St George Girls High School

Year 11

Common Test – 2

2011

Mathematics

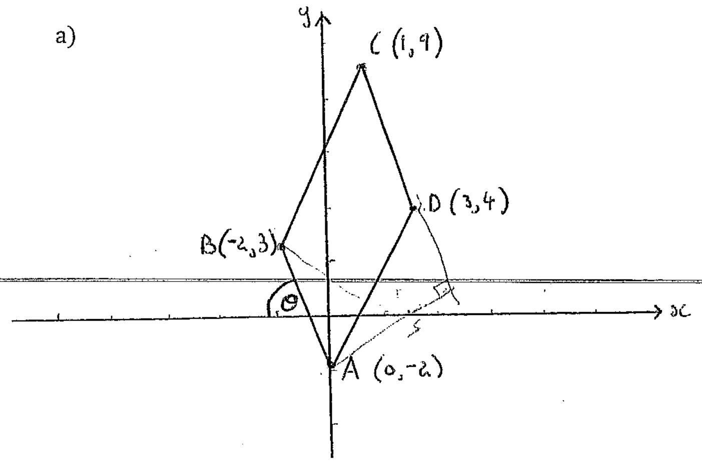
Instructions

1. Working time – 75 minutes
2. All questions should be attempted.
3. Show all working.
4. **Start each question on a new page.**
5. Place Attached Sheet (I) Question 2(e) and (f) and Attached sheet (II) Question 3(b) with relevant question.
6. Marks will be deducted for careless work or poorly presented solutions.
7. On the cover sheet of the answer booklet clearly show:

   a) your name
   b) your mathematics class and teacher

Question 1: (15 Marks) – Start A New Page Marks

a)



(i) Find the gradient of AB. 2

(ii) Find θ correct to the nearest degree. 2

(iii) Find the co-ordinates for M, the mid-point of AC. 1

(iv) Prove $ABCD$ is a parallelogram, using part (iii). 2

(v) Show that the equation of the line DC is $5x + 2y - 23 = 0$ 2

(vi) Find the perpendicular distance from the point A to the line DC. 2

(vii) Find the area of the quadrilateral $ABCD$. 2

b) Suppose that
$$g(x) = \begin{cases} 1 - x^2 & , \text{ for } x < 0 \\ x + 1 & , \quad 0 \le x < 1 \\ \frac{2}{x} & , \text{ for } x \ge 1 \end{cases}$$

Evaluate $g(2) - g(-2)$ 2

Question 2: (15 Marks) – Start A New Page Marks

a) Simplify: 2

$$\frac{1-\sin^2\theta}{\sin(90^\circ-\theta)}$$

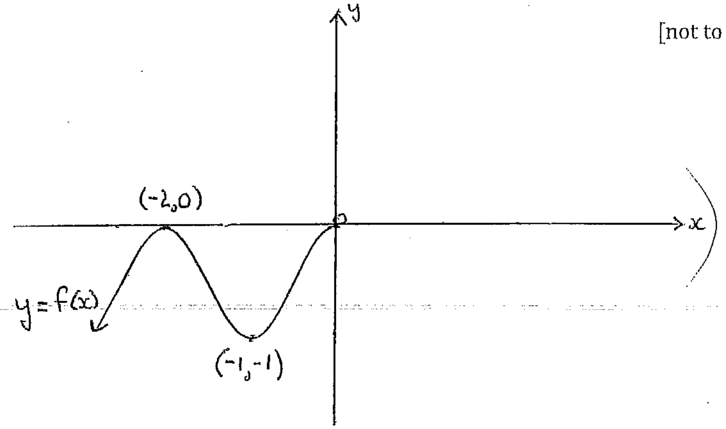
b) If $\cos\beta = \frac{2}{7}$ and $\tan\beta < 0$ find the EXACT value of $\sin\beta$ 2

c) $2\sin^2 x - \sin x = 0$ for $0^\circ \le x^\circ \le 360^\circ$, solve for x. 3

d) Solve for x : $2x^2 - x + 1 < 0$ 2

e) The graph below should be completed on the attached sheet (I) so that it represents an ODD function. 2

[not to scale]



f) (i) Find the points of intersection of $y = x^2 - 3x$ and $y = x + 5$. 2

(ii) Shade the region which shows the intersection of $y \le x^2 - 3x$ and $y > x + 5$ 2

Use axes on attached sheet (I)

Question 3: (15 Marks) – Start A New Page Marks

a) Find:

(i) the natural domain

(ii) the range

for each function

(α) $f(x) = |x+1|$ 2

(β) $y(x) = \frac{1}{\sqrt{16-x^2}}$ 2

b) Using attached sheet (II), sketch on the given axes

(i) $y = f(x-1)$ 2

(ii) $y = -f(x)$ 2

given the graph of $f(x)$ is

c) Let Q be the point of intersection of the lines, L_1 : $2x + 7y - 1 = 0$ and L_2 : $x + 5y + 13 = 0$

Do not find the co-ordinates of Q. Write down the equation of the general line through Q, using $L_1 + k\,L_2 = 0$. Hence, find the equation of the line through Q and the point $P(-1, 2)$. 3

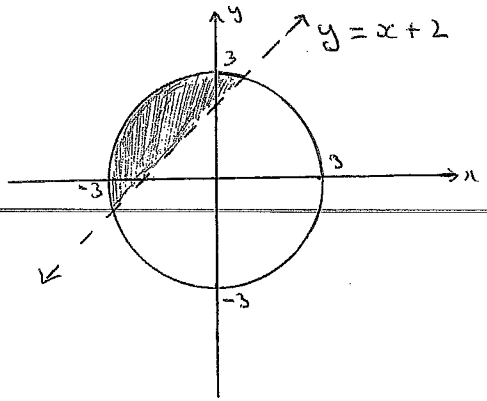
d) Is the perpendicular bisector of the line interval joining $A(3,0)$ to $B(1,4)$ a tangent to the circle $(x-5)^2 + y^2 = 4$? 4

[Hint: find the equation of the perpendicular bisector of AB]

Question 4: (15 Marks) – Start A New Page Marks

a) Write down the inequations which describe the shaded region shown. 2

[not to scale]



b) (i) When asked to solve the simultaneous equations 2

$$y = 3x^3$$
and
$$y = 12x$$

Amy wrote this solution	Beth wrote this solution
$y = 3x^3 \ldots$ Ⓐ	$y = 3x^3 \ldots$ Ⓐ
$y = 12x \ldots$ Ⓑ	$y = 12x \ldots$ Ⓑ
Let Ⓐ = Ⓑ	Let Ⓐ = Ⓑ
$3x^3 = 12x$	$3x^3 = 12x$
$3x^2 = 12$	$3x^3 - 12x = 0$
$x^2 = 4$	$3x(x^2 - 4) = 0$
$x = \pm 2$	$x = 0, -2, 2$
Solution $(-2, -24)$ and $(2, 24)$	Solution $(0, 0)$ and $(-2, -24)$ and $(2, 24)$

Explain why Amy's solution has an error in it.

(ii) Use graphical methods to solve for all values of x which satisfy 2

$$3x^3 < 12x$$

Question 4 (cont'd) Marks

c) If $F(x) = x^2$, find in simplest form $\frac{F(a)-F(b)}{2a+2b}$ 3

d) A plane flew from airport A for 1200 km to airport B on a bearing of 110° T. It then flew 800 km from airport B to airport C on a bearing of 050° T.

(i) With the aid of a diagram (neatly sketched) explain why $\angle ABC = 120°$. 2

(ii) Find the distance between airports A and C [answer to nearest km]. 2

(iii) Find the bearing of airport A from airport C. 2

End of Paper

St George Girls High School

Year 11 Common Test # 2 2011 June

Question 1:

a) (i) $m_{AB} = \frac{3 - ^-2}{-2-0}$

$= \frac{5}{-2}$

$= -2\frac{1}{2}$

(ii) $\tan(180^\circ - \theta) = m_{AB} = -2\frac{1}{2}$

$\therefore \tan(180^\circ - \theta) = -2.5$

$180^\circ - \theta = 111^\circ 48'$

$\therefore \theta = 68.1985...^\circ$ (acute)

ie $\theta = 68^\circ$ (to nearest degree)

S	A
T	C

(iii) $M_{AC}\left(\frac{1+0}{2}, \frac{9 + ^-2}{2}\right)$

ie $M_{AC}\left(\frac{1}{2}, 3\frac{1}{2}\right)$

(iv) $M_{BD}\left(\frac{-2+3}{2}, \frac{3+4}{2}\right)$ ie $M_{BD}\left(\frac{1}{2}, 3\frac{1}{2}\right)$

Since M_{AC} is same as M_{BD} then diagonals BD and AC bisect each other

$\therefore$ ABCD is a parallelogram.

OR $m_{BC} = \frac{9-3}{1-^-2}$ $= \frac{6}{3}$ $= 2$ $= m_{AD}$

$m_{AD} = \frac{4-^-2}{3-0}$ $= \frac{6}{3}$ $= 2$

$m_{DC} = \frac{9-4}{1-3}$ $= \frac{5}{-2}$ $= -2\frac{1}{2}$ $= m_{AB}$

Since opposite sides of quadrilateral ABCD are parallel then ABCD is a parallelogram

(v) Since ABCD is a parallelogram m_{AB} (part i) $= m_{CD}$

$\therefore$ Equation of line DC is $\frac{y-9}{x-1} = \frac{-5}{2}$

$2y - 18 = -5x + 5 \Rightarrow 5x + 2y - 23 = 0$

(vi) $p_{A \to DC} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|5 \times 0 + 2 \times -2 - 23|}{\sqrt{5^2 + 2^2}}$

$= \frac{|-27|}{\sqrt{25+4}}$

Perpendicular distance $= \frac{27}{\sqrt{29}}$ units

(vii) $d_{AB} = \sqrt{(-2-0)^2 + (3-^-2)^2}$

$= \sqrt{4 + 25}$

$= \sqrt{29}$

Area_{ABCD} = base × perpendicular height

$= \sqrt{29} \times \frac{27}{\sqrt{29}}$ units^2

$= 27$ units^2

b) $g(2) - g(-2) = \frac{2}{2} - (1 - (-2)^2)$

$= 1 - (1 - 4)$

$= 1 - ^-3$

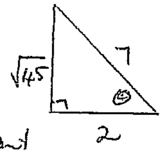
$= 4$

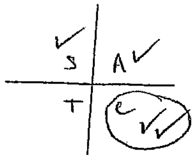
Question 2

a) $\dfrac{1-\sin^2\theta}{\sin(90^\circ-\theta)} = \dfrac{\cos^2\theta}{\cos\theta}$, $\sin^2\theta+\cos^2\theta=1$

$= \cos\theta$, $\cos\theta \neq 0$

b) $\cos\beta = \frac{2}{7}$ and $\tan\beta < 0$





β is an angle in 4th quadrant

$\therefore \sin\beta = -\dfrac{\sqrt{45}}{7}$

c) $2\sin^2 x - \sin x = 0$ $\qquad 0^\circ \leq x^\circ \leq 360^\circ$

$\sin x(2\sin x - 1) = 0$

$\therefore \sin x = 0$ OR $2\sin x - 1 = 0$

$\therefore x^\circ = 0^\circ, 180^\circ, 360^\circ$

$\sin x = \frac{1}{2}$

$x^\circ = 30^\circ, 180^\circ - 30^\circ$

ie $x^\circ = 0^\circ, 30^\circ, 150^\circ, 180^\circ, 360^\circ$

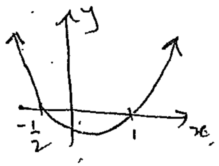
d) $2x \times \begin{smallmatrix}+1\\-1\end{smallmatrix}$, x

$2x^2 - x - 1 < 0$

Consider $2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

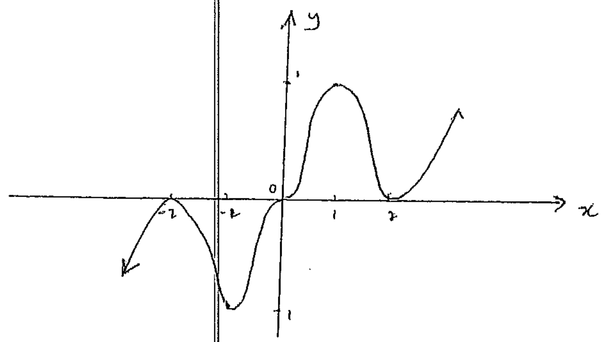
$x = -\frac{1}{2}$ OR $x = 1$



Thus, $2x^2 - x - 1 < 0$

for $-\frac{1}{2} < x < 1$

e)



f) (i) $y = x^2 - 3x$ — (1)

$y = x + 5$ — 2

① = ② $\quad x^2 - 3x = x + 5$

$x^2 - 4x - 5 = 0$

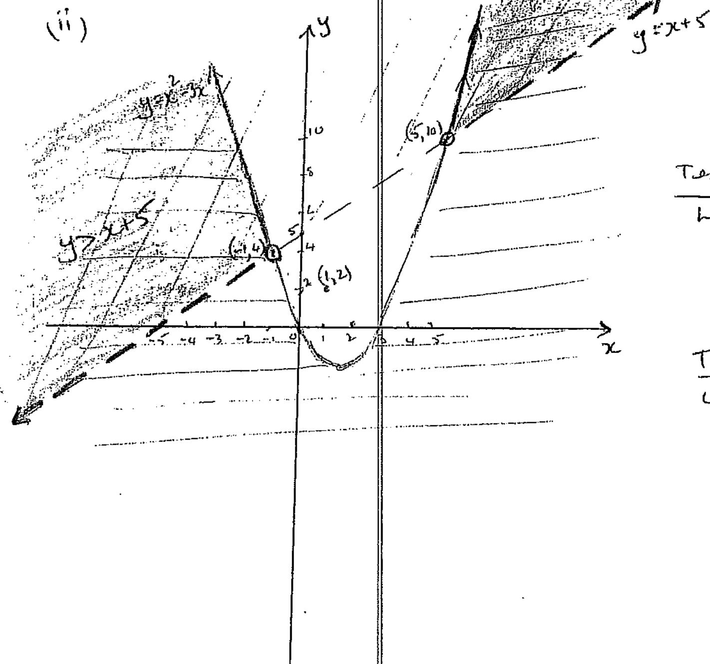
$(x-5)(x+1) = 0$

$x \times \begin{smallmatrix}1\\-5\end{smallmatrix}$, x

$\therefore \begin{cases} x = 5 \\ y = 10 \end{cases}$ OR $\begin{cases} x = -1 \\ y = 4 \end{cases}$

ie Points of intersection are $(5,10)$ and $(-1,4)$

(ii)



$y = x^2 - 3x$

$= x(x-3)$

Test $(1,2)$

LHS $= y$ $\quad$ RHS $= x^2 - 3x$

$= 2$ $\quad = 1^2 - 3\times 1$

$= -2$

$2 > -2$

Test $(0,0)$

LHS $= y$ $\quad$ RHS $= x + 5$

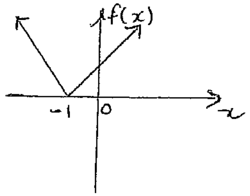
$= 0$ $\quad = 0 + 5$

$= 5$

$0 < 5$

Question 3

a) α) (i) $f(x) = |x+1|$ Domain: all Real numbers

(ii) 

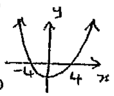
Range: $y \in$ Reals, $y \geq 0$

β) (i) $y(x) = \dfrac{1}{\sqrt{16-x^2}}$

Domain: $16 - x^2 > 0$

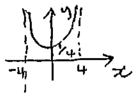
$x^2 - 16 < 0$

$(x-4)(x+4) < 0$



$\therefore -4 < x < 4$

Function is even $\therefore x = 0 \Rightarrow y = \frac{1}{4}$



Range: $y > \frac{1}{4}$

b) Sheet

c) $L_1 + kL_2 = 0 \Rightarrow 2x + 7y - 1 + k(x + 5y + 13) = 0$

Now $P(-1, 2)$ lies on this line

$\therefore 2\times -1 + 7 \times 2 - 1 + k(-1 + 5 \times 2 + 13) = 0$

$-2 + 14 - 1 + k(-1 + 10 + 13) = 0$

$11 + k(22) = 0$

$k = \frac{-11}{22} = -\frac{1}{2}$

$\therefore$ Equation of line PQ is

$2x + 7y - 1 - \frac{1}{2}(x + 5y + 13) = 0$

$4x + 14y - 2 - x - 5y - 13 = 0$

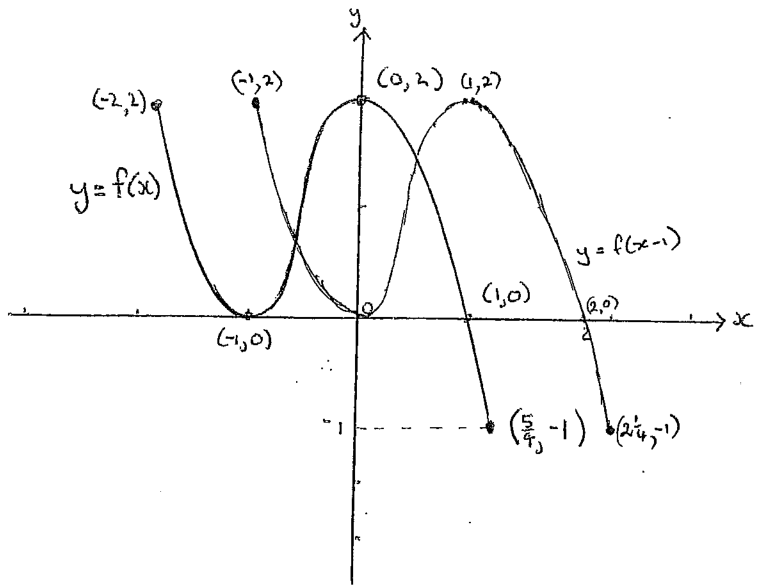
ie $3x + 9y - 15 = 0$

ie $x + 3y - 5 = 0$

Attached – Sheet II

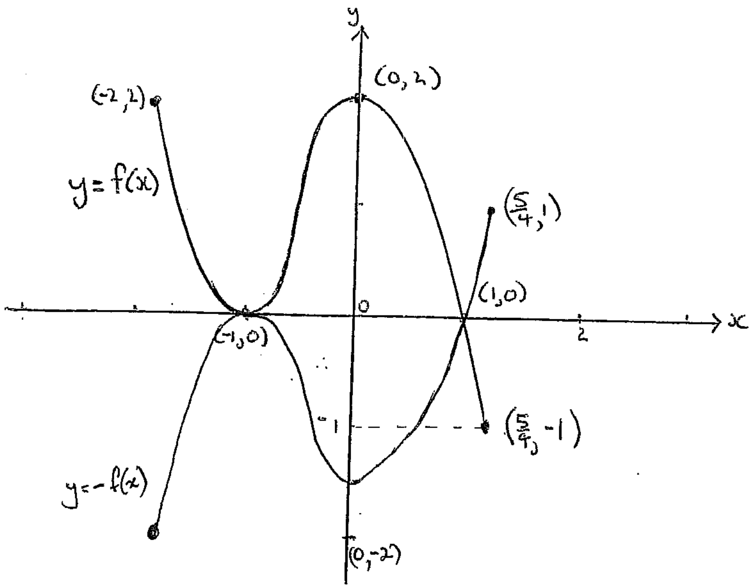
Question 3 b)(i)

$y = f(x-1)$



Question 3 b)(ii)

$y = -f(x)$



d) $m_{AB} = \frac{0-4}{3-1}$ $\quad M_{AB}\left(\frac{3+1}{2}, \frac{0+4}{2}\right)$

$= \frac{-4}{2}$ $\quad (2, 2)$

$= -2$

Gradient of line perpendicular to AB is $\frac{1}{2}$

$\therefore$ Equation of perpendicular bisector AB is

$$y - 2 = \frac{1}{2}(x - 2)$$

$$2y - 4 = x - 2$$

$$0 = x - 2y + 2$$

Centre of circle is $(5, 0)$ radius 2 units.

If perpendicular distance of $(5,0)$ to $x - 2y + 2 = 0$ is 2 units then $x - 2y + 2 = 0$ is tangent

$$P_{\text{Centre} \to AB} = \frac{|5 \times 1 - 2 \times 0 + 2|}{\sqrt{1^2 + (-2)^2}}$$

$$= \frac{|5+2|}{\sqrt{1+4}}$$

$$= \frac{7}{\sqrt{5}} \text{ units} \neq 2$$

$\therefore$ Perpendicular bisector of AB is not tangent to circle.

Question 4

a) $x^2 + y^2 \leq 9 \quad \cap \quad y > x + 2$

and

b) (i) Amy's solution has an error at the step where $3x^3 = 12x$ became $3x^2 = 12$ because she divided by x and if x were zero this is not possible. In this case $x = 0$ is a possible solution since

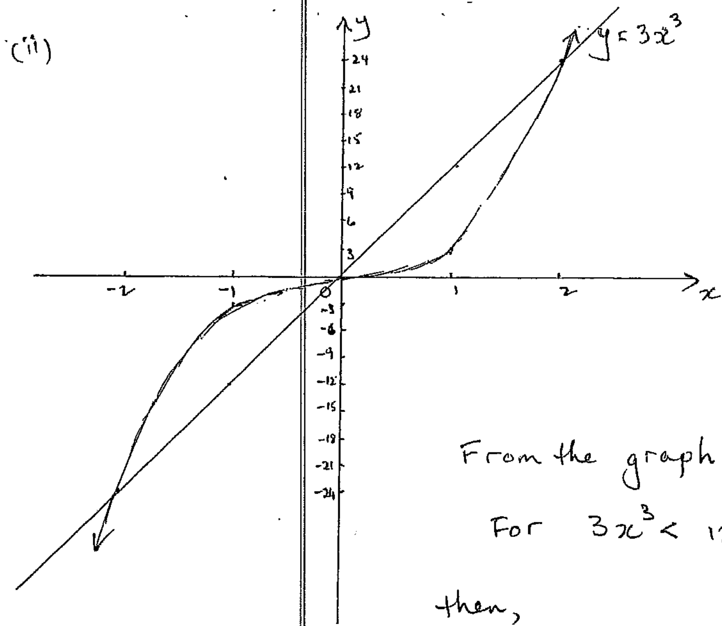
LHS. $= 3x^3$ $\quad$ RHS $= 12x$

$= 3 \times 0^3$ $\quad = 12 \times 0$

$= 0$ $\quad = 0$

LHS = RHS

(ii)



From the graph:

For $3x^3 < 12x$

then,

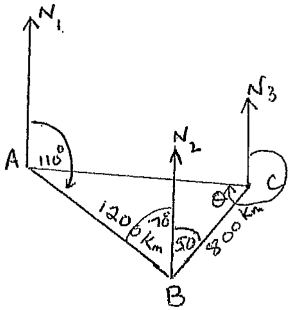
$x < -2$ OR $0 < x < 2$

c) $F(x) = x^2 \therefore \dfrac{F(a) - F(b)}{2a + 2b} = \dfrac{a^2 - b^2}{2a+2b}$

$= \dfrac{(a-b)\cancel{(a+b)}}{2\cancel{(a+b)}}, \quad a+b \neq 0$

$= \dfrac{a-b}{2}$

d) (i)



(i) $\angle ABN_2$ (on second North) $= 70^\circ$

cointerior angles are supplementary $AN_1 \parallel BN_2$

Thus, $\angle ABC = 70^\circ + 50^\circ$
$= 120^\circ$

(ii) Using Cosine Rule in $\triangle ABC$,

$AC^2 = 1200^2 + 800^2 - 2 \times 1200 \times 800 \cos 120^\circ$
$= 3040000$
$AC = 1743.5595\ldots$

ie distance between airports A and C is 1744 Km (nearest Km)

(iii) By Sine Rule in $\triangle ABC$, Let $\theta = \angle ACB$

$\dfrac{\sin\theta}{1200} = \dfrac{\sin 120^\circ}{1743.55\ldots}$

$\sin\theta = \dfrac{1200 \sin 120^\circ}{1743.55\ldots}$

$= 0.59639\ldots$

$\theta = 36.5867\ldots^\circ$

ie $\theta = 36^\circ 35'$

$\therefore$ Bearing of A from C is $360^\circ - (130^\circ - 36^\circ 35')\,T$