



Name: _____

Teacher: _____

SCEGGS Darlinghurst

2007

Year 10

Semester 2 Examination

Mathematics (Pathway 5.3E)

Outcomes Assessed: NS 5.3.2, MS 5.3.2, PAS 5.2.4, PAS 5.3.4, PAS 5.3.2, MS 5.2.2, MS 5.3.1, WMS 5.3.1, WMS 5.3.2, WMS 5.3.4, SGS 5.2.2 and SGS 5.3.3

Task Weighting: 40%

General Instructions

- Time allowed - 2 hours
- **Carefully** read the instructions
- Attempt **all** questions
- Write your name on every page
- Write using blue or black pen
- Draw all diagrams in pencil
- Marks may be deducted for careless or badly arranged work
- All necessary working **must** be shown

	Question	Possible Mark	Mark Awarded
1	Trigonometry	15	
2	Quadratic Equations	13	
3	Surface Area and Volume	10	
4	Graphs of Physical Phenomena	9	
5	Deductive Geometry	9	
6	Graphs	16	
7	Logarithms	8	
	TOTAL	80	

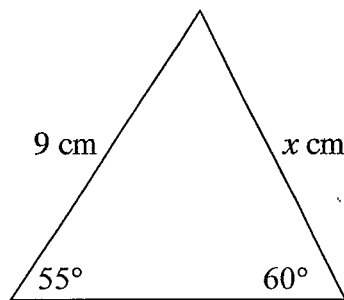
Attempt **all** questions in the spaces provided
 Write your name at the top of each page
 Calculators may be used
 In all questions, show clear working where necessary

Question 1 (15 marks)**Trigonometry****Marks**

- (a) If $\sin \theta = \frac{3}{5}$, find θ correct to the nearest degree.

1

(b)



- (i) Complete the rule:

1

$$\frac{x}{\boxed{}} = \frac{\boxed{}}{\sin 60^\circ}$$

- (ii) Find x correct to one decimal place.

2

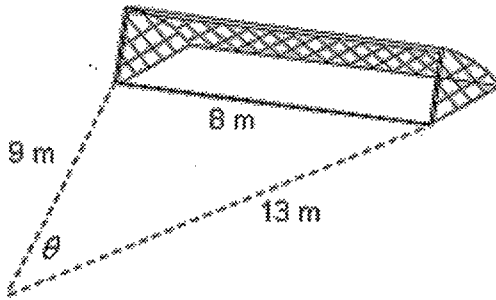
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Question 1 (continued)

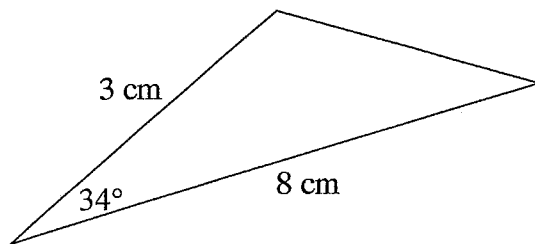
- (c) A soccer player is 9 m from one goal post and 13 m from the other. The goal is 8 m wide. 3

Within what angle, θ , to the nearest degree, must he kick the ball to score?

Use the formula $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



- (d) Using the formula $A = \frac{1}{2}ab \sin C$ find the area of this triangle to 1 decimal place. 2



**NOT
TO
SCALE**

Question 1 continues on the next page

Question 1 (continued)

- (e) The triangle ABC is acute angled and has the following lengths:

$$AC = 12\text{m}, BC = 17\text{m} \text{ and } \angle CAB = 50^\circ.$$

The unknown angle is $\angle ABC$.

- (i) Draw a diagram to represent this situation.

1

- (ii) Find the unknown angle to the nearest minute.

2

- (f) A cross-country runner starts at checkpoint A and runs for 8 km on a bearing of
- 50°T
- to reach checkpoint B, then heads directly east for 10 km to checkpoint C.

3

- (i) Draw a diagram to represent this information.

- (ii) How far is checkpoint C from the starting point A?
-
- Answer correct to 2 decimal places.

Question 2 (13 marks)**Quadratic Equations****Marks**

(a) Solve the following.

(i) $x^2 + 3x = 0$

2

(ii) $y^2 - 36 = 0$

2

(iii) $d^2 - 4d + 3 = 0$

2**Question 2 continues on the next page**

Question 2 (continued)

- (b) Susan solved the following quadratic equation $5x^2 - 2x - 4$ using the quadratic formula.

Her solution is shown below.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-4)}}{2} \quad \text{Line 1}$$

$$x = \frac{2 \pm \sqrt{-4 + 80}}{2} \quad \text{Line 2}$$

$$x = \frac{2 \pm \sqrt{72}}{2} \quad \text{Line 3}$$

- (i) She has made mistake(s). Explain what they are and which line(s) they occur. 2

- (ii) Show the correct solution.

- (c) Sam began to solve a quadratic equation using the quadratic formula. She wrote correctly:

3

$$x = \frac{3 \pm \sqrt{9 + 28}}{2}$$

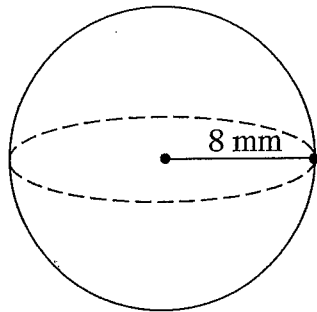
What was the original quadratic equation?

Question 3 (10 marks)

Surface Area and Volume

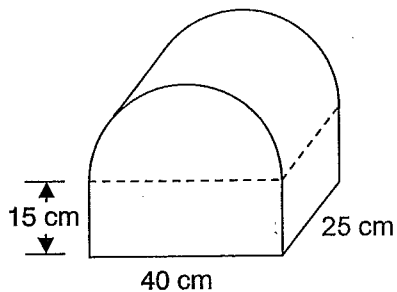
- (a) Calculate the volume of this sphere, giving your answer to the nearest whole number.

2



- (b) Calculate the volume of this prism correct to the nearest whole number.

3



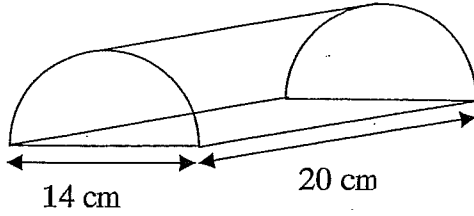
Question 3 continues on the next page

Question 3 (continued)

(c) Calculate the surface area of these closed solids giving your answer to 3 significant figures.

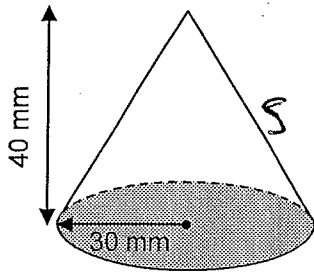
(i)

2



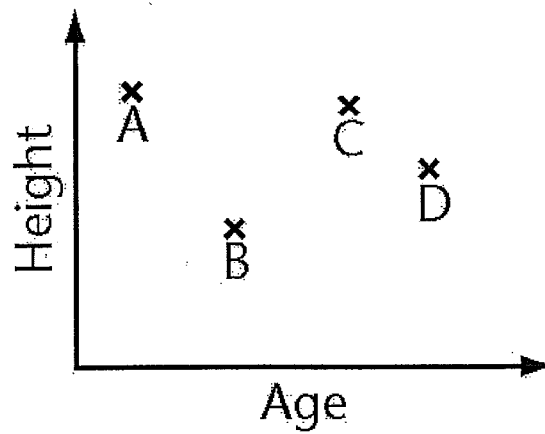
(ii)

3



Question 4 (9 marks) Graphs of Physical Phenomena

(a) The graph below compares four students' heights with their ages.

3

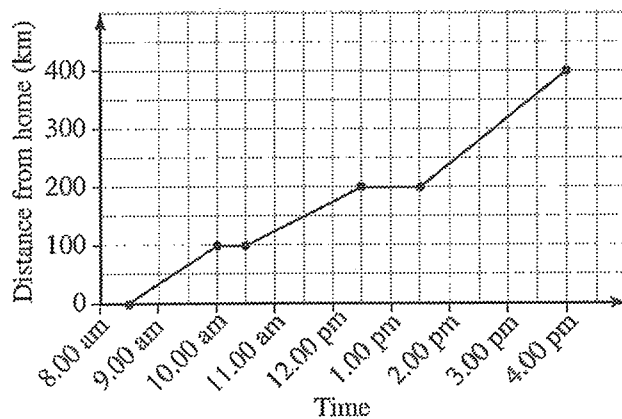
- (i) Which student is the tallest?
- (ii) Which student is the oldest?
- (iii) Name one student that is taller than D and older than B.

Question 4 continues on the next page.

Question 4 (continued)

- (b) The distance–time graph below shows a family’s journey from home to their holiday destination.

4



- (i) At what time did the family leave home?
- (ii) When and where did the family first stop?
- (iii) The family stopped for lunch at the half-way point. For how long did they stop?
- (iv) Between which times was the family travelling fastest?

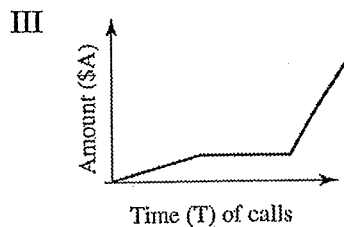
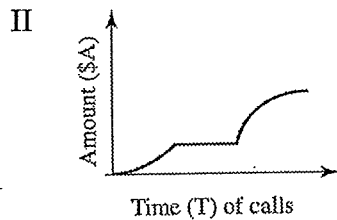
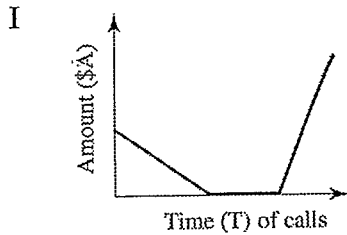
Question 4 continues on the next page

Question 4 (continued)

- (c) Gareth's mobile phone contract requires him to pay \$0.20 per minute for the first \$20 of calls, he then has free calls to the value of \$20. After that he is charged \$0.40 per minute again.

1

- (i) Which of the following graphs best represents the relationship between the amount \$A of his total bill and total time T of calls?



- (ii) Explain your choice.

1

Question 5 (9 marks)

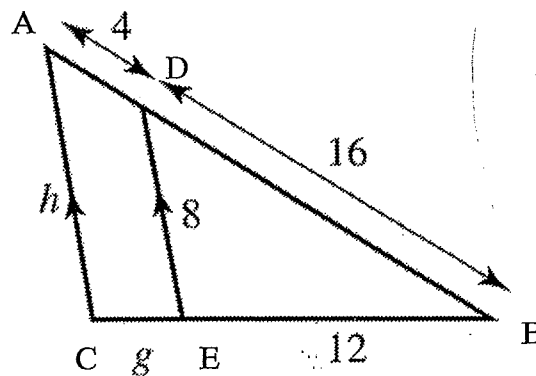
Deductive Geometry

- (a) Comment on the following statement:

1

"A square is always a rectangle, but a rectangle is not always a square."

- (b)
- $\triangle ABC \sim \triangle DBE$
- . Find the value of the pronumerals
- g
- and
- h
- in the figure below.



- (c) True or false?

(i) A rhombus is a parallelogram.

1

(ii) A parallelogram is a rhombus

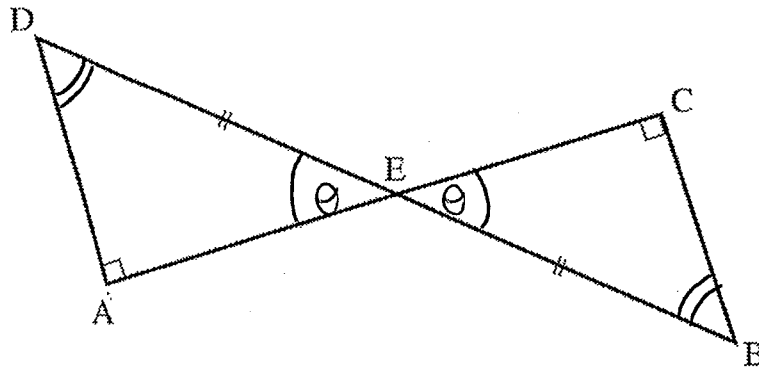
1

Question 5 continues on the next page

Question 5 (continued)

(d) Prove that DB bisects AC.

3



Question 6 (16 marks)

Graphs

- (a) (i) Complete the following table of values for the function:

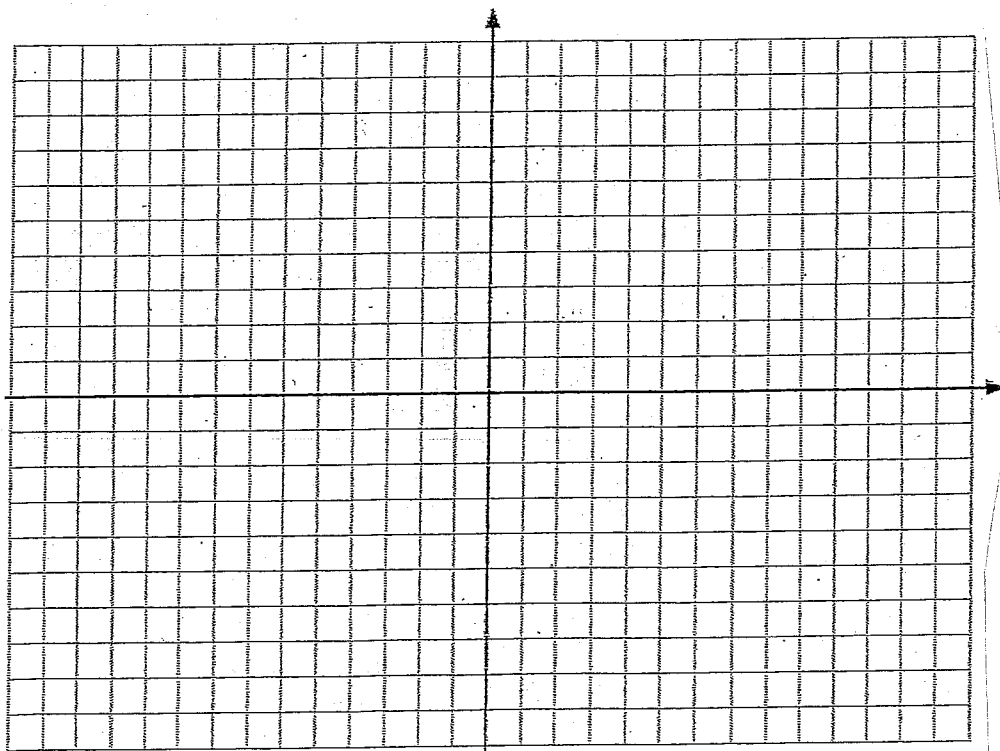
1

$$y = \frac{1}{x-2}$$

x	-3	-2	-1	0	1	1.5	2	2.5	3	4	5
y											

- (ii) Graph the function on the number plane below.

2



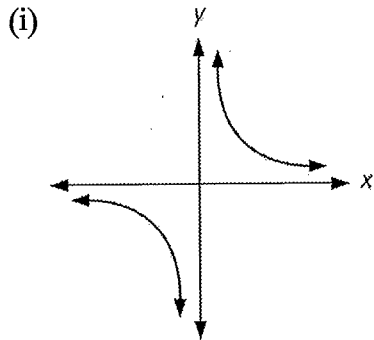
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Question 6 (continued)

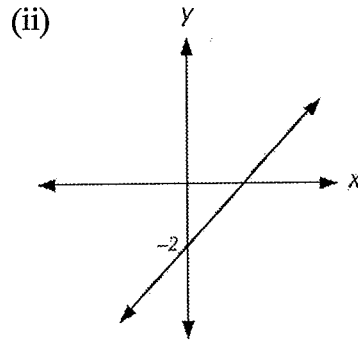
(b) Match the following equations with the graphs below.

5

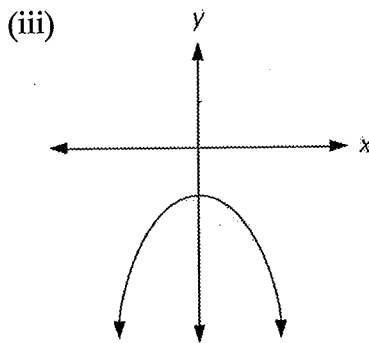
- | | | |
|-------------------|-----------------------|-----------------|
| (1) $y = x^2 - 1$ | (2) $y = \frac{4}{x}$ | (3) $y = x - 2$ |
| (4) $y = 2x - 3$ | (5) $y = -x^2 - 1$ | |



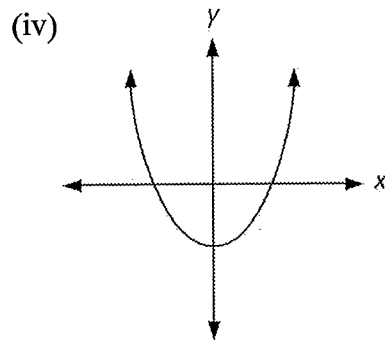
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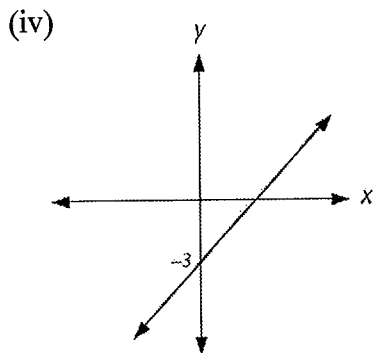
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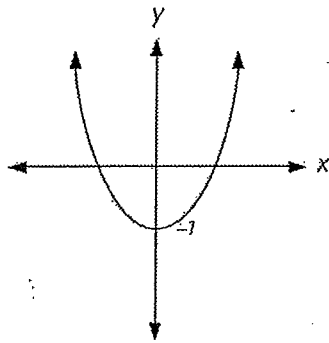
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Question 6 (continued)

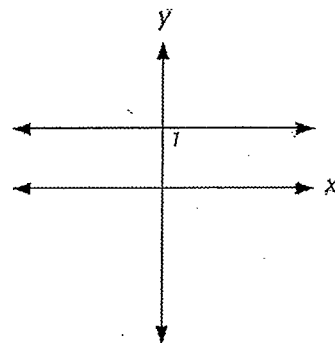
(c) Write down an equation represented in the following graphs

2

(i)



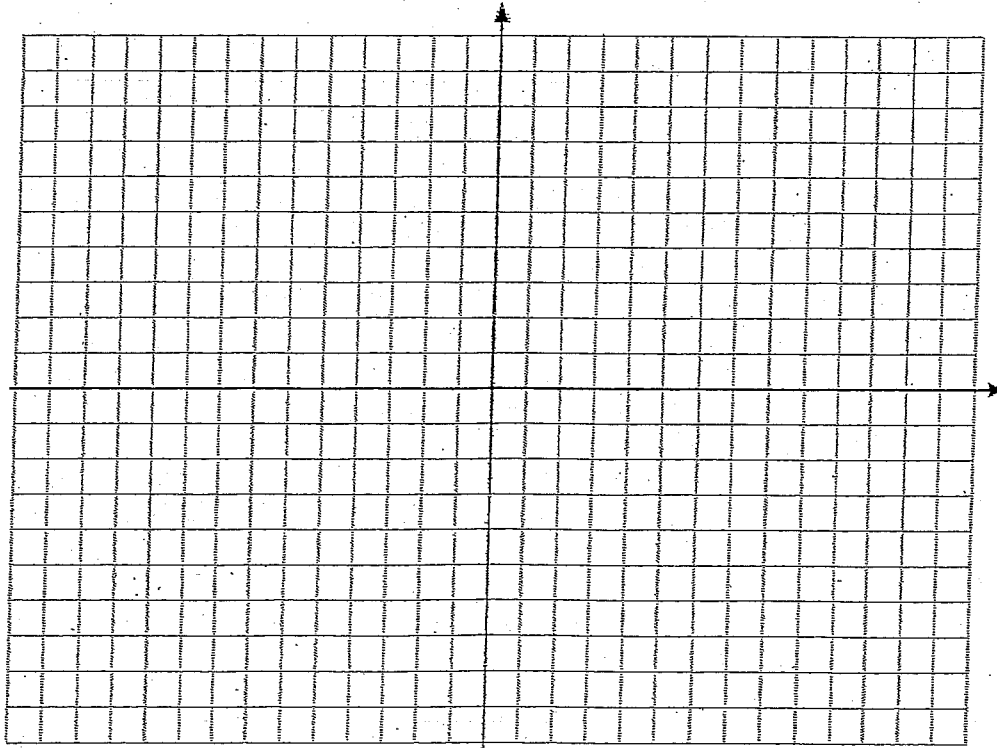
(ii)



(d) Graph the curve $y = x^2 + 3x + 2$ by finding:

3

- (i) x -intercepts
- (ii) y -intercept
- (iii) the vertex



Question 6 continues on the next page

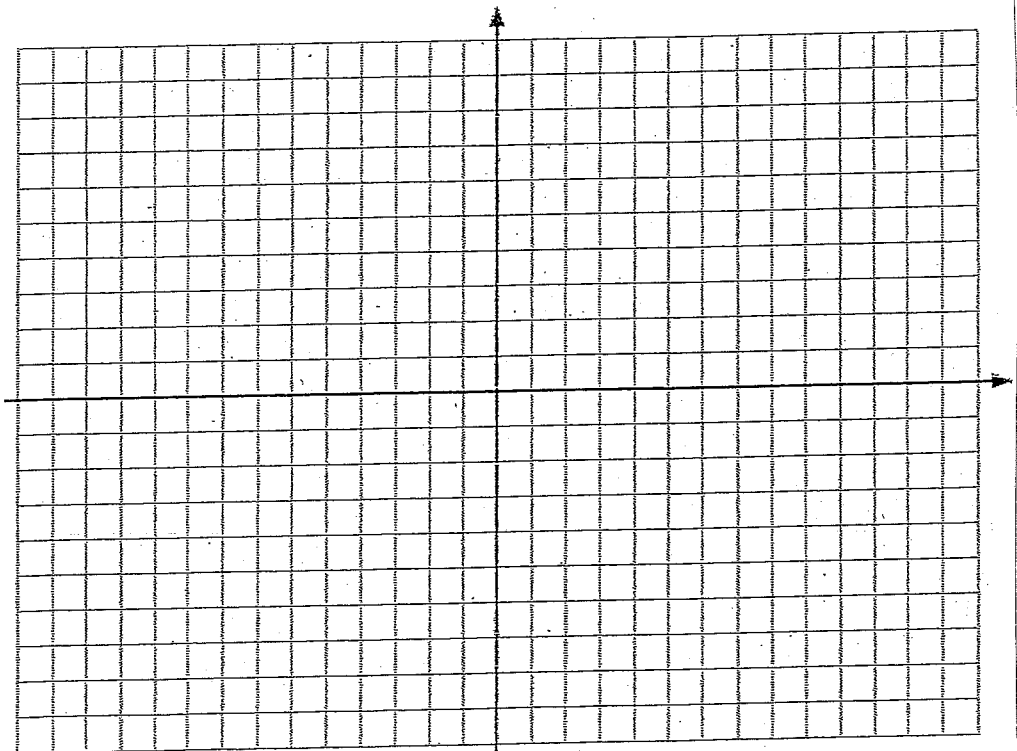
Question 6 (continued)

(e) Graph the following curves on the same set of axis

3

$y = 2^{-x}$

$y = -(2^{-x})$



Question 7 (8 marks)**Logarithms****Marks**(a) Given $\log_a 3 = 0.428$ and $\log_a 4 = 0.572$, evaluate:

(i) $\log_a 12$.

1

(ii) $\log_a 0.75$.

1

(iii) $\log_a 36$

2**Question 7 continues on the next page**

Question 7 (continued)

(b) Evaluate $\log_{11} \frac{1}{11}$.

1

(c) Solve $\log_2 32 = x$.

1

(d) Solve the logarithmic equation $\log_5 25\sqrt{5} = x$

2

End of paper

FORMULAE SHEET

Circumference of a circle = $\pi \times \text{diameter}$
 $[C = \pi d]$

Area of a circle = $\pi \times \text{radius squared}$
 $[A = \pi r^2]$

Area of a parallelogram = $\text{base} \times \text{perpendicular height}$
 $A = bh$

Area of a rhombus = $\text{half the product of the diagonals}$
 $\left[A = \frac{1}{2}xy \right]$

Area of a trapezium = $\text{half the sum of the parallel sides} \times \text{perpendicular height}$
 $\left[A = \left(\frac{a+b}{2} \right) h \right]$

Volume of a prism = $\text{area of cross-section} \times \text{height}$
 $[V = Ah]$

Volume of a cylinder = $\pi \times \text{radius squared} \times \text{height}$
 $[V = \pi r^2 h]$

Pythagoras' theorem states:

In a right-angled triangle,

the hypotenuse squared = *the sum of the squares of the other two sides*
 $[c^2 = a^2 + b^2]$

Attempt all questions in the spaces provided
 Write your name at the top of each page
 Calculators may be used
 In all questions, show clear working where necessary

Question 1 (15 marks)

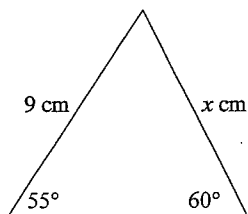
Trigonometry

Marks

- (a) If $\sin \theta = \frac{3}{5}$, find θ correct to the nearest degree. 1

37° ✓

- (b)



- (i) Complete the rule: 1

$$\frac{x}{\sin 55^\circ} = \frac{9}{\sin 60^\circ}$$

- (ii) Find x correct to one decimal place. 2

$$x = \frac{9 \sin 55^\circ}{\sin 60^\circ}$$

$$x = 8.5 \text{ cm}$$

Question 1 continues on the next page

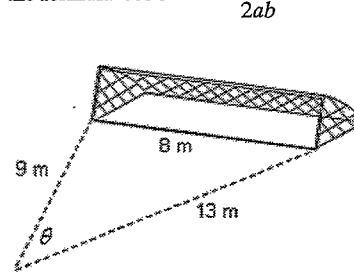
Question 1 (continued)

Marks

- (c) A soccer player is 9 m from one goal post and 13 m from the other. 3
 The goal is 8 m wide.

Within what angle, θ , to the nearest degree, must he kick the ball to score?

Use the formula $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



$$\cos \theta = \frac{9^2 + 13^2 - 8^2}{2 \times (9)(13)}$$

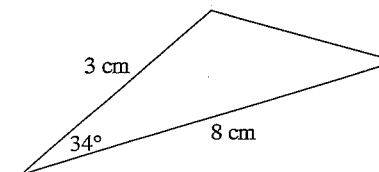
$$\cos \theta = 0.79$$

$$\theta = 37^\circ$$

$$\theta = 37^\circ$$

✓✓✓

- (d) Using the formula $A = \frac{1}{2}ab \sin C$ find the area of this triangle to 1 decimal place. 2



NOT TO SCALE

$$A = \frac{1}{2} (3 \times 8) (\sin 34)$$

$$A = 6.7 \text{ cm}^2$$

✓✓

Question 1 continues on the next page

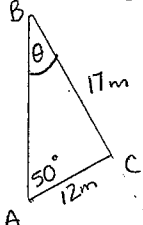
Question 1 (continued)

(e) The triangle ABC is acute angled and has the following lengths:

AC = 12m, BC = 17m and $\angle CAB = 50^\circ$.

The unknown angle is $\angle ABC$.

(i) Draw a diagram to represent this situation. 1



(ii) Find the unknown angle to the nearest minute. 2

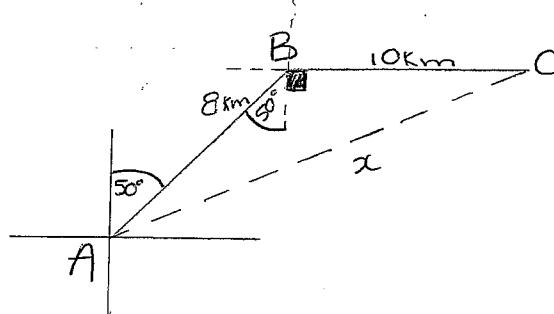
$$\frac{12m}{\sin \theta} = \frac{17m}{\sin 50^\circ}$$

$$\therefore \theta = 32^\circ 44'$$

$$\therefore \sin \theta = \frac{12m \sin 50^\circ}{17m} = 0.54$$

(f) A cross-country runner starts at checkpoint A and runs for 8 km on a bearing of $50^\circ T$ to reach checkpoint B, then heads directly east for 10 km to checkpoint C. 3

(i) Draw a diagram to represent this information.



(ii) How far is checkpoint C from the starting point A?
Answer correct to 2 decimal places.

$\angle ABC = 50^\circ + 90^\circ$ (parallel lines, alternate angles)
 $= 140^\circ$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 8^2 + 10^2 - 2(80) \cos 140^\circ$$

$$x^2 = 286.57 \quad x = 16.93 = CA$$

Question 2 (13 marks)

Quadratic Equations

(a) Solve the following.

(i) $x^2 + 3x = 0$ 2

$$x(x+3) = 0$$

$$x = 0 \quad \checkmark$$

$$(x+3) = 0$$

$$x = -3 \quad \checkmark$$

(ii) $y^2 - 36 = 0$ 2

$$(y-6)(y+6) = 0$$

$$y = \pm 6 \quad \checkmark$$

(iii) $d^2 - 4d + 3 = 0$ 2

$$P = 3$$

$$S = -4$$

$$F = -1, -3$$

$$(d-1)(d-3) = 0$$

$$d = 3, 1 \quad \checkmark$$

Question 2 continues on the next page

Question 2 (continued)

- (b) Susan solved the following quadratic equation $5x^2 - 2x - 4$ using the quadratic formula.

Her solution is shown below.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-4)}}{2} \quad \text{Line 1}$$

$$x = \frac{2 \pm \sqrt{-4 + 80}}{2} \quad \text{Line 2}$$

$$x = \frac{2 \pm \sqrt{72}}{2} \quad \text{Line 3}$$

- (i) She has made mistake(s). Explain what they are and which line(s) they occur. 2

Line 1 = the denominator must be 2(a) which is 2(5) not just 2.

Line 2 = when a negative is squared it becomes positive $(-2)^2 = 4$ not -4

Line 3 = if she were to do $-4 + 80$ the

- (ii) Show the correct solution answer would be 76 not 72.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(5)(-4)}}{2(5)}$$

$$= \frac{2 \pm \sqrt{4 + 80}}{10} = \frac{2 \pm \sqrt{84}}{10} = \frac{2 \pm \sqrt{84}}{5}$$

- (c) Sam began to solve a quadratic equation using the quadratic formula. She wrote correctly. 3

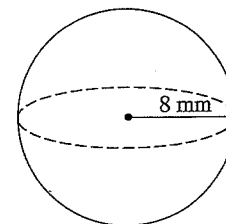
$$x = \frac{3 \pm \sqrt{9 + 28}}{2}$$

What was the original quadratic equation?

$$y = x^2 - 3x - 7$$

Question 3 (10 marks) Surface Area and Volume

- (a) Calculate the volume of this sphere, giving your answer to the nearest whole number. 2

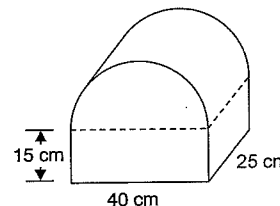


$$\frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \pi 8^3$$

$$= \cancel{2144.6} \rightarrow 2145$$

- (b) Calculate the volume of this prism correct to the nearest whole number. 3



$$\frac{\pi r^2 h}{2} + (15)(40)(25)$$

$$= \frac{\pi (20)^2 (25)}{2} + 15000$$

$$V = 30708 \text{ cm}^3$$

$$V = \cancel{307} \text{ cm}^3$$

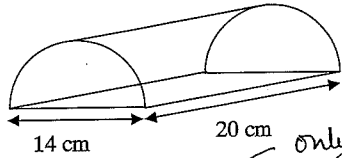
Learn how to do conversions.

Question 3 continues on the next page

Question 3 (continued)

(c) Calculate the surface area of these closed solids giving your answer to 3 significant figures.

(i)



only half a cylinder

$$SA = \pi r^2 + 2\pi r \times \text{Oh} \div 2$$

$$SA = \pi 7^2 + (2\pi(7))(20)$$

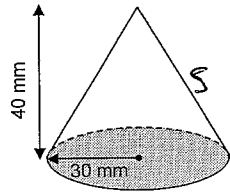
$$SA = 153.9 + 879.6$$

$$SA = 1033.5 \approx 1030 \text{ cm}^2 \text{ (3 sig. fig)}$$

type. ✓

2

(ii)



Pythagoras theorem

$$s^2 = 30^2 + 40^2$$

$$s = 50$$

incorrect formula

$$SA = (\cancel{\frac{1}{2}} \pi r \times s) + \pi r^2$$

$$SA = 2(\pi \times 30 \times 50) + \pi(30)^2$$

$$SA = 12252 \text{ mm}^2$$

$$SA = 12200 \text{ mm}^2 \text{ (3 sig. fig)}$$

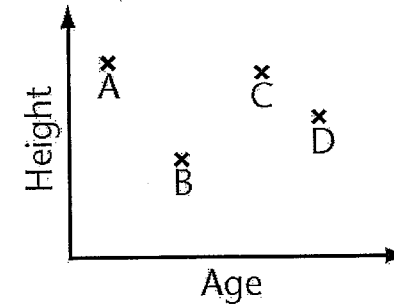
3

Question 4 (9 marks)

Graphs of Physical Phenomena

(a) The graph below compares four students' heights with their ages.

3



(i) Which student is the tallest?

A ✓

(ii) Which student is the oldest?

D ✓

(iii) Name one student that is taller than D and older than B.

C ✓

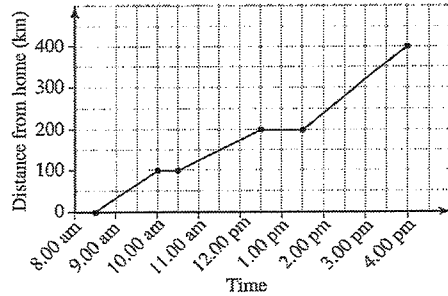
Question 4 continues on the next page

Question 4 (continued)

Marks

- (b) The distance–time graph below shows a family’s journey from home to their holiday destination.

4



- (i) At what time did the family leave home?

8.30 ✓

- (ii) When and where did the family first stop?

at 10. am till 11. am ✓
100 km away from home

- (iii) The family stopped for lunch at the half-way point. For how long did they stop?

2 hours X

- (iv) Between which times was the family travelling fastest?

2 pm - 4.00 pm ✓
1.30 X

Question 4 continues on the next page

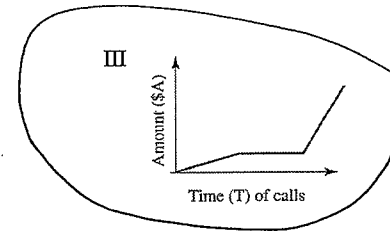
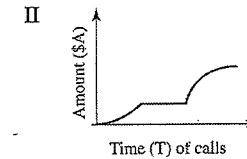
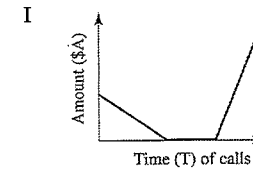
Question 4 (continued)

Marks

- (c) Gareth’s mobile phone contract requires him to pay \$0.20 per minute for the first \$20 of calls, he then has free calls to the value of \$20. After that he is charged \$0.40 per minute again.

1

- (i) Which of the following graphs best represents the relationship between the amount \$A of his total bill and total time T of calls?



- (ii) Explain your choice.

1

here is point where e pays nothing extra or a certain amount of time
 because for every time he is billed it is at a constant rate, which II does not show, and he never gains any money from his plan, which eliminates the negative slope of I
 (so the slant for the 2nd part of the graph must be III)

Question 5 (9 marks) Deductive Geometry

(a) Comment on the following statement:

1

~~A square is always a rectangle; but a rectangle is not always a square.~~

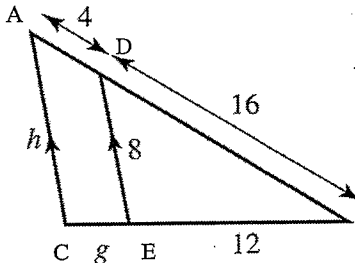
The properties of a rectangle are

- opposite sides parallel,
- all angles equal
- opposite sides equal.

~~which are the same properties of a square, except a square has all sides equal which a rectangle does not.~~

(b) $\triangle ABC \sim \triangle DBE$. Find the value of the pronumerals g and h in the figure below. ~~Does not~~

$$\frac{20}{16} = \frac{g+12}{12}$$



$$\frac{20}{16} = \frac{h}{8}$$

$$h = \frac{8 \times 20}{16}$$

$$h = 10 \text{ units}$$

$$\frac{10}{8} = \frac{g+12}{12}$$

$$\frac{20}{8} = g+12$$

$$15 = g+12 \rightarrow g = 3 \text{ units}$$

(c) True or false?

(i) A rhombus is a parallelogram.

1

True

(ii) A parallelogram is a rhombus

1

~~False~~

~~True~~

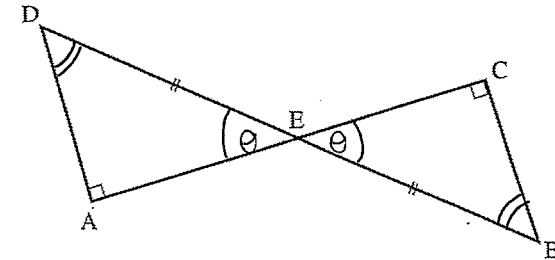
False

Question 5 continues on the next page

Question 5 (continued)

(d) Prove that DB bisects AC.

3



$$\triangle DAE \cong \triangle ECB$$

- $DE = EB$ (given)

- $\angle DEA = \angle CEB$ (vertically opposite angles are equal)

- $\angle ECB = \angle EAD$ (given)

$\therefore \angle ADE = \angle CBE$ (angle sum of a triangle is 180°)

\therefore if triangles are congruent, (ASA)

$AE = EC \rightarrow \therefore DB$ bisects AC

Question 6 (16 marks)

Graphs

(a) (i) Complete the following table of values for the function:

1

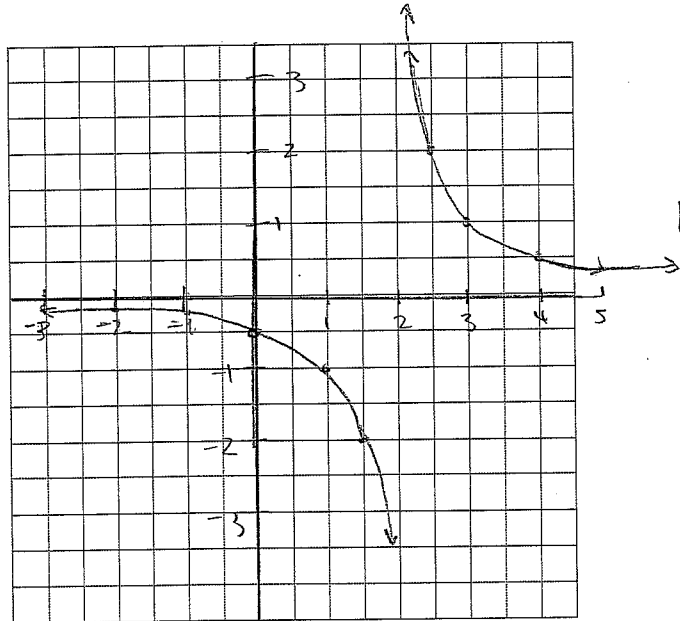
$$y = \frac{1}{x-2}$$

x	-3	-2	-1	0	1	1.5	2	2.5	3	4	5
y	$-\frac{1}{5}$	$-\frac{1}{4}$	$-\frac{1}{3}$	$-\frac{1}{2}$	-1	-2	0	2	1	$\frac{1}{2}$	$\frac{1}{3}$

X

(ii) Graph the function on the number plane below.

2



Question 6 continues on the next page

Question 6 (continued)

(b) Match the following equations with the graphs below.

5

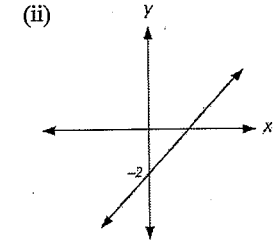
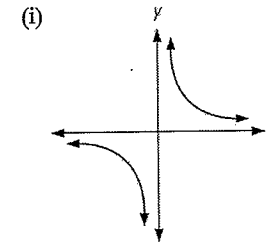
(1) $y = x^2 - 1$

(2) $y = \frac{4}{x}$

(3) $y = x - 2$

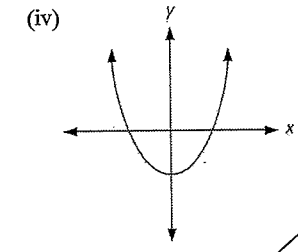
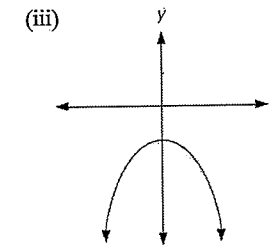
(4) $y = 2x - 3$

(5) $y = -x^2 - 1$



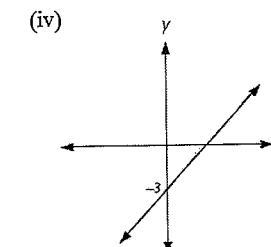
2

3



5

1

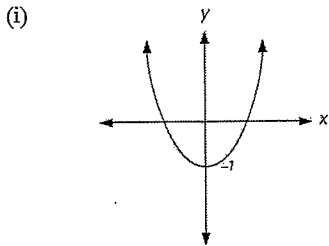


4

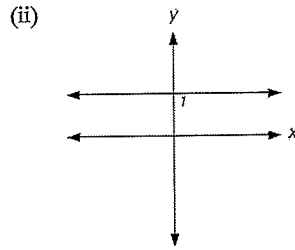
Question 6 continues on the next page

Question 6 (continued)

(c) Write down an equation represented in the following graphs 2



$y = x^2 - 1$ ✓



$y = 1$ ✓

(d) Graph the curve $y = x^2 + 3x + 2$ by finding:

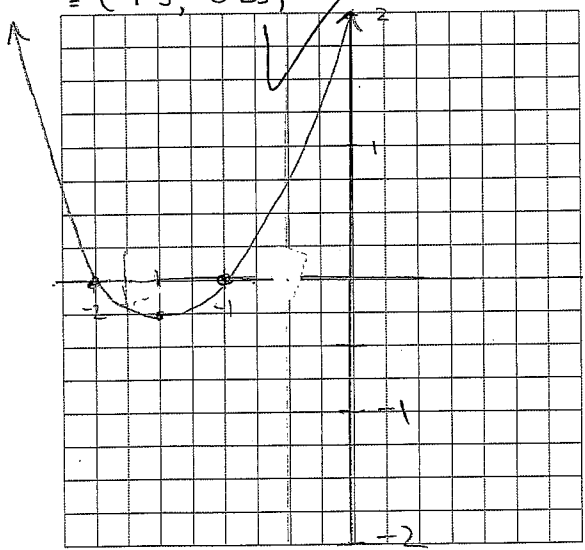
(i) x-intercepts $-2, -1$ ✓

(ii) y-intercept $+2$ ✓

(iii) the vertex $y = (-1.5)^2 + 3(-1.5) + 2$
 $= (-1.5, -0.25)$

$P = 2$
 $S = 3$
 $F = 3$

$(x+2)(x+1)$
 $x = -2, -1$ 3

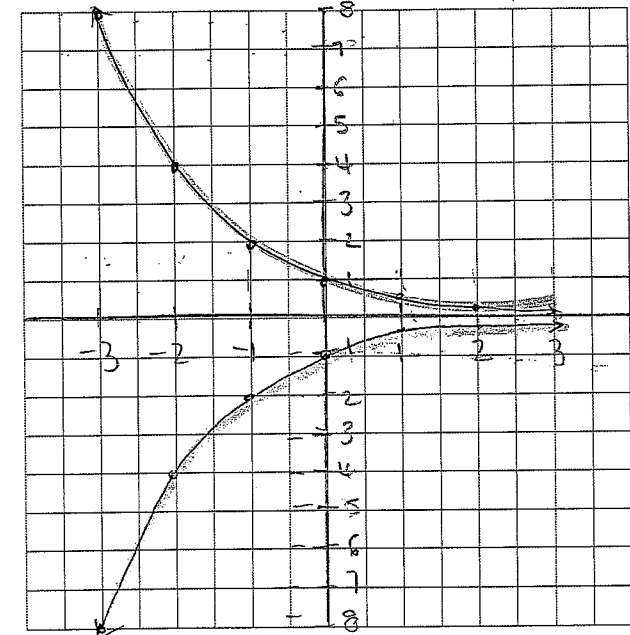


Question 6 continues on the next page

Question 6 (continued)

(e) Graph the following curves on the same set of axis 3

~~$y = 2$~~
 ~~$y = -(2^{-x})$~~



✓✓✓

Question 7 (8 marks)

Logarithms

Marks

- (a) Given
- $\log_a 3 = 0.428$
- and
- $\log_a 4 = 0.572$
- , evaluate:

(i) $\log_a 12$.

1

$$\log_a 3 + \log_a 4 = \log_a 12$$

$$0.428 + 0.572 = 1$$

$$\log_a 12 = 1$$

(ii) $\log_a 0.75$.

1

$$\log_a 3 - \log_a 4 = \log_a 0.75$$

$$0.428 - 0.572 = -0.144$$

(iii) $\log_a 36$

2

$$\log_a 3 + \log_a 4 + \log_a 3$$

$$\log_a 12 + \log_a 3 = \log_a 36$$

~~\log_a~~

$$1 + 0.428 = 1.428$$

$$\log_a 36 = 1.428$$

Question 7 continues on the next page

Question 7 (continued)

Marks

- (b) Evaluate
- $\log_{11} \frac{1}{11}$
- .

1

$$-\log_{11} 11 = -1$$

- (c) Solve
- $\log_2 32 = x$
- .

1

$$= \log_2 2^5 = x = 5$$

- (d) Solve the logarithmic equation
- $\log_5 25\sqrt{5} = x$

2

$$\log_5 25\sqrt{5} = x$$

$$\frac{1}{2} \log_5 25 \times 5 = x$$

$$\frac{1}{2} \log_5 125 = x$$

$$\frac{1}{2} \log_5 5^3 = x$$

$$\log_5 5^{\frac{3}{2}} = x$$

End of paper

$$x = \frac{3}{2}$$

$$x = 1.5$$