13/5/04

YEAR 11 2UNIT COURSE

LOGARITHMS

1. Evaluate each of the following:

(a) $\log_3 1$ (b) $\log_2 8$ (c) $\log_7 \sqrt{7}$ (d) $\log_2 \left(\frac{1}{8}\right)$

(e) $\log_4 8$ (f) $\log_8 16$ (g) $\log_{10} 10^{-4}$ (h) $\log_2 2\sqrt{2}$

2. If $\log_m 2 = p$ and $\log_m 5 = q$, find expressions for each of the following in terms of p and q:

(a) $\log_m 16$ (b) $\log_m 10$ (c) $\log_m 0.4$ (d) $\log_m 2m$

(e) $\log_m 20$ (f) $\log_m \left(\frac{8\sqrt{2}}{25}\right)$ (g) $\log_2 5$ (h) $\log_4 25$

3. Simplify:

(a) $\log_{10} 2 + \log_{10} 50$ (b) $\log\left(\frac{x}{y}\right) + \log\left(\frac{y}{x}\right)$

(c) $\log\left(\frac{ab}{c}\right) + \log\left(\frac{bc}{a}\right) + \log\left(\frac{ac}{b}\right)$

4. Solve for x: (write both sides in terms of the same base).

(a) $8^x = 16$ (b) $4^{x+3} = 8$ (c) $16^x = \frac{1}{8}$

(d) $25^{1-x} = \frac{1}{125}$ (e) $36^x = \left(\frac{1}{216}\right)^2$ (f) $4^{x+2} = \frac{1}{2}$

5. Solve for x: (when unknown is an index: take logs of both sides)

(a) $3^{x-1} = 5$ (b) $2^{1-x} = 3$ (c) $2^{x+1} = 3^{x-1}$

(d) $3^{1-x} = 5^{x+3}$ (e) $5^{2x-1} = \sqrt{7}$

"YEAR 11 2UNIT COURSE: LOGARITHMS"

14/5/04. 1a) $\log_3 1$

let $\log_3 1 = x$

$\therefore 3^x = 1$

$\therefore x = 0$ ✓

$\therefore \log_3 1 = 0.$ ✓

b) let $\log_2 8 = x$

$\therefore 2^x = 8$

$\therefore x = 3.$ ✓ $\therefore \log_2 8 = 3$ ✓

c) let $\log_7 \sqrt{7} = x$

$\therefore 7^x = \sqrt{7}$

$\therefore x = \frac{1}{2}.$ ✓ $\therefore \log_7 \sqrt{7} = \frac{1}{2}$ ✓

d) let $\log_2\left(\frac{1}{8}\right) = x$

$\therefore 2^x = \frac{1}{8}$

$\therefore x = -3$ ✓ $\therefore \log_2\left(\frac{1}{8}\right) = -3$ ✓

e) let $\log_4 8 = x$

$\therefore 4^x = 8 \Rightarrow (2^2)^x = 2^3 \Rightarrow 2x = 3$

$\therefore x = 3/2.$ ✓ $\therefore \log_4 8 = 3/2.$

f) let $\log_8 16 = x$

$\therefore 8^x = 16$

$\therefore 2^{3x} = 2^4$

$\therefore x = \frac{4}{3}$ ✓ $\therefore \log_8 16 = \frac{4}{3}.$ ✓

g) let $\log_{10} 10^{-4} = x$

$\therefore 10^x = 10^{-4}$ ✓

$\therefore x = -4$ $\therefore \log_{10} 10^{-4} = -4$ ✓

h) let $\log_2 2\sqrt{2} = x$

$\therefore 2^x = 2\sqrt{2}$

$\therefore x = 1 + \frac{1}{2} = 3/2$ ✓

$\therefore \log_2 2\sqrt{2} = 3/2$ ✓

$\log_m 2 = p$, $\log_m 5 = q$, then:

a) $\log_m 16 = \log_m 2^4$ ✓

$= 4 \log_m 2$

$= 4p.$ ✓

b) $\log_m 10 = \log_m (2 \times 5)$

$= \log_m 2 + \log_m 5$ ✓

$= p + q.$ ✓

c) $\log_m 0.4 = \log_m\left(\frac{2}{5}\right)$ ✓

$= \log_m 2 - \log_m 5$

$= p - q$ ✓

d) $\log_m 2m = \log_m 2 + \log_m m$ ✓

$= p + 1$ ✓

e) $\log_m 20 = \log_m (2^2 \times 5)$

$= 2\log_m 2 + \log_m 5$ ✓

$= 2p + q$ ✓

f) $\log_m\left(\frac{8\sqrt{2}}{25}\right) = \log_m\left(\frac{2^{3+\frac{1}{2}}}{5^2}\right)$ ✓

$= \frac{7}{2}\log_m 2 - 2\log_m 5$

$= \frac{7p}{2} - q$ ✓

g) $\log_2 5 = \frac{\log_m 5}{\log_m 2}$ ✓ $= \frac{q}{p}$ ✓

h) $\log_4 25 = \frac{\log_m 25}{\log_m 4}$

$= \frac{\log_m 5^2}{\log_m 2^2}$

$= \frac{2\log_m 5}{2\log_m 2}$ ✓ $= \frac{2q}{2p} = \frac{q}{p}$ ✓

3a) $\log_{10} 2 + \log_{10} 50 = \log_{10} 100 = \log_{10} 10^2$

$= 2.$

b) $\log\left(\frac{x}{y}\right) + \log\left(\frac{y}{x}\right) = \log 1 = 0$ ✓

c) $\log\left(\frac{ab}{c}\right) + \log\left(\frac{bc}{a}\right) + \log\left(\frac{ac}{b}\right)$

$= \log\left(\frac{\cancel{a}b}{\cancel{c}} \times \frac{\cancel{b}\cancel{c}}{\cancel{a}} \times \frac{ac}{\cancel{b}}\right)$ ✓

$= \log abc.$ ✓

4a) $8^x = 16$ ✓

ie. $2^{3x} = 2^4$ ✓

$\therefore x = \frac{4}{3}$ ✓

b) $4^{x+3} = 8$

ie. $2^{2x+6} = 2^3$ ✓

$\therefore 2x + 6 = 3$

$\therefore x = -3/2$ ✓

c) $16^{x} = \frac{1}{8}$ ✓

ie. $2^{4x} = 2^{-3}$

$\therefore 4x = -3$ ✓

$\therefore x = \frac{-3}{4}$

d) $25^{1-x} = \frac{1}{125}$ ✓

ie. $5^{2-2x} = 5^{-3}$

$\therefore 2-2x = -3$

$\therefore 2x = 5$ ✓

$\therefore x = \frac{5}{2}$ ✓

e) $36^{x} = \left(\frac{1}{216}\right)^2$

ie. $6^{2x} = 6^{-6}$ ✓

$\therefore 2x = -6$

$\therefore \underline{x = -3}$ ✓

f) $4^{x+2} = \frac{1}{2}$

ie. $2^{2x+4} = 2^{-1}$

$\therefore 2x+4 = -1$ ✓

$\therefore 2x = -5$

$\therefore x = -\frac{5}{2}$ ✓

5a) $3^{x-1} = 5$

ie. $\log_{10} 3^{x-1} = \log_{10} 5$

ie. $x-1 \log_{10} 3 = \log_{10} 5$

$\therefore x-1 = \frac{\log_{10} 5}{\log_{10} 3}$ ✓

$\therefore x = 2.46$ (to 2dp). ✓

b) $2^{1-x} = 3$

ie. $\log_{10} 2^{1-x} = \log_{10} 3$

ie. $(1-x)\log_{10} 2 = \log_{10} 3$

$\therefore (1-x) = \frac{\log_{10} 3}{\log_{10} 2}$ ✓

$\therefore x = -0.58$ (to 2dp) ✓

c) $2^{x+1} = 3^{x-1}$

ie. $\log_{10} 2^{x+1} = \log_{10} 3^{x-1}$

ie. $(x+1)\log_{10} 2 = (x-1)\log_{10} 3$

ie. $\frac{x+1}{x-1} = \frac{\log_{10} 3}{\log_{10} 2}$

$\therefore \frac{x+1}{x-1} = 1.585$

$\therefore x+1 = 1.585(x-1)$

$= 1.585x - 1.585$

$\therefore 2.585 = 0.585x$

$\therefore \frac{2.585}{0.585} = x$

$\therefore x = 4.42$ (to 2 dp).

(d) + (e) are done in the same way.