Year 11 Extension 1 Assignment 3 due:

HAND in your solutions of A4 paper.
Write neatly and staple your pages together

1. Solve for x:

(a) $\dfrac{3}{2-\frac{x}{2}} = 2$ (b) $2x^2+5x=2$ (c) $(x+5)(x+6)=42$

2. Sketch the solutions to the following on separate number lines.

(a) $2\frac{1}{3} \leqslant \dfrac{2+3x}{2}$ (b) $|5x-9|>21$ (c) $2x^2-7x<0$

(d) $\dfrac{2}{x-1} \geqslant 4-x$

3. (a) Show that $x^2+2x+5=0$ has no solution. quadratic

(b) Find the average of the roots of the equation $2x^2+14x+17=0$

(c) Solve $t^4+2t^3-8t-16=0$

(d) Solve $\dfrac{2}{x-1} - \dfrac{1}{x+1} = 1$

4. In the right angled triangle ABC, one of the sides adjacent to the right angle is 4cm longer than the other side. If the triangle has an area of 96cm^2, find the length of the hypotenuse.

5. Solve $t^2-6t=5$, giving answers in exact form. Hence, find the roots of $(t-1)^2-6(t-1)=5$

6. Solve for x:

(a) $|2x-11| = 3x-4$ (b) $\dfrac{|x-3|}{3-x} = x$

(c) $|x-3| + |x+4| = |x-2|$

7. A number consists of 2 digits whose sum is 11. If the digits are reversed, the second number is 63 smaller than the first. Use simultaneous equations to find the number.

8. How many points do the following curves have in common? What is/are they?

$x^2-xy+y^2=21$ and $3x-2y=2$

9. Find the values of x, y and z for this set of simultaneous equations

$$\begin{cases} 2x+3y+4z=-4 \\ 3x+4y-2z=17 \\ 4x+6y-3z=25 \end{cases}$$

10. If x and y are real numbers such that $0<y<x$ and $x^2+y^2=6xy$, then the value of $\sqrt{\dfrac{x+y}{x-y}}$ is a surd.

Determine the value of this surd.

Ashley Cheung

Yr 11 Ext 1 Assignment 3.

Great work.

1a) $\frac{6}{4-x} = 2 \quad (x \neq 4)$ ✓

$(4-x)6 = 4(4-x)^2$

$4(4-x)^2 - 6(4-x) = 0$

$(4-x)(16-4x-6) = 0$

$(4-x)(10-4x) = 0$

$2(4-x)(5-2x) = 0$

$x = 4$ or $x = 2\frac{1}{2}$

$\boxed{\begin{aligned} 6 &= 8-2x \\ 2x &= 2 \\ x &= 1 \end{aligned}}$ ✓✓

1b) $2x^2 + 5x - 2 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-5 \pm \sqrt{25+16}}{4}$

$= \frac{-5 \pm \sqrt{41}}{4}$ ✓

1c) $x^2 + 11x + 30 = 42$

$x^2 + 11x - 12 = 0$

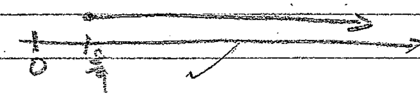
$(x+12)(x-1) = 0$

$\therefore x = -12$ or $x = 1$ ✓

2a) $4\frac{2}{3} \leq 2 + 3x$

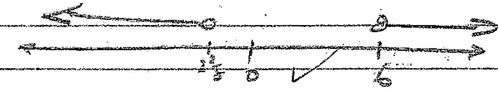
$2\frac{2}{3} \leq 3x$

$\frac{8}{9} \leq x$ ✓



2b)

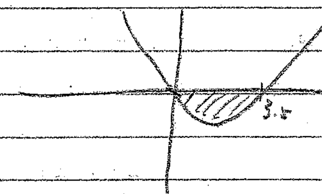
$5x - 9 > 21$	$9 - 5x > 21$
$5x > 30$	$-5x > 12$
$x > 6$ ✓	$5x < -12$
	$x < -2\frac{2}{5}$ ✓

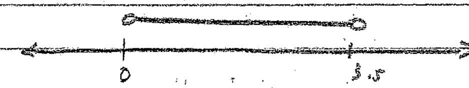


2c) $2x^2 - 7x < 0$

$x(2x-7) < 0$

$\therefore 0 < x < 3.5$ ✓





2d) $2(x-1) \geq (4-x)(x-1)^2$ ✓

$0 \geq (4-x)(x-1) - 2(x-1)$

$0 \geq (x-1)(4-x-2)$

$0 \geq (x-1)(2-x)$

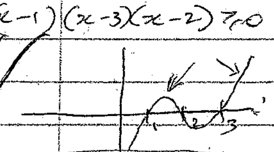
$1 < x < 2$

$(x-1)\{(4-x)(x-1)-2\} \leq 0$

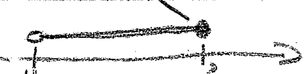
$(x-1)\{-x^2+5x-6\} \leq 0$

$(x-1)\{x^2-5x+6\} \geq 0$

$(x-1)(x-3)(x-2) \geq 0$ ✓✓



$\boxed{\therefore x \leq 1 \cup 2 \leq x \leq 3}$



3a) $\Delta = b^2 - 4ac$

$= 4 - 20$

$= -16$

Since $\Delta < 0$ no real solutions.

3b) $\dfrac{\alpha + \beta}{2} = \dfrac{-\frac{b}{a}}{2}$

$= \dfrac{-\frac{14}{2}}{2}$

$= -3.5$

3c) $(t+2)(t-2)(t^2-2t+4)$ are factors

$$\begin{array}{r} t^3 - 8 \\ t+2 \overline{\smash{)}\, t^4 + 2t^3 - 8t - 16} \\ t^4 + 2t^3 \\ \hline 0 - 8t - 16 \\ -8t - 16 \\ \hline 0 \end{array}$$

$$\begin{array}{r} t^2 - 2t + 4 \\ t-2 \overline{\smash{)}\, t^3 - 8} \\ t^3 - 2t^2 \\ \hline -2t^2 - 8 \\ -2t^2 + 4t \\ \hline 4t - 8 \\ 4t - 8 \\ \hline 0 \end{array}$$

$\therefore t = 2$ & $t = -2$ are the only solutions.

OR

$t^4 + 2t^3 - 8t - 16$

$= t^3(t+2) - 8(t+2)$

$= (t^3 - 8)(t+2)$

$= (t-2)(t^2+2t+4)(t+2)$

$= 0$ when $t = 2$ or -2

(Note $t^2+2t+4=0$ has no solutions

$\Delta = 4 - 16 = -12$

3d) $\dfrac{2(x+1) - (x-1)}{(x+1)(x-1)} = 1$ $\quad x \neq 1$ & $x \neq -1$

$(x+1)(x-1)(x+3) = (x+1)^2(x-1)^2$ ← Only necessary with inequalities

$(x+1)(x-1)[(x+1)(x-1) - (x+3)] = 0$

$(x+1)(x-1)[x^2 - 1 - x - 3] = 0$

$(x+1)(x-1)(x^2 - x - 4) = 0$

$\therefore x = \cancel{1}, \cancel{-1}, \dfrac{1 \pm \sqrt{17}}{2}$

OR

$\therefore \dfrac{x+3}{x^2-1} = 1$

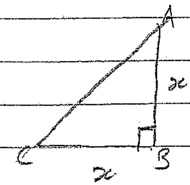
$\therefore x + 3 = x^2 - 1$

$0 = x^2 - x - 4$

$x = \dfrac{1 \pm \sqrt{1+16}}{2}$

$= \dfrac{1+\sqrt{17}}{2}, \dfrac{1-\sqrt{17}}{2}$

4)



$A = 96 = \dfrac{x(x+4)}{2}$

$192 = x^2 + 4x$

$x^2 + 4x - 192 = 0$

$(x-12)(x+16) = 0$

$\therefore x = 12$ since $x \geq 0$

$\therefore AC^2 = (x+4)^2 + x^2$

$= 16^2 + 12^2$

$= 400$

$\therefore AC = 20$

good

5) $t^2 - 6t - 5 = 0$

$t = \dfrac{6 \pm \sqrt{36+20}}{2} = \dfrac{6 \pm \sqrt{56}}{2} = 3 \pm \sqrt{14}$

5 cont'd) $(t-1)(t-1-6) = 5$

$(t-1)(t-7) = 5$

$t^2 - 8t + 7 = 5$

$t^2 - 8t + 2 = 0$

$t = \dfrac{8 \pm \sqrt{64-8}}{2}$

$= 4 \pm \sqrt{14}$ ✓

$\boxed{OR}$ ∴ roots of $(t-1)^2 - 6(t-1) = 5$

are $t-1 = 3 + \sqrt{14}$

and $t-1 = 3 - \sqrt{14}$

$\therefore t = 4+\sqrt{14}$ or $4-\sqrt{14}$

6a) $|2x-11| = 3x-4$

$2x-11 = 3x-4$ or $11-2x = 3x-4$ ✓

$\boxed{x = -7}$ ✗ $15 = 5x$

But $x \neq -7$ when tested. $x = 3$ ✓ is only sol^n.

6b) $\dfrac{|x-3|}{3-x} = x$ $x \neq 3$

for $x > 3$

$\dfrac{|x-3|}{3-x} = -1$

$\therefore x = -1$

but $x > 3$

∴ not a solution

for $x < 3$

$\dfrac{|x-3|}{3-x} = 1$ ✓

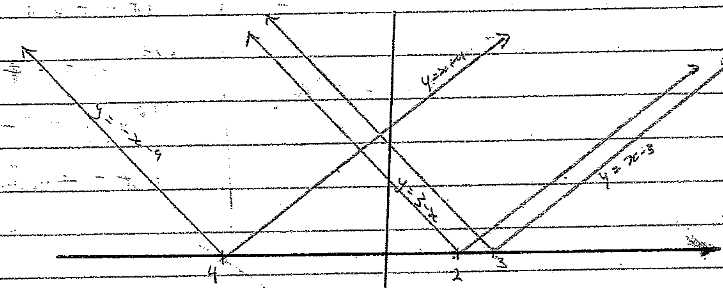
$\therefore x = 1$

∴ 1 solution

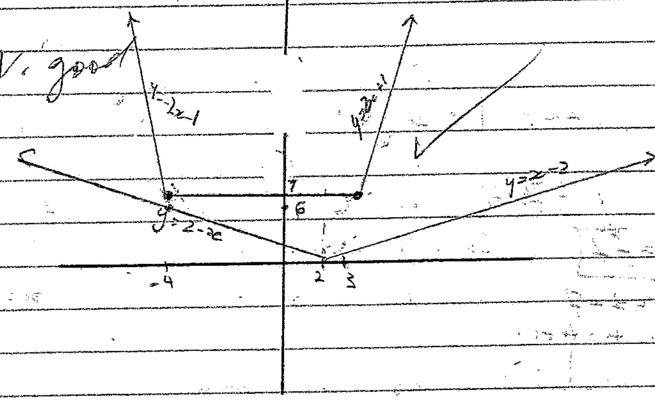
$x = 1$ ✓

V-good

6c) $|x-3| + |x+4| = |x-2|$



V. good ✓



$y = -x-4+3-x$ $= -(2x+1)$

$y = x+4+3-x$ $= 7$

$y = x+4+x-3$ $= 2x+1$

$x-2 = 7$, $x = 9$

$2-x = 7$, $x = -5$

∴ no intersection

since $|x-2| \neq |2x+1|$ for $x > 3$ & $x < -4$

&

$|x-2| \neq 7$ for $-4 < x < 3$

∴ no solutions. ✓

7) $\frac{T \mid U}{x \mid y} \Rightarrow 10x + y$

Tens	units
x	y
y	x

$\therefore N_1 = 10x + y$

$N_2 = 10y + x$

$N_1 - 63 = N_2$

$\therefore 10x + y - 63 = 10y + x$

$9x - 9y = 63$

$x - y = 7$ ✓

Also $x + y = 11$

$\therefore x + y = 11$

$x - y = 7$

$\therefore 2y = 4$

$y = 2$

$\therefore x = 9$ ✓

$\therefore$ the number is 92 ✓ V. good.

8) see ⟶ Next page ⟶

9) $2x + 3y + 4z = -4$ — (1)

$3x + 4y - 2z = 17$ — (2)

$4x + 6y - 3z = 25$ — (3)

(1)×2 → $4x + 6y + 8z = -8$ — (4)

(4) − (3) $\therefore 11z = -33$

$z = -3$

9 cont'd) $2x + 3y - 12 = -4 \Rightarrow 2x + 3y = 8$

$3x + 4y + 6 = 17 \quad\quad 3x + 4y = 11$

$6x + 9y = 24$

$6x + 8y = 22$

$y = 2$

$x = 1$

$\therefore x = 1,\ y = 2,\ z = -3$

10) $(x+y)^2 = x^2 + y^2 + 2xy = 8xy$

$(x-y)^2 = x^2 + y^2 - 2xy = 4xy$ ✓

$\therefore \frac{(x+y)^2}{(x-y)^2} = \frac{8xy}{4xy}$

$\frac{x+y}{x-y} = \frac{\sqrt{8xy}}{\sqrt{4xy}}$ ✓ good

$= \sqrt{2}$

$\sqrt{\frac{x+y}{x-y}} = \sqrt{\sqrt{2}}$ ✓

8) $x^2 - xy + y^2 = 21 \leftarrow$ & $y = \frac{3x-2}{2}$ substitute

$x^2 - x\frac{(3x-2)}{2} + \frac{(3x-2)^2}{4} = 21 \quad (\times 4)$

$4x^2 - 2x(3x-2) + (9x^2 - 12x + 4) = 84$

$4x^2 - 6x^2 + 4x + 9x^2 - 12x + 4 - 84 = 0$

$7x^2 - 8x - 80 = 0$

$(7x + 20)(x - 4) = 0$

$\therefore x = 4$ or $x = -\frac{20}{7}$

$y = 5 \quad\quad y = -\frac{37}{7}$ } 2 points of intersection