Year 11   Extension 1   Assignment 3   due:

HAND in your solutions of A4 paper.
Write neatly and staple your pages together

1. Solve for x:

(a) $\dfrac{3}{2 - \frac{x}{2}} = 2$ (b) $2x^2 + 5x = 2$ (c) $(x+5)(x+6) = 42$

2. Sketch the solutions to the following on separate number lines.

(a) $2\frac{1}{3} \le \dfrac{2 + 3x}{2}$ (b) $|5x - 9| > 21$ (c) $2x^2 - 7x < 0$

(d) $\dfrac{2}{x-1} \ge 4 - x$

3. (a) Show that $x^2 + 2x + 5 = 0$ has no solution. quadratic

(b) Find the average of the roots of the equation
$$2x^2 + 14x + 17 = 0$$

(c) Solve $t^4 + 2t^3 - 8t - 16 = 0$

(d) Solve $\dfrac{2}{x-1} - \dfrac{1}{x+1} = 1$

4. In the right angled triangle ABC, one of the sides adjacent to the right angle is 4cm longer than the other side. If the triangle has an area of 96cm², find the length of the hypotenuse.

5. Solve $t^2 - 6t = 5$, giving answers in exact form.
Hence, find the roots of $(t-1)^2 - 6(t-1) = 5$

6. Solve for x:

(a) $|2x - 11| = 3x - 4$ (b) $\dfrac{|x-3|}{3-x} = x$

(c) $|x-3| + |x+4| = |x-2|$

7. A number consists of 2 digits whose sum is 11. If the digits are reversed, the second number is 63 smaller than the first.
Use simultaneous equations to find the number.

8. How many points do the following curves have in common? What is/are they?
$$x^2 - xy + y^2 = 21 \quad \text{and} \quad 3x - 2y = 2$$

9. Find the values of x, y and z for this set of simultaneous equations
$$\begin{cases} 2x + 3y + 4z = -4 \\ 3x + 4y - 2z = 17 \\ 4x + 6y - 3z = 25 \end{cases}$$

10. If x and y are real numbers such that $0 < y < x$ and $x^2 + y^2 = 6xy$, then the value of $\sqrt{\dfrac{x+y}{x-y}}$ is a surd.

Determine the value of this surd.

Yr 11   Ext 1   Assignment 3.

1a) $\dfrac{6}{4-x} = 2$ $(x \neq 4)$ ✓

$(4-x)6 = 4(4-x)^2$

$4(4-x)^2 - 6(4-x) = 0$

$(4-x)(16-4x+6) = 0$

$(4-x)(16-4x) = 0$

$2(4-x)(5-2x) = 0$

$x = 4 \quad x \quad x = 2\tfrac{1}{2}$

$$6 = 8 - 2x$$
$$2x = 2$$
$$x = 1 \quad \checkmark\checkmark$$

1b) $2x^2 + 5x - 2 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-5 \pm \sqrt{25 + 16}}{4}$

$= \dfrac{-5 \pm \sqrt{41}}{4}$ ✓

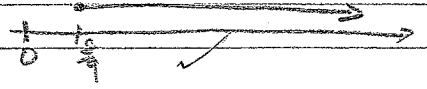**1c)** $x^2 + 11x + 30 = 42$

$x^2 + 11x - 12 = 0$

$(x + 12)(x - 1) = 0$

$\therefore x = -12 \text{ or } x = 1$ ✓

2a) $4\tfrac{2}{3} \leq 2 + 3x$

$2\tfrac{2}{3} \leq 3x$
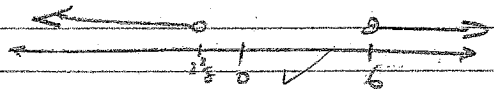
$\dfrac{8}{9} \leq x$ ✓

2b)

$5x - 9 > 21$	$9 - 5x > 21$
$5x > 30$	$-5x > 12$
$x > 6$	$5x < -12$
✓	$x < -2\tfrac{2}{5}$ ✓

2c) $2x^2 - 7x < 0$

$x(2x - 7) < 0$

$\therefore 0 < x < 3.5$ ✓
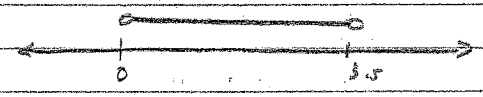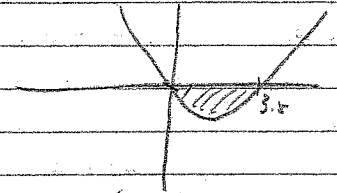
2d) $2(x-1) \geq (4-x)(x-1)^2$

$(x-1)\{4-x(x-1) - 2\} \leq 0$

$(x-1)\{-x^2 + 5x - 6\} \leq 0$

$(x-1)\{x^2 - 5x + 6\} \geq 0$

$(x-1)(x-3)(x-2) \geq 0$
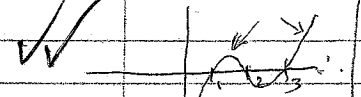
$1 \leq x \leq 2$

$\boxed{\therefore x \leq 1 \cup 2 \leq x \leq 3}$

3a)

$$\Delta = b^2 - 4ac$$

$$= 4 - 20$$

$$= -16$$

Since $\Delta < 0$ no real solutions. ✓

3b)

$$\frac{\alpha + \beta}{2}, \quad \frac{-\frac{b}{a}}{2}$$

$$= \frac{-\frac{14}{4}}{2}$$

$$= -3.5 \checkmark$$

3c)

$(t+2)(t-2)(t^2 - 2t + 4)$ are factors

$$
\begin{array}{r}
t^3 - 8 \\
t+2 \,\overline{\big)\, t^4 + 2t^3 - 8t - 16} \\
\underline{t^4 + 2t^3} \\
-0 - 8t - 16 \\
\underline{-8t - 16} \\
0
\end{array}
$$

$$
\begin{array}{r}
t^2 - 2t + 4 \\
t-2 \,\overline{\big)\, t^3 - 8} \\
\underline{t^3 - 2t^2} \\
-2t^2 - 8 \\
\underline{-2t^2 + 4t} \\
4t - 8 \\
\underline{4t - 8} \\
0
\end{array}
$$

∴ $t = 2$ & $t = -2$ are the only solutions.

OR

$t^4 + 2t^3 - 8t - 16$
$= t^3(t+2) - 8(t+2)$
$= (t^3 - 8)(t+2)$
$= (t-2)(t^2 + 2t + 4)(t+2)$
$= 0$ when $t = 2$ or -2 ✓✓

(Note $t^2 + 2t + 4 = 0$ has no solutions $\Delta = 4 - 16 = -12$

3d)

$$\frac{2(x+1) - (x-1)}{(x+1)(x-1)} = 1 \qquad x \neq 1 \,\&\, x \neq -1$$

OR

$$\therefore \frac{x+3}{x^2 - 1} = 1$$
$$\therefore x + 3 = x^2 - 1$$
$$0 = x^2 - x - 4$$
$$x = \frac{1 \pm \sqrt{1 + 16}}{2}$$
$$= \frac{1 + \sqrt{17}}{2}, \frac{1 - \sqrt{17}}{2}$$

$(x+1)(x-1)(x+3) = (x+1)^2(x-1)^2$ ← Only necessary with inequality

$(x+1)(x-1)[(x+1)(x-1) - (x+3)] = 0$

$(x+1)(x-1)[x^2 - 1 - x - 3] = 0$
$(x+1)(x-1)(x^2 - x - 4) = 0$
∴ $x = \cancel{1}, \cancel{-1}, \frac{1 \pm \sqrt{17}}{2}$ ✓

4)

$$A = 96 = \frac{x(x+4)}{2} \checkmark$$

$$192 = x^2 + 4x$$
$$x^2 + 4x - 192 = 0 \checkmark$$
$$(x-12)(x+16) = 0 \checkmark$$
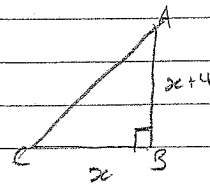
∴ $x = 12$ since $x \geqslant 0$ ✓

∴ $AC^2 = (x+4)^2 + x^2$

$$= 16^2 + 12^2$$

$$= 400$$

∴ $AC = 20$ ✓   good

5) $t^2 - 6t - 5 = 0$

$$\frac{6 \pm \sqrt{36 + 20}}{2} = \frac{6 \pm \sqrt{56}}{2} = 3 \pm \sqrt{14}$$

5 cont'd) $(t-1)(t-1-6) = 5$ | OR 2 roots of
$(t-1)(t-7) = 5$ | $(t-1)^2 - 6(t-1) = 5$
$t^2 - 8t + 7 = 5$ | are $t - 1 = 3 + \sqrt{14}$
$t^2 - 8t + 2 = 0$ | and $t - 1 = 3 - \sqrt{14}$
$t = \dfrac{8 \pm \sqrt{64-8}}{2}$ | $\therefore t = 4 + \sqrt{14}$ or $4 - \sqrt{14}$

$= 4 \pm \sqrt{14}$

(6a) $|2x - 11| = 3x - 4$

$-2x - 11 = 3x - 4$ or $11 - 2x = 3x - 4$

$\boxed{x = -7}$ $5x \ne 0$ $\cancel{\times}$ $15 = 5x$
But $x \ne -7$ $x = \frac{7}{5}$ $x = 3 \checkmark$ is only sol^n.
when tested $\checkmark$

6b) $\dfrac{|x-3|}{3-x} = x$ $x \ne 3$

for $x > 3$ | for $x < 3$

$\dfrac{|x-3|}{3-x} = -1$ | $\dfrac{|x-3|}{3-x} = 1$ $\checkmark$

$\therefore x = 1$ or $\therefore x = 1$
but $x > 3$

$\therefore$ not a solution | $\therefore$ 1 solution
$x = 1$ $\checkmark$

V-good

6c) $|x-3| + |x+4| = |x-2|$

V. good

$y = -x-4 + 3-x$ | $y = x+4 + 3-x$ | $y = x+4 + x-3$
$= -(2x+1)$ | $= 7$ | $= 2x+1$

$x - 2 = 7$ | $2 - x = 7$
$x = 9$ | $x = -5$

$\therefore$ no intersection
since $|x-2| \ne |2x+1|$ for $x > 3$ & $x < -4$
&
$|x-2| \ne 7$ for $-4 < x < 3$

$\therefore$ no solutions. $\checkmark$

7) $\dfrac{}{x\,|\,y} \Rightarrow 10x+y$

	Tens	Units

$\therefore N_1 = 10x + y \quad\leftarrow\quad x\,|\,y$

$N_2 = 10y + x \quad\leftarrow\quad y\,|\,x$

$N_1 - 63 = N_2$

$\therefore 10x + y - 63 = 10y + x$

$9x - 9y = 63$

$x - y = 7 \quad\checkmark$

Also $x + y = 11$

$\therefore x + y = 11$

$x - y = 7$

$\therefore 2y = 4$

$y = 2$

$\therefore x = 9 \quad\checkmark$

$\therefore$ the number is 92 $\quad\checkmark\quad$ V. good

8) see $\longrightarrow$ Next page $\longrightarrow$

9) $2x + 3y + 4z = -4 \quad —(1)$

$3x + 4y - 2z = 17 \quad —(2)$

$4x + 6y - 3z = 25 \quad —(3)$

$(1) \times 2 \rightarrow 4x + 6y + 8z = -8 \quad —(4)$

$(4)-(3) \quad \therefore 11z = -33$

$z = -3$

---

9 cont'd) $2x + 3y - 12 = -4 \Rightarrow 2x + 3y = 8$

$3x + 4y + 6 = 17 \qquad\qquad 3x + 4y = 11$

$6x + 9y = 24$

$6x + 8y = 22$

$y = 2$

$x = 1$

$\therefore x = 1, \quad y = 2, \quad z = -3$

10) $(x+y)^2 = x^2 + y^2 + 2xy = 8xy$

$(x-y)^2 = x^2 + y^2 - 2xy = 4xy$

$\therefore \dfrac{(x+y)^2}{(x-y)^2} = \dfrac{8xy}{4xy}$

$\dfrac{x+y}{x-y} = \dfrac{\sqrt{8xy}}{\sqrt{4xy}} \qquad\qquad$ V. good

$= \sqrt{2}$

$\sqrt{\dfrac{x+y}{x-y}} = \sqrt{\sqrt{2}} \qquad\qquad \checkmark$

8) $x^2 - xy + y^2 = 21 \quad \& \quad y = \dfrac{3x-2}{2}$ substitute

$x^2 - x\left(\dfrac{3x-2}{2}\right) + \dfrac{(3x-2)^2}{4} = 21 \quad (\times 4)$

$4x^2 - 2x(3x-2) + (9x^2 - 12x + 4) = 84$

$4x^2 - 6x^2 + 4x + 9x^2 - 12x + 4 - 84 = 0$

$7x^2 - 8x - 80 = 0$

$(7x + 20)(x - 4) = 0$

$\therefore x = 4 \quad \text{or} \quad x = \dfrac{-20}{7} \quad \Big\} \text{ 2 points of intersection}$

$y = 5 \qquad\qquad y = -3\frac{5}{7}$