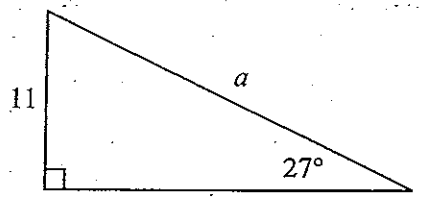
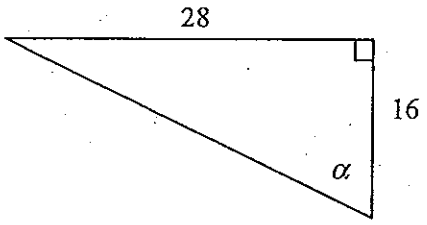
Test Year 10 Extended Trigonometry March 2006

Question 1: *10 marks*

(a) Find the length of a in the following. (correct to 1 decimal place) *2* MS 5.1.2

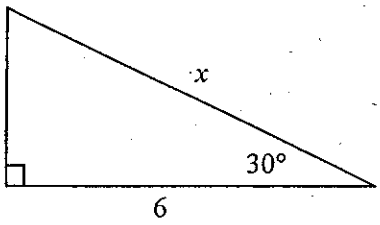


(b) Find the size of α in the following. (correct to the nearest minute) *2*



(c) Charity is flying a kite at a height of 32 metres. If the angle of elevation of the kite with the ground is 32°, find the length of the string from the ground to the kite. (to the nearest cm) *3*

(d) Find the exact value of x in simplest surd form: *3*



Question 2: *16 marks*

(a) Find the following ratios in exact form: MS 5.3.2

(i) $\cos 210°$ (ii) $\tan 330°$ *4*

(b) If $\sin\theta = -\frac{1}{\sqrt{2}}$ what quadrants could the angles be in? Find the two angles. *3*

(c) Simplify $\frac{\cos(90° - \theta)}{\sin(180 + \theta)}$ *2*

(d) If $\cos x = -\frac{1}{2}$ and $\sin x > 0$ find the exact value of $\tan x$ *1*

(e) Find the value of x in the following:

(i) $\cos x = \sin 57°$ *1*

(ii) $\cos(5x° - 20°) = \sin 120°$ *2*

(f) Show that $(\tan 30°)^2 + 1 = \frac{1}{(\cos 30°)^2}$ *3*

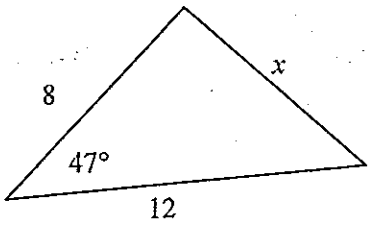
Question 3: *18 marks*

(a) When the angle of elevation of the sun is 60° a tree 42 metres high casts a shadow x metres long. What is the exact length of the shadow? Leave your answer in simplest surd form) *3*

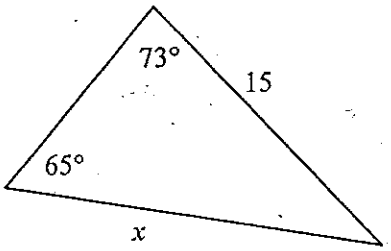
(b) Calculate the value of x in the following. Leave your answer correct to 1 decimal place:

6

(i)



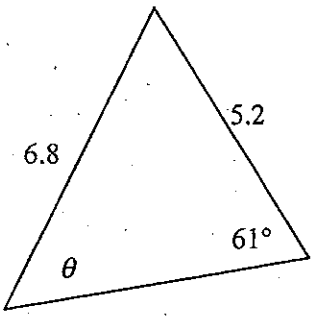
(ii)



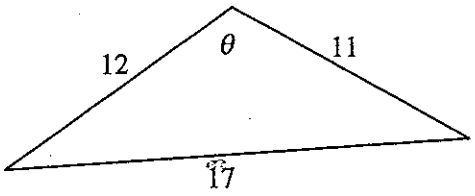
(c) Find the value of θ in the following. Leave your answer correct to the nearest minute.

6

(i)

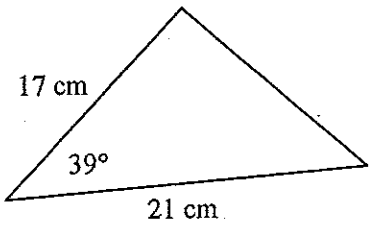


(ii)



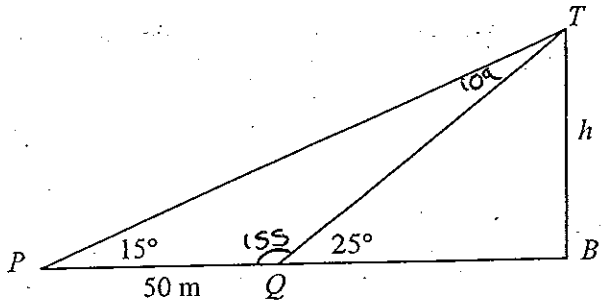
(d) Find the area of the triangle below, correct to 3 significant figures.

3



Question 4: *10 marks*

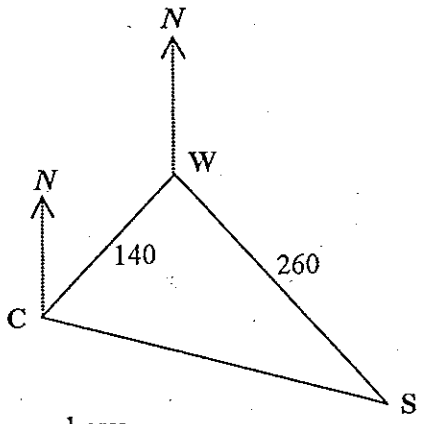
(a) Prudence is sitting in a boat at P, where the angle of elevation of the top of a vertical cliff BT is 15°. She then rows 50 metres closer to the cliff to Q, where the angle of elevation of T is 25°.



(i) Show that $TQ = \dfrac{50\sin 15°}{\sin 10°}$ *2*

(ii) Hence find the height h of the cliff to the nearest cm. *2*

(b) A ship sailed 140 kms from Port Catherine (C) to Port Waverley (W) on a bearing of 050°T. It then sailed 260 kms from Port Waverley to Port Scots (S) on a bearing of 130°T.



(i) Copy and complete the diagram above. *1*

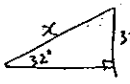
(ii) Explain why $\angle CWS = 100°$ *1*

(iii) Find the distance between Port Scots and Port Catherine. (to the nearest kilometre) *3*

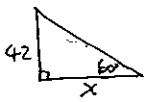
(iv) What is the bearing of Port Scots from Port Catherine? *1*

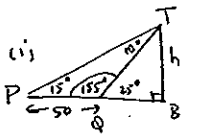
End of Test

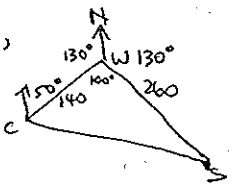
MS 5.2.3 MS 5.3.2

Qn	Solutions	Marks	Comments+Criteria
1	(a) $\sin 27° = \frac{11}{a}$		
	$a = \frac{11}{\sin 27°}$	✓	
	$= 24.229\ldots$	✓	1 ROE
	$\div 24.2$		
	(b) $\tan\alpha = \frac{28}{16}$		
	$\alpha = \tan^{-1}\frac{7}{4}$	✓	
	$= 60.255\ldots$	✓	1 ROE
	$\div 60° 15' \left[18.43''\right]$		
	(c)  $\sin 32° = \frac{32}{x}$		
	$x = \frac{32}{\sin 32°}$	✓	
	$= 60.386\ldots$	✓	
	$\div 60.39$ m	✓	−1 ROE
	(d) $\cos 30° = \frac{6}{x}$		
	$x = \frac{6}{\cos 30°}$	✓	
	$= \frac{6}{\frac{\sqrt{3}}{2}}$	✓	
	$= \frac{12}{\sqrt{3}}$		
	$= \frac{12\sqrt{3}}{3} = 4\sqrt{3}$ u	✓	

Qn	Solutions	Marks	Comments+Criteria
2	(a) (i) $\cos 210°$ −+		
	$= -\cos 30$	✓	
	$= -\frac{\sqrt{3}}{2}$	✓	
	(ii) $\tan 330°$ +−		
	$= -\tan 30$	✓	
	$= -\frac{1}{\sqrt{3}}$	✓	
	(b) $\sin\theta = -\frac{1}{\sqrt{2}}$ +−		
	Q3 + Q4	✓	
	$\therefore \theta = 225°, 315°$	✓✓	
	$\left[= 180°+45°, 360°-45\right]$		
	(c) $\frac{\cos(90-\theta)}{\sin(180+\theta)} = \frac{\sin\theta}{-\sin\theta}$	✓	
	$= -1$	✓	
	(d) $\cos x = -\frac{1}{2}$ $\sin x > 0$		
	$\therefore$ Q2		
	$\therefore x = 120°$		
	$\therefore \tan x = \tan 120$		
	$= -\tan 60$	✓	
	$= -\sqrt{3}$		
	(e) (i) $x = 33°$	✓	
	(ii) $\cos(5x-20) = \sin 120°$		accept $5x-20+120=90$
	$= \sin 60°$	✓✓	$x = -2$
	$5x-20+60 = 90$		
	$5x = 50$		
	$x = 10$		

Qn	Solutions	Marks	Comments+Criteria
2	(f) LHS $= (\tan 30)^2 + 1$		1 correct exact values
	$= \left(\frac{1}{\sqrt{3}}\right)^2 + 1 = \frac{4}{3}$		1 correct eval^n LHS
	RHS $= \frac{1}{(\cos 30)^2} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{4}{3}$	✓✓	1 " " RHS
	$= $ LHS		
	$\therefore (\tan 30)^2 + 1 = \frac{1}{(\cos 30)^2}$		
3(a)	 $\tan 60° = \frac{42}{x}$	✓	
	$x = \frac{42}{\tan 60}$		
	$= \frac{42}{\sqrt{3}}$		
	$= \frac{42\sqrt{3}}{3}$	✓	
	$= 14\sqrt{3}$ u	✓	
(b)	(i) $x^2 = 8^2 + 12^2 - 2\cdot 8\cdot 12\cdot\cos 47°$	✓	
	$= 77.0563\ldots$	✓	
	$x = \sqrt{77.056\ldots} = 8.7781\ldots$		
	$\div 8.8$	✓	
	(ii) $\frac{x}{\sin 73°} = \frac{15}{\sin 65°}$	✓	
	$x = \frac{15\sin 73}{\sin 65}$	✓	
	$= 15.827\ldots \div 15.8$	✓	

Qn	Solutions	Marks	Comments+Criteria
3	(c) (i) $\frac{\sin\theta}{5.2} = \frac{\sin 61°}{6.8}$	✓	
	$\sin\theta = \frac{5.2\sin 61}{6.8}$		
	$= 0.66882\ldots$	✓	
	$\theta = \sin^{-1} 0.66882.$		
	$= 41.9765\ldots$	✓	1 ROE
	$\div 41° 58' 35'' \div 41° 59'$		
	(ii) $\cos\theta = \frac{12^2+11^2-17^2}{2\times 12\times 11}$	✓	
	$= \frac{-24}{264}$	✓	
	$\theta = \cos^{-1}\frac{-24}{264}$		
	$= 95.2159\ldots$	✓	or $180-(84° 47' 2\ldots$
	$\div 95° 13'$		
	(d) $\text{Area}_\triangle = \frac{1}{2}.17.21.\sin 39°$	✓	
	$= 112.3336\ldots$	✓	
	$\div 112$ cm^2	✓	3sf RO
4(a)			
	(i) in $\triangle PQT$ $\frac{TQ}{\sin 15} = \frac{50}{\sin 10}$	✓	

Qn	Solutions	Marks	Comments+Criteria
	$\therefore TQ = \frac{50\sin 15}{\sin 10}$	✓	
(ii)	$\sin 25 = \frac{h}{TQ}$	✓	
	$\therefore h = \sin 25 . TQ$		
	$= \sin 25 . \left(\frac{50\sin 15}{\sin 10}\right)$		
	$= 31.495\ldots$	✓	
	$\doteq 31.50$ m		
(b) (i)		✓	
(ii)	$\angle NWC = 130°$ ($\because$ cointerial $\measuredangle$s on // lines)		
	$\therefore \angle CWS = 360° - 130° - 130°$	✓	
	$= 100°$		
(iii)	$CS^2 = 140^2 + 260^2 - 2.140.260.\cos 100$	✓	
	$= 99841.58733\ldots$	✓	
	$CS = \sqrt{\quad} = 315.977$	✓	
	$\doteq 316$ km		
(iv)	$\cos \hat{WCS} = \frac{140^2+316^2-260^2}{2\times 140\times 316}$		
	$= 0.5860\ldots$		
	$\hat{WCS} \doteq 54°$		
	$\therefore$ bearing is $50° + 54° = 104°$T		