unless otherwise instructed.
- Start a new page for each question.
- All necessary working should be shown for every question.
- Allow about 2 hours 45 minutes for this section.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$

Trial HSC Examination – Mathematics 2012

Multiple Choice Answer Sheet

Name ______________________________

Completely fill the response oval representing the most correct answer.

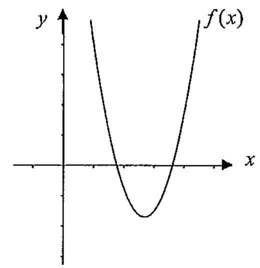
1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

1. An angle measuring $4^\circ 20'$ is converted to radians. The answer written in scientific notation correct to 2 significant figures is:

(A) 0.076 (B) 7.6×10^{2}

(C) 75.6×10^{-3} (D) 7.6×10^{-2}

2. For the function $f(x)=ax^2+bx+c$ sketched below, which of the following statements is true?



(A) $a>0$, $c>0$ and $b^2-4ac=0$

(B) $a>0$, $c>0$ and $b^2-4ac>0$

(C) $a>0$, $c<0$ and $b^2-4ac<0$

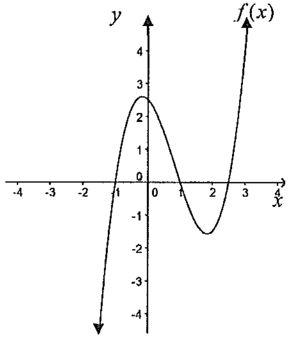
(D) $a>0$, $c>0$ and $b^2-4ac<0$

3. The value of $\dfrac{4.56+1.78}{\sqrt{3.09-2.05}}$ is closest to:

(A) 10 (B) 6

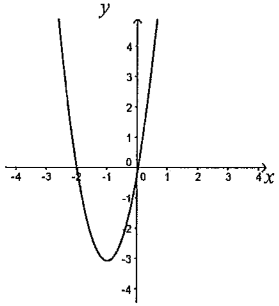
(C) 4 (D) 9

4. Examine the features of the graph of $f(x)$ supplied.

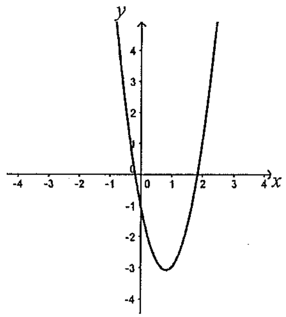


Which of the graphs below best represents $f'(x)$?

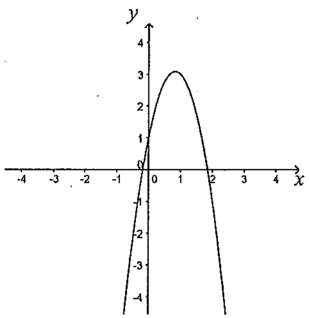
(A)



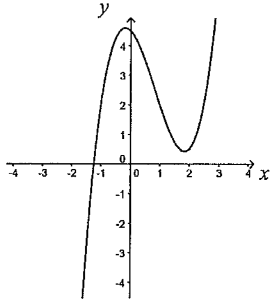
(B)



(C)



(D)



5. The line ℓ passes through the point $P(2,6)$ and has a gradient of $-\frac{1}{3}$.

The equation of ℓ is written in general form as:

(A) $3y = 20 - x$ (B) $y = -\frac{1}{3}x + 6\frac{2}{3}$

(C) $x + 3y - 16 = 0$ (D) $x + 3y - 20 = 0$

6. In a standard deck of cards there are 26 red cards and 26 black cards. If 3 cards are chosen at random from the deck, without replacement, the likelihood that they are all black is:

(A) $\frac{3}{52}$ (B) $\frac{1}{8}$ (C) $\frac{2}{17}$ (D) $\frac{3}{140608}$

7. The lines $9x - 2y + 20 = 0$ and $3x + y - 10 = 0$ intersect at point A which is on the y-axis. The coordinates of A are:

(A) (0, 10) (B) (0,–10) (C) (10, 0) (D) $(3\frac{1}{3},0)$

8. Solving the equation $3^{2x} - 10(3^x) + 9 = 0$ gives 2 solutions for x. Which pair of solutions below is correct?

(A) $x = 1$ or $x = 0$ (B) $x = \log_3 3$ or $x = \log_3 1$

(C) $x = 0$ or $x = 2$ (D) $x = \log_e 9$ or $x = \log_e 1$

9. To find the area enclosed by the curve $y = 4x(x-1)(x-2)$ and the x - axis we would need to use which integral?

(A) $\int_0^2 (4x(x-1)(x-2))dx$

(B) $\left|\int_2^0 (4x(x-1)(x-2))dx\right|$

(C) $2\int_1^2 (4x(x-1)(x-2))dx$

(D) $\left|\int_0^1 (4x(x-1)(x-2))dx\right| + \left|\int_1^2 (4x(x-1)(x-2))dx\right|$

10. A curve $f(x)$ is known to have gradient function $f'(x) = \sin^2 x \cos x$.

It is also known that this curve passes through the point $\left(\frac{\pi}{2}, 1\right)$.

Using the fact that $\frac{d}{dx}\left(\sin^3 x\right) = 3\sin^2 x \cos x$, identify the correct equation for this curve:

(A) $y = \sin^3 x \cos^2 x$

(B) $y = 3\sin^3 x - 2$

(C) $y = \frac{2}{3}\sin^3 x + \frac{1}{3}$

(D) $y = \frac{1}{3}\sin^3 x + \frac{2}{3}$

End of Section I

Section II

Total marks (90)
Attempt Questions 11-16
Allow about 2 hours 45 minutes for this section

Answer all questions, starting each question on a new sheet of paper with your name and question number at the top of the page. Do not write on the back of sheets.

Question 11 (15 marks) Use a Separate Sheet of paper **Marks**

a) For the series 5, -1, -7,...

i) Which term will be equal to -61? **1**

ii) Find the sum of the first 20 terms. **1**

b) Simplify $\frac{3x}{x+2} - \frac{5x+19}{x^2+5x+6}$ **2**

c) Solve $6x^2 + x = 2$ **2**

d) Find the point on the curve $y = 5x^2 - 4x + 1$ where the gradient of the tangent equals 6. **2**

e) Given that $\left(2+\sqrt{48}\right)\left(3\sqrt{12}+\sqrt{75}\right) = a\sqrt{3} + b$, find the values of a and b. **2**

f) Differentiate with respect to x:

i) $3x^2 \sin x$ **2**

ii) $\log_e(x^2 - 1)$ **1**

g) Evaluate $\int_{-1}^{1} (3x+4)^3\, dx$ **2**

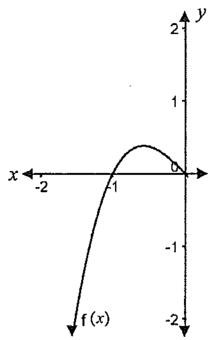
End of Question 11

Question 12 (15 Marks) Use a Separate Sheet of paper **Marks**

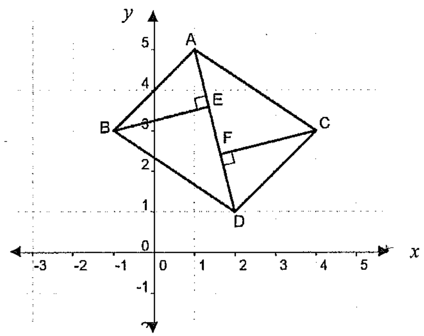
a) The graph below shows the function $f(x)$ for the region $x \leq 0$.

i) Given that $f(x)$ is the odd function $f(x) = x^3 - x$, sketch $f(x)$ for the region $-2 < x < 2$. **1**

ii) Hence or otherwise, state how many turning points $f'(x)$ has. **1**



b) In the diagram below, $A(1,5), B(-1,3), C(4,3)$ and $D(2,1)$ form a quadrilateral.



NOT TO SCALE

i) Show that $AC \parallel BD$. **2**

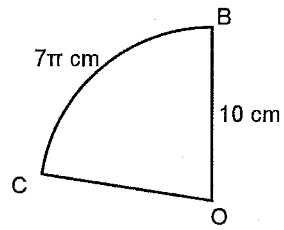
ii) Given that $AB \parallel CD$, prove that $\Delta BAE \equiv \Delta CDF$. **3**

iii) Given that the AD has equation $4x + y - 9 = 0$, calculate the length of interval BE. **2**

iv) Evaluate the area of $ABCD$. **2**

Marks

c) COB is a sector in a circle with centre O, and radius 10 cm.
The length of arc CB is 7π cm.



NOT TO SCALE

Calculate the exact area of sector COB. **2**

d) For the expression $\sum_{n=1}^{5} n^2 + 3$

i) Express it as a series. **1**

ii) Evaluate the expression. **1**

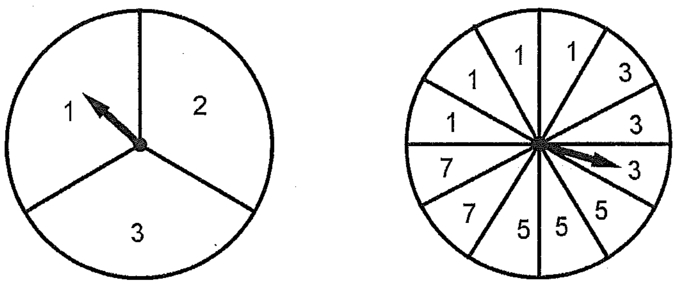
End of Question 12

Question 13 (15 Marks) Use a Separate Sheet of paper **Marks**

a) A point $P(x, y)$ moves such that it is always 3 units away from the point $(1, 4)$. Describe the locus of P and state its equation. **2**

b) The diagram below shows two spinners.

1st Spinner 2nd Spinner



Each of the three outcomes on the first spinner are equally likely.
On the second spinner there are 12 equally likely sectors for the arrow to land on with four possible outcomes.

In a game, both spinners are spun simultaneously.
The players score is the sum of the two numbers that the spinners land on.

In the diagram above, the player's score would be 4 since the first spinner landed on 1 and the second spinner landed on 3.

A player wins if their score is an odd number greater than 6.

i) Draw a probability tree showing all possible outcomes. **3**

ii) What is the probability that a player will win on a single turn? **2**

c) For the curve $y = x^3 + \frac{3}{2}x^2 - 6x + 7$,

i) Find the stationary points and determine their nature. **3**

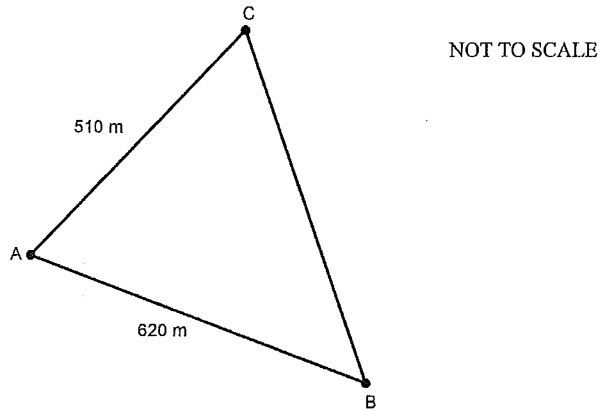
ii) Find any inflexion points. **2**

iii) Sketch the curve showing all important features. **3**

End of Question 13

Question 14 (15 Marks) Use a Separate Sheet of paper **Marks**

a) Anthony, Bethany and Carl decided to meet for lunch at Bethany's house.
Carl's house (C) is situated on a bearing of $046°T$ from Anthony's house (A).
From Bethany's house (B), the bearing to Carl's house is $341°T$.



The distance from Anthony's house to Bethany's is 620 m.
The distance from Anthony's house to Carl's is 510 m.

i) What is the bearing to the nearest minute of Bethany's house from Anthony's? **3**

ii) Use the Cosine Rule to show that the distance from Carl's house to Bethany's is given by: **1**

$$BC = \sqrt{644500 - 632400 \cos 66°48'} \text{ metres.}$$

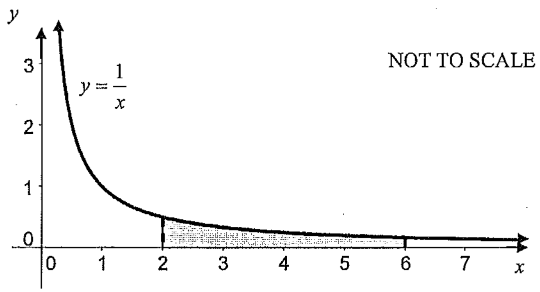
b) The quadratic function $2x^2 - x - 15$ has roots α and β.

i) Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ **2**

ii) Express the quadratic function in the form $Px(x-2) + Q(x-2) + R$. **3**

Marks

c) For the curve $y = \frac{1}{x}$:



i) Use the Trapezoidal Rule with five function values to approximate the area bounded by the curve, the x-axis and the lines $x = 2$ and $x = 6$. **2**

ii) Calculate the same area by evaluating $\int_2^6 \frac{1}{x}\,dx$. **1**

iii) Explain the difference between your answers to part i) and part ii). **1**

iv) The area above has been revolved about the x-axis. Evaluate the exact volume of the solid formed by this revolution. **2**

End of Question 14

Question 15 (15 Marks) Use a Separate Sheet of paper **Marks**

a) The closure of several manufacturing businesses in a small town has caused the town's population (P) to decline over the last decade. At the start of 2002 the population was 15 000. At the start of 2012 it was estimated to be 9 500.

Assume that the population (P) decay is proportional to P, so that $\frac{dP}{dt} = -kP$, where k is a positive constant and (t) is the time in years.

i) Show that $P = 15000e^{-kt}$ satisfies the differential equation. **1**

ii) Find the value of k correct to 4 decimal places. **2**

iii) If the population continues to decrease at the same rate, in what year will the population drop to 5 000? **2**

b) Mr and Mrs Kelly are buying their first home. They have chosen a house valued at \$345 000. After paying a 10% deposit they need to borrow \$310 500 from a bank to pay for the house.

The interest rate charged by their bank is 6.6% per annum, compounded monthly over a 20 year term. Let A_n be the balance owing on the loan after n months. Let the monthly repayments be M.

i) Find an expression for A_n. **2**

ii) Show that the monthly repayments are calculated using the formula: **2**

$$M = \frac{0.0055(1.0055)^{240} \times 310500}{(1.0055)^{240} - 1}.$$

iii) What is the total amount that the Kellys will pay for their house? **1**

c) i) Show that $\log_3[(x-1)(x+4)] = \frac{\ln(x-1) + \ln(x+4)}{\ln 3}$ **2**

ii) State the domain for the function $f(x) = \log_3[(x-1)(x+4)]$. **1**

d) Solve the equation $\cos x = \sqrt{3}\sin x$ for $-\pi < x < \pi$. **2**

End of Question 15

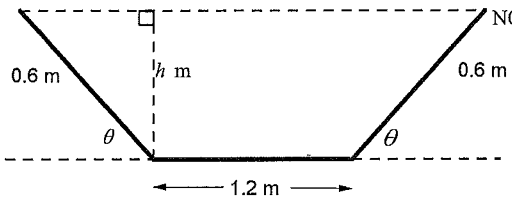
Question 16 (15 Marks) Use a Separate Sheet of paper **Marks**

a) A water overflow channel is to be dug next to a dam.

The base of the channel is to be 1.2 metres wide and 15 metres long.

The channel is dug such that the left and right sides are on an angle of θ with the horizontal. The cross-sectional view of the channel is a trapezium with two sides of length 0.6 m.

The diagram below shows a cross-sectional view of the channel.



i) Find an expression for h in terms of θ. **1**

ii) Show that the volume of the channel is given by **2**

$$V = 5.4(2\sin\theta + \sin\theta\cos\theta) \text{ cubic metres}$$

iii) Find the value of the angle θ so that the volume of the channel is a **maximum**. Give your answer correct to the nearest minute. **4**

b) A rock is dropped from the window of a building. The window is 100 metres above ground level. The acceleration of the rock is approximately -9.8m/s^2 at any time.

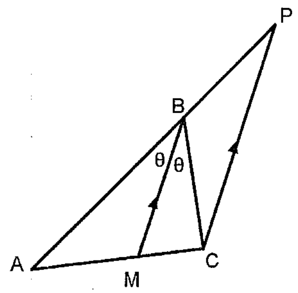
i) Let ground level be the origin and derive equations for velocity and displacement. **2**

ii) How long will it take for the rock to hit the ground? **1**

iii) At what speed will the rock hit the ground? **1**

Marks

c) In $\Delta ABC, BM$ bisects $\angle ABC$.
$PC \parallel BM$.



NOT TO SCALE

i) Prove that ΔBPC is isosceles with $BP = BC$. **2**

ii) Hence show that $\dfrac{AM}{MC} = \dfrac{BA}{BC}$ **2**

End of Examination

WESTERN REGION

2012
TRIAL HSC
EXAMINATION

Mathematics

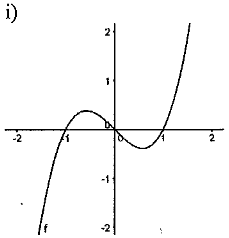
SOLUTIONS

Multiple Choice Answers

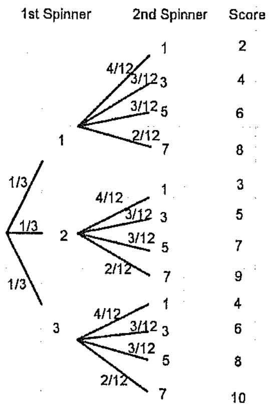
	A	B	C	D
1.	○	○	○	●
2.	○	●	○	○
3.	○	●	○	○
4.	○	●	○	○
5.	○	○	○	●
6.	○	○	●	○
7.	●	○	○	○
8.	○	○	●	○
9.	○	○	○	●
10.	○	○	○	●

Question 11	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
a)	i) $T_n = a+(n-1)d$ $-61 = 5-6(n-1)$ $-61 = 5-6n+6$ $-61 = 11-6n$ $-72 = -6n$ $n = 12$	1	
	ii) $S_n = \frac{n}{2}(2a+(n-1)d)$ $S_{20} = \frac{20}{2}(10+19(-6))$ $= -1040$	1	
b)	$\frac{3x}{x+2} - \frac{5x+19}{x^2+5x+6} = \frac{3x}{x+2} - \frac{5x+19}{(x+2)(x+3)}$ $= \frac{3x(x+3)}{(x+2)(x+3)} - \frac{5x+19}{(x+2)(x+3)}$	1	
	$= \frac{3x^2+9x}{(x+2)(x+3)} - \frac{5x+19}{(x+2)(x+3)}$ $= \frac{3x^2+9x-5x-19}{(x+2)(x+3)}$ $= \frac{3x^2+4x-19}{(x+2)(x+3)}$	1	
c)	$6x^2+x-2=0$ $\frac{(6x+4)(6x-3)}{6} = 0$ $\frac{2(3x+2)3(2x-1)}{6} = 0$ $(3x+2)(2x-1) = 0$	1	Factorised correctly.
	$3x+2=0$ or $2x-1=0$ $3x=-2$ or $2x=1$ $x = -\frac{2}{3}$ or $x = \frac{1}{2}$	1	2 correct solutions

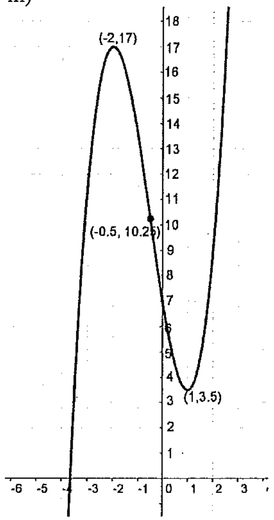
Question 11	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
d)	$y = 5x^2-4x+1$ $y' = 10x-4$ The gradient of the tangent is 6 when y'= 6 $10x-4=6$	1	Let y'= 6
	$10x=10$ $x=1$ When $x=1$, $y = 5(1)^2-4(1)+1$ $=2$ $\therefore$ The point where the gradient is 6, is $(1,2)$	1	2 correct coordinates
e)	$(2+\sqrt{48})(3\sqrt{12}+\sqrt{75}) = 6\sqrt{12}+2\sqrt{75}+3\sqrt{576}+\sqrt{3600}$	1	
	$= 12\sqrt{3}+10\sqrt{3}+3\times24+60$ $= 22\sqrt{3}+132$ $\therefore a=22, b=132$	1	
f)	i) let $u=3x^2$ let $v=\sin x$ $u'=6x$ $v'=\cos x$ $y'=vu'+uv'$	1	
	$\frac{d}{dx}(3x^2\sin x) = 6x\sin x+3x^2\cos x$ $= 3x(2\sin x+x\cos x)$	1	
	ii) $\frac{d}{dx}(\log_e(x^2-1)) = \frac{2x}{x^2-1}$	1	
g)	$\int_{-1}^{1}(3x+4)^3\,dx = \left[\frac{(3x+4)^4}{12}\right]_{-1}^{1}$	1	
	$= \frac{1}{12}\left[(3\times1+4)^4-(3\times-1+4)^4\right]$ $= \frac{1}{12}(7^4-1^4)$ $= \frac{1}{12}(2400)$ $= 200$	1	
		/15	

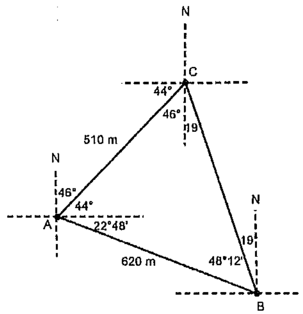
Question 12	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
a)	i)  ii) Since $f(x)$ has 2 turning points, and is cubic, $f'(x)$ will be quadratic and have 1 turning point.	1 1	
b)	i) $m_{AC}=\frac{3-5}{4-1}=-\frac{2}{3}$ $m_{BD}=\frac{1-3}{2--1}=-\frac{2}{3}=m_{AC}$ Since AC and BD have equal gradients, they are parallel.	2	1 gradients 1 for equal and $\parallel$
	ii) $ABCD$ is a parallelogram (2 pairs of opposite sides are parallel) $\therefore AB=CD$ (opposite sides in parallelograms are equal in length) $\angle BAE=\angle CDF$ (alternate angles on parallel lines AB and CD) $\angle BEA=\angle CFD=90°$ (given) $\therefore \Delta BAE\equiv\Delta CFD$ (RHS)	3	Or any correct alternate solution 1 for each reason
	iii) $d=\frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $=\frac{\lvert 4(-1)+1(3)-9\rvert}{\sqrt{4^2+1^2}}$ $=\frac{10}{\sqrt{17}}\times\frac{\sqrt{17}}{\sqrt{17}}$ $=\frac{10\sqrt{17}}{17}$ *units*	1 1	

Question 12	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
b)	iv) The area of parallelogram $ABCD$ is equal to twice the area of ΔABD $d_{AD}=\sqrt{(2-1)^2+(1-5)^2}$ $=\sqrt{1+16}$ $=\sqrt{17}$ $AreaABCD=2\times\frac{1}{2}\times AD\times BE$ $=\sqrt{17}\times\frac{10\sqrt{17}}{17}$ =10 square units	1 1	
c)	$l=r\theta$ $7\pi=10\theta$ $\theta=\frac{7\pi}{10}$ Area$=\frac{1}{2}r^2\theta$ $=\frac{1}{2}\times10^2\times\frac{7\pi}{10}$ $=35\pi$ square units	1 1	
d)	i) $\sum_{n=1}^{5}n^2+3=(1^2+3)+(2^2+3)+(3^2+3)+(4^2+3)+(5^2+3)$ $=4+7+12+19+28$ ii) 70	1 1	
		/15	

Question 13	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
a)	The locus of point P is a circle with centre (1,4) and radius 3 units.	1	
	Its equation is $(x-1)^2+(y-4)^2=9$ or In general form $x^2-2x+y^2-8y+8=0$	1	
b)	i) 	1 1 1	layout correct probabilities shown accurate sample space
	ii) P(odd and >6)= P(7 or 9) $=\left(\frac{1}{3}\times\frac{3}{12}\right)+\left(\frac{1}{3}\times\frac{2}{12}\right)$ $=\frac{5}{36}$	2	1 1

Question 13	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
c)	i) $y=x^3+\frac{3}{2}x^2-6x+7,$ Stationary points at y'= 0 $3x^2+3x-6=0$ $3(x^2+x-2)=0$ $(x+2)(x-1)=0$ $x=-2, x=1$	1	
	When $x=-2, y=(-2)^3+\frac{3}{2}(-2)^2-6(-2)+7$ $=17$ When $x=1, y=1^3+\frac{3}{2}(1)-6(1)+7$ $=3\frac{1}{2}$	1	
	Look at y'' to determine nature of stationary points. $y''=6x+3$ When $x=-2, y''=6(-2)+3$ $<0 \therefore$ maximum turning point When $x=1, y''=6(1)+3$ $>0 \therefore$minimum turning point. Therefore there are stationary points at (-2, 17) local maximum and (1, $3\frac{1}{2}$) local minimum.	1	
	ii) Possible inflexions when y''=0 Let $6x+3=0$ $6x=-3$ $x=-\frac{1}{2}$ Test for concavity change either side of possible inflexion. When $x=0, y''=6(0)+3$ >0 When $x=-\frac{3}{4}, y''=6(-\frac{3}{4})+3$ <0 Therefore concavity changes and we have an inflexion point at $x=-\frac{1}{2}$	1	testing concavity change

Question 13	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
c)	ii) continued When $x=-\frac{1}{2}, y=(-\frac{1}{2})^3+\frac{3}{2}(-\frac{1}{2})^2-6(-\frac{1}{2})+7$ $=10\frac{1}{4}$		
	So inflexion at $(-\frac{1}{2},10\frac{1}{4})$	1	correct coordinates
	iii) 	1 1 1	inflexion turning points y-intercept at 7
		/15	

Question 14	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
a)	i) 	1	for angle C
	$\frac{\sin B}{510}=\frac{\sin 65°}{620}$ $\sin B = 510\times\frac{\sin 65°}{620}$ $B=\sin^{-1}\left(510\times\frac{\sin 65°}{620}\right)$	1	1 for angle B or A
	$\approx 48°12'$ Bearing of B from A $= 90°+22°48'$ $= 112°48'$	1	1 for correct bearing
	ii) $BC^2 = 510^2+620^2-2\times510\times620\cos 66°48'$ $=260100+384400-632400\cos 66°48'$ $=644500-632400\cos 66°48'$ $\therefore BC=\sqrt{644500-632400\cos 66°48'}$ metres	1	

Question 14	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
b)	i) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$		
	$=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$	1	
	$=\frac{\left(\frac{1}{2}\right)^2-2\left(\frac{-15}{2}\right)}{\left(\frac{-15}{2}\right)^2}$	1	
	$=\frac{61}{225}$		
	ii) $2x^2-x-15\equiv Px(x-2)+Q(x-2)+R$ $=Px^2-2Px+Qx-2Q+R$ $=Px^2+(Q-2P)x-2Q+R$	1	
	$\therefore P=2$ (1) $Q-2P=-1$ (2) $-2Q+R=-15$ (3)	1	
	Sub (1) into (2) : $Q-2(2)=-1$ $Q-4=-1$ $Q=3$ (4)		
	Sub (4) into (3) $-2(3)+R=-15$ $-6+R=-15$ $R=-9$		
	$\therefore 2x^2-x-15\equiv 2x(x-2)+3(x-2)-9$	1	

Question 14	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
c)	i) $\int_2^6 \frac{1}{x}dx\approx\frac{1}{2}\left(\frac{1}{2}+2\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right)+\frac{1}{6}\right)$	1	
	$=\frac{67}{60}$ ≈ 1.1167 square units	1	
	ii) $\int_2^6 \frac{1}{x}dx=[\ln x]_2^6$ $=\ln 6-\ln 2$ $=\ln 3$ ≈ 1.0986 square units	1	
	iii) The function is concave up between $x=2$ and $x=6$, so Trapezoidal Rule will over estimate.	1	
	iv) $V=\pi\int_2^6 \frac{1}{x^2}dx$ $=\pi\left[-\frac{1}{x}\right]_2^6$	1	
	$=\pi\left(-\frac{1}{6}-\left(-\frac{1}{2}\right)\right)$ $=\frac{\pi}{3}$ cubic units	1	
		/15	

Question 15	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
a)	i) Substituting $P=15000e^{-kt}$ into the differential equation $\frac{dP}{dt}=-kP$ gives: LHS= $\frac{dP}{dt}$ $=\frac{d}{dt}(15000e^{-kt})$ $=-k15000e^{-kt}$ $=-kP$ =RHS And, when $t=0, P=15000e^{0}=15000$ ii) $P=15000e^{-kt}$ $9500=15000e^{-10k}$ $e^{-10k}=\frac{9500}{15000}$ $-10k=\ln\left(\frac{9500}{15000}\right)$ $k=-\frac{\ln\left(\frac{9500}{15000}\right)}{10}=0.04567584$ ≈ 0.0457 iii) $P=15000e^{-0.0457t}$ $5000=15000e^{-0.0457t}$ $e^{-0.0457t}=\frac{5000}{15000}$ $t=-\frac{\ln\left(\frac{5000}{15000}\right)}{.04567584}$ $t=24.05$ years ∴ The population will drop to 5000 in the year 2026.	1 1 1 1 1	
b)	i) $A_1=1.0055(310500)-M$ $A_2=(1.0055)^2(310500)-(1.0055)M-M$ $A_3=(1.0055)^3(310500)-(1.0055)^2M-(1.0055)M-M$ $A_n=(1.0055)^n(310500)-M(1+1.0055+(1.0055)^2+...+(1.0055)^{n-1})$	1 1	

Question 15	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
b)	ii) $A_{240}=0$ $(1.0055)^{240}(310500)-M(1+1.0055+(1.0055)^2+...+(1.0055)^{239})=0$ $M=\frac{(1.0055)^{240}(310500)}{(1+1.0055+(1.0055)^2+...+(1.0055)^{239})}$ The denominator is a geometric series with $a=1, r=1.0055, n=240$. $S_n=\frac{a(r^n-1)}{r-1}$ $=\frac{((1.0055)^{240}-1)}{0.0055}$ $\therefore M=\frac{(1.0055)^{240}(310500)}{\frac{(1.0055)^{240}-1}{0.0055}}$ $=\frac{(1.0055)^{240}(310500)\times 0.0055}{(1.0055)^{240}-1}$ iii) M=\$2333.32 Total Repayments = 240×2333.32 = \$559996.99 Total Cost = Deposit + repayments = \$594496.99	1 1 1	
c)	i) $\log_3[(x-1)(x+4)]=\log_3(x-1)+\log_3(x+4)$ $=\frac{\ln(x-1)}{\ln 3}+\frac{\ln(x+4)}{\ln 3}$ $=\frac{\ln(x-1)+\ln(x+4)}{\ln 3}$ ii) Domain : all $x>1$	1 1 1	
d)	$\cos x=\sqrt{3}\sin x$ $1=\sqrt{3}\tan x$ $\tan x=\frac{1}{\sqrt{3}}$ $x=\frac{\pi}{6},-\frac{5\pi}{6}$	1 1	
		/15	

Question 16	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
a)	i) $h = 0.6\sin\theta$ m	1	
	ii) Area of trapezium $= \frac{1}{2}h(a+b)$ where $a = 1.2, b = 1.2 + 1.2\cos\theta, h = 0.6\sin\theta$		
	$A = \frac{1}{2}(0.6\sin\theta)(1.2 + 1.2 + 1.2\cos\theta)$	1	
	$= (0.6\sin\theta)(1.2 + 0.6\cos\theta)$ $= 0.6\sin\theta(0.6)(2 + \cos\theta)$ $= 0.36(2\sin\theta + \sin\theta\cos\theta)$ $V = 15 \times 0.36(2\sin\theta + \sin\theta\cos\theta)$ $= 5.4(2\sin\theta + \sin\theta\cos\theta)$	1	
	iii) Stationary points at V'=0 $V' = 5.4(2\cos\theta + 2\cos^2\theta - 1)$ Let $2\cos^2\theta + 2\cos\theta - 1 = 0$ $\cos\theta = \frac{-2 \pm \sqrt{2^2 - 4(2)(-1)}}{4}$ $= \frac{-2 \pm \sqrt{12}}{4}$ $= \frac{-1 \pm \sqrt{3}}{2}$	1	
	$\cos\theta = \frac{-1-\sqrt{3}}{2}$ or $\cos\theta = \frac{-1+\sqrt{3}}{2}$	1	
	No solutions $\theta = 68°32'$	1	
	(For the situation, $0° < \theta < 90°$) Check that this is a maximum turning point. $V'' = 5.4(-2\sin\theta - 4\cos\theta\sin\theta)$ When $\theta = 68°32', A'' = 5.4(-2\sin 68°32' - 4\sin 68°32'\cos 68°32')$ $< 0 \therefore$ maximum The volume is a maximum when $\theta = 68°32'$..	1	

Question 16	Trial HSC Examination - Mathematics		2012
Part	**Solution**	**Marks**	**Comment**
b)	i) $\ddot{x} = -9.8$ $\dot{x} = -9.8t + C$ When $t = 0, \dot{x} = 0 \therefore C = 0$ $\dot{x} = -9.8t$	1	
	$x = -4.9t^2 + C$ When $t = 0, x = 100 \therefore C = 100$ $x = -4.9t^2 + 100$	1	
b)	ii) When the rock hits ground displacement will be zero. $-4.9t^2 + 100 = 0$ $t^2 = \frac{100}{4.9}$ $t = \sqrt{\frac{100}{4.9}}$ ≈ 4.52 seconds	1	Ignore rounding
	iii) When $t = 4.52, \dot{x} = -9.8(4.52)$ $= 44.27$ m/s	1	
c)	i) $BM \parallel PC$ (given) $\angle P = \angle ABM = \theta$ (corresponding angles on parallel lines)	1	
	$\angle PCB = \angle CBM = \theta$ (alternate angles on parallel lines) $\therefore \Delta BPC$ is isosceles with $BP = BC$.	1	
	ii) $\frac{AM}{MC} = \frac{BA}{BP}$ (ratio of intercepts on parallel lines) But $BP = BC$ (as shown above)	1	
	$\therefore \frac{AM}{MC} = \frac{BA}{BC}$	1	
		/15	