Student Name: ____________________

St. Catherine's School
Waverley

22nd June 2011

PRELIMINARY ASSESSMENT TASK 3

Weighting 25%

Extension 1 Mathematics

Time allowed: *60 minutes*
Total marks: *50 marks*

INSTRUCTIONS

- Marks for each part of a question are indicated
- All questions should be attempted in the booklet provided
- All necessary working should be shown
- Start each question on a new page
- Approved scientific calculators and drawing templates may be used badly arranged work.
- Diagrams should be drawn using PENCIL AND RULER

Question 1 (14 Marks) **Marks**

(a) The equation $x^2-6x+2=0$ has roots α and β. Find the values of

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ **2**

(b) Find the value(s) for k for which the equation $kx^2+(k-3)x+1=0$ has real roots. **2**

(c) Write $5x^2-2x+6$ in the form $Ax(x-1)+B(x-2)+C$. **3**

(d) Solve the quadratic equation $9^x-12.3^x+27=0$ **3**

(e) Find $\lim_{x\to 8}\left(\frac{x^2-5x-24}{x-8}\right)$ **2**

Question 2 (11 Marks)

(a) Suppose that P is the point $(-4,7)$ and Q is the point $(1,-3)$.

(i) Find the point R which divides the interval PQ internally in the ratio $c:1$. **2**

(ii) If the point R is $(-2, 3)$ find the coordinates of the point T that divides PQ externally in the ratio $c:1$. **3**

(b) The two lines $x-3y+7=0$ and $2x+y-3=0$ intersect at a point.

(i) Find the equation of the line passing through the intersection which also passes through the point (2, -3) **2**

(ii) Find the acute angle between the two intersecting lines to the nearest minute **2**

(c) When the line $3x-4y+7=0$ is closest to the point $P(1,-3)$ how far is it from the point P? **2**

Question 3 (12 Marks)

(a) If $f(x)=\frac{3}{2-x}$, find $f'(x)$. **2**

(b) Find $\frac{dy}{dx}$ given $y=\frac{5x^2}{x+1}$ **2**

(c) Find the derivative of $\sqrt{x}-\frac{1}{x^2}$. **2**

(d) Find the equation of the normal to the curve $y=\sqrt{x^3-2x}$ at the point where $x=-1$. **3**

(e) Find the values of x where the curve $y=(x-2)^3(2x-3)^4$ is horizontal **3**

Question 4 (13 marks)

(a) A The variable point $P(x, y)$ moves so that it is equidistant from the fixed point S (3, 3) and the fixed line $y + 1 = 0$. Let L be the point $(x, -1)$

(i) Sketch the information in the question letting $P(x, y)$ be in the 1st quadrant. **1**

(ii) By letting $PS^2 = PL^2$, find the equation of the locus of the point P **2**

(iii) Describe the locus geometrically **2**

(b) For the curves $f(x)=x^2$ and $g(x)=(x-2)^2$;

(i) Their point of intersection **1**

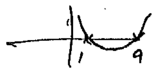
(ii) The acute angle between the curves at their point of intersection. Give your answer to the nearest minute. **2**

(c) The acute angle between the lines $y=3x+2$ and $y=8x-2$ is θ.

(i) Show that $\tan\theta=\frac{1}{5}$. **2**

(ii) The acute angle between the lines $y=3x-7$ and $y=mx-4$ is also θ. Find the value of m. **3**

YEAR 11 EXTENSION 1 MATHEMATICS SOLUTIONS
TASK 2

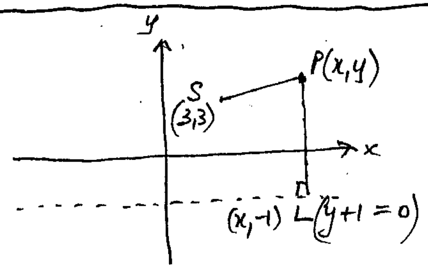
Qn	Solution	Marks	Comments: Criteria
Q1	a) $x^2-6x+2=0$		
	(i) $\alpha+\beta=6$	1	
	(ii) $\alpha\beta=-2$	1	
	(iii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$	$\frac{1}{2}$	
	$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	$\frac{1}{2}$	
	$=\frac{36+4}{-2}$		
	$=-20$	1	
(b)	$kx^2+(k-3)x+1=0$ $\Delta\geq 0$: $(k-3)^2-4k\geq 0$ $k^2-10k+9\geq 0$ $(k-9)(k-1)\geq 0$  $k\leq 1$ or $k\geq 9$		$\frac{1}{2}$ for DISCR $\frac{1}{2}$ for ≥ 0 1 for solution
(c)	$5x^2-2x+6\equiv Ax(x-1)+B(x-2)+C$		
	$A=5$	1	
	Let $x=2$ $\quad 22=10+C$ $\therefore C=12$	1	
	let $x=0$ $\quad 6=-2B+12$ $\therefore B=3$	1	
	$\therefore 5x^2-2x+6\equiv 5x(x-1)+3(x-2)+12$ *		* $-\frac{1}{2}$ if not in this form
d)	$9^x-12.3^x+27=0$		
	$3^{2x}-12.3^x+27=0$	$\frac{1}{2}$	
	$(3^x-3)(3^x-9)=0$	$\frac{1}{2}$	
	$\therefore 3^x=3,9$	1	
	$\therefore x=1,2$	1	

Qn	Solution	Marks	Comments: Criteria
e)	$\lim_{x\to 8}\frac{x^2-5x-24}{x-8}$		
	$=\lim_{x\to 8}\frac{\cancel{(x-8)}(x+3)}{\cancel{(x-8)}}$	1	
	$=11$	1	
Q2 a)	$P(-4,7)$ $\quad c:1$ $\quad R$ $\quad Q(1,-3)$		
	(i) $R\left(\frac{-4+c}{c+1},\ \frac{7-3c}{c+1}\right)$		
	(ii) If R is the point $(-2,3)$ $\frac{-4+c}{c+1}=-2$ OR $\frac{7-3c}{c+1}=3$ $-4+c=-2c-2$ $\quad 7-3c=3c+3$ $3c=2$ $\quad 4=6c$ $c=\frac{2}{3}$ $\quad \frac{2}{3}=c$		
	$\therefore R$ divides PQ internally in ratio $\frac{2}{3}:1$ OR $2:3$	1	
	$\therefore T$ divides PQ externally in ratio $\frac{2}{3}:1$ OR $2:3$		
	$P(-4,7)$ $\quad -2:3$ $\quad Q(1,-3)$ $\therefore T\left(\frac{-12-2}{1},\ \frac{21+6}{1}\right)$ $T(-14,27)$	2	-1 for incorrect ratio
	OR let ratio be $c:-1$ (or $-c:1$) $\left(\frac{c+4}{c-1},\frac{-3c-7}{c-1}\right)$ OR $\left(\frac{-c-4}{-c+1},\frac{3c+7}{-c+1}\right)$		2

then sub $c=\frac{2}{3}$

Qn	Solutions	Marks	Comments+Criteria
Q2) b)	$x-3y+7=0 \qquad 2x+y-3=0$		OR
	(i) $x-3y+7+k(2x+y-3)=0$	½	$2x-6y=-14$ —(1)
	$(2,-3)$ $2+9+7+k(4-3-3)=0$		$2x+y=3$ —(2)
	$k=\frac{-18}{-2}$		$7y=17$ ½
	$k=9$	½	$y=\frac{17}{7}$
	$\therefore$ the required line is		$2x=3-\frac{17}{7}$
	$x-3y+7+9(2x+y-3)=0$	½	$x=\frac{2}{7}$ ½
	$x-3y+7+18x+9y-27=0$		$\left(\frac{2}{7},\frac{17}{7}\right)$ and $(2,-3)$
	$19x+6y-20=0$	½	$\therefore m=\frac{-19}{6}$ ½
			$y+3=-\frac{19}{6}(x-2)$
			$19x+6y-20=0$ ½
	(ii) $m_1=\frac{1}{3} \qquad m_2=-2$		
	$\tan\theta=\left\lvert\frac{\frac{1}{3}+2}{1+\frac{1}{3}\cdot-2}\right\rvert$	1	
	$=\lvert 7\rvert$		
	$\therefore \theta=\tan^{-1}(7)$		
	$=81^\circ 52'$	1	$-\frac{1}{2}$ for rounding
c)	$3x-4y+7=0 \qquad P(1,-3)$		
	$d=\left\lvert\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right\rvert$		
	$=\frac{3+12+7}{\sqrt{9+16}}$	1	
	$=\frac{22}{5}$ units	1	

Qn	Solution	Marks	Comments: Criteria
Q3 a)	$f(x)=\frac{3}{2-x} = 3(2-x)^{-1}$	½	
	$\therefore f'(x)=-3(2-x)^{-2}\cdot -1$	1	
	$=\frac{3}{(2-x)^2}$	½	
b)	$y=\frac{5x^2}{x+1}$		
	$\frac{dy}{dx}=\frac{(x+1).10x-5x^2.1}{(x+1)^2}$	1	
	$=\frac{10x^2+10x-5x^2}{(x+1)^2}$		
	$=\frac{5x^2+10x}{(x+1)^2}$ OR $\frac{5x(x+2)}{(x+1)^2}$	1	
c)	$\frac{d}{dx}\left(\sqrt{x}-\frac{1}{x^2}\right)=\frac{d}{dx}\left(x^{1/2}-x^{-2}\right)$	½	
	$=\frac{1}{2}x^{-1/2}+2x^{-3}$	1	
	$=\frac{1}{2\sqrt{x}}+\frac{2}{x^3}$	½	
d)	$y=\sqrt{x^3-2x}=(x^3-2x)^{1/2}$	½	
	$y'=\frac{1}{2}(x^3-2x)^{-1/2}.(3x^2-2)$		
	$=\frac{3x^2-2}{2\sqrt{x^3-2x}}$	½	
	at $(-1,1)$ $y'=\frac{1}{2}$	½	
	$\therefore$ gradient of normal at $(-1,1)$ is -2	½	
	$y-1=-2(x+1)$		
	$y-1=-2x-2$	1	
	$2x+y+1=0$ is equation of normal		

Qn	Solutions	Marks	Comments+Criteria
3)e)	$y=(x-2)^3(2x-3)^4$		
	$y'=(2x-3)^4\cdot 3(x-2)^2+(x-2)^3\cdot 4(2x-3)^3\cdot 2$	1	
	$=(2x-3)^3(x-2)^2\left[3(2x-3)+8(x-2)\right]$		
	$=(2x-3)^3(x-2)^2(14x-25)$	1	-1 for no $x=\frac{25}{14}$
	$=0$ for horizontal		
	$x=\frac{3}{2},\ 2,\ \frac{25}{14}$	1	
4)a) (i)		1	$-\frac{1}{2}$ for any missing feature
(ii)	$PS^2=(x-3)^2+(y-3)^2$		
	$PL^2=(y+1)^2$	1	
	Now $(x-3)^2+(y-3)^2=(y+1)^2$		
	$(x-3)^2=\cancel{y^2}+2y+1-\cancel{y^2}+6y-9$		
	$(x-3)^2=8y-8$		
	$(x-3)^2=4.2(y-1)$	1	
	OR $x^2-6x-8y+17=0$		
(iii)	Locus is a parabola with	½	
	vertex $(3,1)$	½	
	focal length 2	½	
	focus $(3,3)$ given	½	
	directrix $y=-1$		

Qn	Solution	Marks	Comments: Criteria
Q4 a)	$f(x)=x^2 \qquad g(x)=(x-2)^2$		
(i)	$x^2=x^2-4x+4$	½	
	$4x=4$		
	$x=1 \quad y=1$		
	∴ point of intersection $(1,1)$	½	
(ii)	$f'(x)=2x \qquad g'(x)=2(x-2)$	½	
	at $(1,1)$ $f'(x)=2 \qquad g'(x)=-2$		
	$\therefore \tan\theta=\left\|\frac{2+2}{1-4}\right\|$		
	$=\left\|-\frac{4}{3}\right\|$	½	
	$=\frac{4}{3}$		
	$\therefore \theta=53°8'$	1	
c) (i)	$y=3x+2 \qquad y=8x-2$		
	$\tan\theta=\left\|\frac{3-8}{1+24}\right\|$		
	$=\frac{1}{5}$		
(ii)	$y=3x-7 \qquad y=mx-4$		
	$\tan\theta=\left\|\frac{3-m}{1+3m}\right\|=\frac{1}{5}$	1	
	$\therefore \frac{3-m}{1+3m}=\pm\frac{1}{5}$	1	-1 for only one solution
	$\therefore 15-5m=1+3m$ OR $15-5m=-1-3m$	1	
	$14=8m \qquad 16=2m$		
	$\frac{7}{4}=m \qquad 8=m$		