

J.M.J.Ch

MARCELLIN COLLEGE RANDWICK



MATHEMATICS

YEAR 10 Stage 5.3

ASSESSMENT TASK 2 2017

Weighting: 30% of Assessment Mark

STUDENT NAME: _____

MARK: /75

TEACHER: MATHS A
 LOBO

MATHS B
 ROBINS

Time Allowed: 75 minutes

Directions:

- Answer all questions in the space provided.
- Show all necessary working. Where more than one mark is allocated to a question, full marks may not be awarded for answers only.
- Marks may not be awarded for careless or badly arranged work.
- Calculators may be used.

Q1 Algebra	14 /14	Q4 Quadratic Equations	1 /15
Q2 Further Equations	9 /9	Q5 Financial Maths	/9
Q3 Surds and Indices	17 /17	Q6 Linear Relationships	/11
TOTAL			775

Question 1

Algebra

(14 marks)

a) Fully factorise: $4ab^2 - 8ab$ 1 mark

b) Expand and simplify: $7 - 2(a - 4)$ 2 marks

c) Factorise then simplify: $\frac{5m+10}{m^2-m-2} \div \frac{m^2-4}{3m+3}$ 3 marks

d) Solve: $\frac{a}{4} - \frac{a+2}{3} = 9$ 3 marks

Question 2**Further Equations****(9 marks)**

a) Solve the pair of simultaneous equations:

$$\begin{aligned}2x + 3y &= -14 \\ x + 3y &= -4\end{aligned}$$

2

b) Solve the pair of simultaneous equations:

$$\begin{aligned}x^2 + y^2 &= 9 \\ x + y &= 3\end{aligned}$$

3

c) Solve $8x^3 + 1 = 0$

2

d) Rearrange the formula to make m the subject:

$$bm - y = z - cm$$

2

e) Solve $7-3n < 4$ 2 marks

f) Factorise then simplify: $\frac{1}{x^2-1} + \frac{2}{x+1}$ 3 marks

Question 3

Surds and Indices

(17 marks)

a) Simplify:

7

i. $(5x^3y^2)^2$

ii. $6a^4b + 18a^3b^2$

iii. $\left(\frac{2}{3x}\right)^{-2}$

iv. $(16y^4)^{\frac{3}{4}}$

b) Simplify:

6

i. $\sqrt{12} - \sqrt{27}$

ii. $2\sqrt{3} \times 5\sqrt{12}$

iii. $\frac{16\sqrt{12}}{2\sqrt{3}}$

c) Expand and simplify

$$(\sqrt{2}+1)(\sqrt{2}-1)$$

2

d) Express the following with a rational denominator:

$$\frac{2}{\sqrt{5}+1}$$

2

Question 4 **Quadratic Equations**

(15 marks)

a) Solve the following equations:

6

i. $x^2 - x = 0$

ii. $4x^2 - 1 = 0$

iii. $m^2 = 6 - 5m$

b) Solve by completing the square:

3

$x^2 + 4x - 1 = 0$ (Leave your answer in surd form)

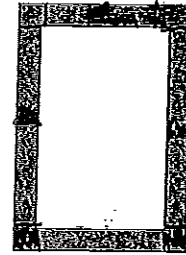
- c) Using the Quadratic Formula, solve the following equation, leaving your answer in the *simplest surd form*.

2

$$3x^2 - 2x - 2 = 0$$

- d) A rectangular swimming pool 12m by 8m is surrounded by a concrete path of a constant width. If the area of the path is $224m^2$, find the width of the path.

4



Question 5

Financial Maths

(9 marks)

a) James takes out a loan of \$4000 for a period of 2 years at a simple interest rate of 11.5% per annum. How much will he need to pay back per month?

2

b) Jessica invested an amount for 10 years at 5% p.a. simple interest. He earned \$750 in interest on his investment. How much did he invest?

2

c) A new car is purchased for \$25 000 and depreciated by the 10% p.a. Find, to the nearest dollar, the value of the car after 7 years.

2

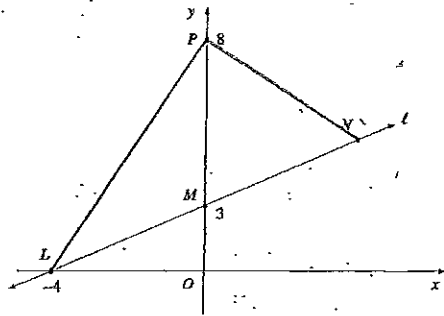
d) Michael and Lilly plan to have \$20 000 in an investment account in 15 years time for their grandson's university fees. The interest rate is 3% per annum compounded monthly. Calculate the amount that they will need to deposit into the account now in order to achieve their plan.

3

Question 6

Linear Relationships

(11 marks)



a) Show that the equation of LM is $3x - 4y + 12 = 0$.

2

b) Show that the point $(16, 15)$ lies on the line LM .

1 mark

c) Show that $\triangle PLM$ is an isosceles triangle.

2

d) Calculate the gradient of PL .

2

e) M is the midpoint of the interval LN . Find the coordinates of N .

2

SOLUTIONS

Question 1

Algebra

(14 marks)

<p>a) Fully factorise: $4ab^2 - 8ab$</p> $4ab(b-2)$ <p style="text-align: right;">✓</p>	1
<p>b) Expand and simplify: $7-2(a-4)$</p> $7-2a+8$ $=15-2a$ <p style="text-align: right;">✓</p>	2
<p>c) Factorise then simplify: $\frac{5m+10}{m^2-m-2} \div \frac{m^2-4}{3m+3}$</p> $\frac{5(m+2)}{(m+1)(m-2)} \div \frac{(m-2)(m+2)}{3(m+1)}$ $\frac{5(m+2)}{(m+1)(m-2)} \times \frac{3(m+1)}{(m-2)(m+2)}$ $\frac{15}{(m-2)^2} = \frac{15}{(m-2)(m-2)}$ <p style="text-align: right;">✓</p>	3
<p>d) Solve: $\frac{a}{4} - \frac{a+2}{3} = 9$</p> $\frac{a \times 3}{4 \times 3} - \frac{a+2}{3 \times 4} = 9$ $\frac{3a}{12} - \frac{(4a+8)}{12} = 9$ $\frac{3a-4a-8}{12} = 9$ $-a-8 = 108$ $-a = 116$ $a = -116$ <p style="text-align: right;">✓</p>	3

<p>e) Solve $7-3n < 4$</p> $\begin{array}{r} 7-3n < 4 \\ -7 \quad -7 \\ \hline -3n < -3 \\ \div -3 \quad \div -3 \\ \hline n > 1 \end{array}$ <p style="text-align: right;">✓</p>	2
<p>f) Factorise then simplify: $\frac{1}{x^2-1} + \frac{2}{x+1}$</p> $\frac{1}{(x-1)(x+1)} + \frac{2x(x-1)}{(x+1)}$ $\frac{1+2x-2}{(x-1)(x+1)} = \frac{-1+2x}{(x+1)(x+1)}$ <p style="text-align: right;">✓</p>	3

Question 2

Further Equations

(9 marks)

a) Solve the pair of simultaneous equations:

$$\begin{aligned} 2x+3y &= -14 \quad (1) & 2x+3y &= -14 \\ x+3y &= -4 \quad (2) & x &= -4-3y \end{aligned}$$

sub x sub y

$$\begin{aligned} 2(-4-3y)+3y &= -14 & x &= -4-3(2) \\ -8-6y+3y &= -14 & x &= -4-6 \\ -8-3y &= -14 & x &= -10 \\ -3y &= -6 & y &= 2 \end{aligned}$$

2

b) Solve the pair of simultaneous equations:

$$\begin{aligned} x^2+y^2 &= 9 \quad (1) & x^2+y^2 &= 9 & (3-y)^2+y^2 &= 9 \\ x+y &= 3 \quad (2) & x &= 3-y & 9-6y+y^2+y^2 &= 9 \\ & & & & 2y^2-6y+9 &= 9 \\ & & & & 2y^2-6y &= 0 \\ & & & & 2y(y-3) &= 0 \\ & & & & y &= 0, 3 \end{aligned}$$

Sub into (2)

$$\begin{aligned} x+y &= 3 \\ x &= 3-y \end{aligned}$$

When $y=0$, $x=3$
 $y=3$, $x=0$

3

3

c) Solve $8x^3+1=0$

$$(a+b)(a^2-ab+b^2)$$

$$(2x+1)(4x^2-2x+1) = 0$$

$$2x+1 = 0$$

$$xz = \frac{1}{z}$$

2

d) Rearrange the formula to make m the subject:

$$bm - y = z - cm$$

$$bm + cm = z + y$$

$$m(b+c) = z+y$$

$$m = \frac{z+y}{b+c}$$

2

Question 3

Surds and Indices

(17 marks)

a) Simplify:

i. $(5x^3y^2)^2$

$25x^6y^4$ ✓

ii. $6a^4b + 18a^3b^2$

$\frac{18a^3b}{3 \cdot 18a^3b^2} = \frac{a}{3b}$ ✓

iii. $\left(\frac{2}{3x}\right)^{-2}$ flip

$\left(\frac{3x}{2}\right)^2 = \frac{9x^2}{4}$ ✓

iv. $(16y^4)^{\frac{3}{4}}$

$\sqrt[4]{(16y^4)^3} = \sqrt[4]{4096y^{12}} = 8y^3$ ✓

7

b) Simplify:

i. $\sqrt{12} - \sqrt{27}$

$\sqrt{4 \cdot 3} - \sqrt{9 \cdot 3}$

$2\sqrt{3} - 3\sqrt{3} = -\sqrt{3}$ ✓

ii. $2\sqrt{3} \times 5\sqrt{12}$

$10\sqrt{36} = 10 \cdot 6 = 60$ ✓

iii. $\frac{16\sqrt{12}}{2\sqrt{3}}$

$\frac{8 \cdot 2\sqrt{3} \cdot \sqrt{4}}{2\sqrt{3}} = 8\sqrt{4} = 16$ ✓

6

c) Expand and simplify

$(\sqrt{2}+1)(\sqrt{2}-1)$

$\sqrt{2}\sqrt{2} - \sqrt{2} + \sqrt{2} - 1$

$= 2 - 1$

$= 1$ ✓

2

d) Express the following with a rational denominator:

$\frac{2}{\sqrt{5}+1}$

$\frac{2 \times (\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)}$

$\frac{2\sqrt{5}-2}{5-1} = \frac{2\sqrt{5}-2}{4} = \frac{2(\sqrt{5}-1)}{4}$ ✓

$= \frac{\sqrt{5}-1}{2}$

2

Question 4

Quadratic Equations

(15 marks)

a) Solve the following equations:

6

i. $x^2 - x = 0$

~~$x^2 - x + \frac{1}{4} = \frac{1}{4}$~~

$(x - \frac{1}{2})^2 = \frac{1}{4}$ $x = \frac{1}{2} \pm \sqrt{\frac{1}{4}}$

$x = \frac{1}{2} \pm \frac{1}{2}$ $\therefore x = 1, 0$ 2

ii. $4x^2 - 1 = 0$

~~$x = \frac{1}{4} + \frac{1}{8}$~~
 ~~$x = \frac{1}{8} - \frac{1}{8}$~~

~~$4x^2 - 1 = 0$~~
 ~~$\frac{4x^2}{4} - \frac{1}{4} = 0$~~
 ~~$x^2 - \frac{1}{4} = 0$~~

~~$x = \frac{1}{4}$~~

~~$x - \frac{1}{8} = \sqrt{\frac{1}{64}}$~~ $x = \frac{1}{8} \pm \frac{1}{8}$

~~$x^2 - \frac{1}{4} + \frac{1}{64} = \frac{1}{64}$~~

~~$(x - \frac{1}{8})^2 = \frac{1}{64}$~~

~~$x - \frac{1}{8} + \frac{1}{64} = \frac{1}{64}$~~

~~$(2x^2 - 1)(2x + 1) = 0$~~ $\therefore x = \pm \frac{1}{2}$ 2

~~$(x - \frac{1}{8})^2 = \frac{1}{64}$~~

~~$(x - \frac{1}{8})^2 = \frac{1}{64}$~~

iii. $m^2 = 6 - 5m$

$m^2 - 6 + 5m = 0$

$m^2 + 5m - 6 = 0$

$(m - 1)(m + 6) = 0$

$m = 1, -6$ 2

b) Solve by completing the square:

3

$x^2 + 4x - 1 = 0$ (Leave your answer in surd form)

$x^2 + 4x - 1 = 0$

$x^2 + 4x = 1$

$x^2 + 4x + 4 = 1 + 4$

~~$(x + 2)^2 = 5$~~

$x + 2 = \pm\sqrt{5}$

$x = \pm\sqrt{5} - 2$ 3

c) Using the Quadratic Formula, solve the following equation, leaving your answer in the simplest surd form.

2

$3x^2 - 2x - 2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{1 \pm \sqrt{7}}{3}$

$2 \pm \sqrt{4 - 4(3x - 2)}$

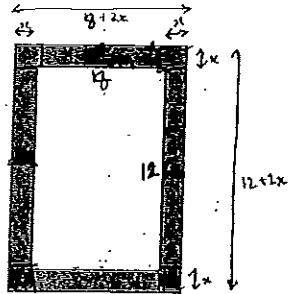
$x = \frac{2 \pm 2\sqrt{7}}{6}$

$x = \frac{2 \pm \sqrt{28}}{2(3)}$

$x = \frac{2 \pm \sqrt{28}}{6}$

$x = \frac{2 \pm \sqrt{4 \cdot 7}}{6}$

- d) A rectangular swimming pool 12m by 8m is surrounded by a concrete path of a constant width. If the area of the path is $224m^2$, find the width of the path.



$$\text{Area}_{\text{path}} = A_{\text{Large rectangle}} - A_{\text{small rectangle}}$$

$$\begin{aligned} A_{\text{large rectangle}} &= (12+2x)(8+2x) \\ &= 96 + 40x + 4x^2 \end{aligned}$$

$$\begin{aligned} A_{\text{small rectangle}} &= 12 \times 8 \\ &= 96 \end{aligned}$$

$$\begin{aligned} \text{Area}_{\text{path}} &= 96 + 40x + 4x^2 - 96 \\ &= 40x + 4x^2 = 224 \end{aligned}$$

$$x^2 + 10x - 56 = 0$$

$$(x+14)(x-4) = 0 \quad (x > 0)$$

$$x = 4 \text{ m.}$$

4

Question 5

Financial Maths

(9 marks)

- a) James takes out a loan of \$4000 for a period of 2 years at a simple interest rate of 11.5% per annum. How much will he need to pay back per month?

$$\begin{aligned} I &= PRN \\ I &= 4000 \times \frac{0.115}{12} \times 24 \\ &= \cancel{\$38.33} \text{ (2dp)} \times \frac{4000}{24} \\ &= \cancel{4000 \times \frac{0.115}{12} \times 24} \end{aligned}$$

$\frac{4000}{24} = \$205 \text{ /month}$

2

- b) Jessica invested an amount for 10 years at 5% p.a. simple interest. He earned \$750 in interest on his investment. How much did he invest?

$$750 = P \times \frac{0.05}{100} \times 10$$

$$750 = 0.5P$$

$$P = \$1500$$

2

- c) A new car is purchased for \$25 000 and depreciated by the 10% p.a. Find, to the nearest dollar, the value of the car after 7 years.

$$\begin{aligned} A &= P(1-R)^n \\ A &= 25000(1-0.10)^7 \end{aligned}$$

$$A = \$11957.42$$

$$= \$11957$$

2

- d) Michael and Lilly plan to have \$20 000 in an investment account in 15 years time for their grandson's university fees.
The interest rate is 3% per annum compounded monthly.
Calculate the amount that they will need to deposit into the account now in order to achieve their plan.

3

$$A = P(1+R)^n$$

$$20000 = P\left(1 + \frac{0.03}{12}\right)^{15 \times 12}$$

$$20000 = P(1.0025)^{180}$$

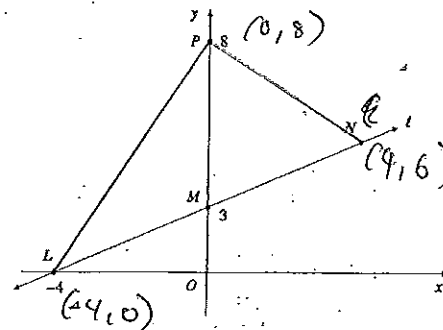
$$20000 = P \times 1.567431725$$

$$P = \$12759.73 \text{ (2dp)}$$

Question 6

Linear Relationships

(11 marks)



- a) Show that the equation of LM is $3x - 4y + 12 = 0$.

2

$$\frac{4y}{4} = \frac{3x + 12}{4}$$

$$y = \frac{3}{4}x + 3$$

$$y = \frac{3}{4}(-4) + 3$$

$$y = 0 \quad (-4, 0)$$

$$y = \frac{3}{4}(x) + 3$$

when $y = 0$ $x = -4$
when $x = 0$ $y = 3$

$$y = \frac{3}{4}(0) + 3$$

\therefore equation of LM

$$\text{is } 3x - 4y + 12 = 0$$

- b) Show that the point (16, 15) lies on the line LM.

1

sub in $x = 16, y = 15$

$$y = \frac{3}{4}x + 3$$

$$15 = \frac{3}{4}(16) + 3$$

$$15 = 15$$

Thus (16, 15) lies on the line LM.

c) Show that $\triangle PLM$ is an isosceles triangle. 2

In PM $(0,8)(0,3)$

$$d = \sqrt{(0-0)^2 + (3-8)^2}$$

$$= 5$$

In ML $(0,3)(-4,0)$

$$d = \sqrt{(0-3)^2 + (-4-0)^2}$$

$$= \sqrt{9 + 16}$$

$\therefore \triangle PLM$ is
an isosceles triangle
as $ML = PM$
with a distance
of 5

d) Calculate the gradient of PL . 2

PL $(0,8)(-4,0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0-8}{-4-0} \quad m = \frac{-8}{-4}$$

$m = 2$

e) M is the midpoint of the interval LN . Find the coordinates of N . 2

$M = (0,3)$ $L = (-4,0)$

$$0 = \frac{x_2 + x_1}{2}$$

$$3 = \frac{y_2 + y_1}{2}$$

$$6 = y_2 + 0$$

$$6 = y_2$$

$$0 = x_2 + x_1$$

$$0 = x_2 - 4$$

$$4 = x_2$$

$N = (4,6)$

f) Show that $\angle NPL$ is a right angle. 2

In $\angle NPL$

~~$LN = 10$~~

$$LN = 10$$

$$PN = 4.47 \text{ (2dp)}$$

$$PL = 8.944$$

using Pythagoras theorem

$$8.944^2 + 4.47^2 = 10^2$$

$$100 = 100$$

$\therefore \angle NPL$ is a right angle.
as $a^2 + b^2 = c^2$
is true

END OF EXAMINATION

FORMULA SHEET

Simple Interest

$$I = PRN$$

Compound Interest

$$A = P(1 + r)^n$$

Depreciation

$$A = P(1 - r)^n$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Midpoint Formula

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Gradient Formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point Gradient Formula

$$y - y_1 = m(x - x_1)$$