Sydney Girls High School

MATHEMATICS

Year 11

Assessment Task 1

2015

Time Allowed: 50 minutes + 5 minutes reading time

Topics: Basic Arithmetic and Algebra

Total: 44 marks

Instructions:

- There are FOUR (4) questions which are of equal value.
- Attempt all questions.
- Show all necessary working. Marks may be deducted for badly arranged work or incomplete working.
- All answer must be written to the simplest exact form.
- Start each question on a new page.
- Write on one side of the paper only.
- Diagrams are NOT to scale.
- Board-approved calculators may be used.
- Write your student number at the top of each question and clearly number each question.

Student 's Name:_____ **Teacher's Name:**_____

Question 1 [11 marks]

Start each question on a new page.

(a) Evaluate in scientific notation $0.4 \times 0.07 \times 0.0003$ [1 mark]

(b) Evaluate $\sqrt{\frac{7+\pi}{11}}$ to 3 significant figures. [1 mark]

(c) Write the following with a fractional index. [1 mark]

$$\sqrt[5]{a^3}$$

(d) Showing all working, write a simplified fraction for $4.\dot{1}\dot{8}$ [2 marks]

(e) Rationalise the denominator then simplify the following. [2 marks]

$$\frac{2-\sqrt{10}}{2\sqrt{6}}$$

(f) Simplify in exact form [2 marks]

(i) $(\sqrt{5}-3)^2$

(ii) $\frac{\sqrt{15}}{\sqrt{12}}$

(g) Write the following without fractional indices. [2 marks]

$$\left(\frac{25a^4}{b^6}\right)^{-\frac{3}{2}}$$

End of Question 1

Question 2 [11 marks]

Start each question on a new page.

(a) If $\sqrt{45} + \sqrt{80} = a\sqrt{5}$, find a. [1 mark]

(b) Solve for x [3 marks]

(i) $4(x+5) = -6(3-x)$

(ii) $\frac{x+2}{4} + \frac{x-3}{5} = 1$

(iii) $4 + \sqrt{x-2} = 7$

(c) Fully factorise [5 marks]

(i) $15b - 3ab$

(ii) $m^2 + 6m + 9$

(iii) $2x^2 - 11x - 21$

(iv) $p^4 + 27p$

(v) $x^2 + 7x - y^2 - 7y$

(d) Find the exact value of $\frac{A^2B^3}{C^2}$ where [2 marks]

$$A = \left(\frac{5}{2}\right)^2 \quad B = \left(\frac{2}{3}\right)^3 \quad C = \left(\frac{5}{3}\right)^3$$

End of Question 2

Question 3 [11 marks]

Start each question on a new page.

(a) Solve for x by factorising [2 mark]

$$6x^2 - 19x - 7 = 0$$

(b) Simplify [7 marks]

(i) $a(1-b) - b(1-a)$

(ii) $\frac{6m^3n}{5mn^3} \div \frac{3m^5}{mn^2}$

(iii) $\frac{3}{x^2+7x+10} - \frac{1}{x+5}$

(iv) $\frac{8-x^3}{x-2}$

(v) $\frac{a^2-b^2}{3a^2-3ab} \times \frac{2a^2+2ab}{a^2+2ab+b^2}$

(c) Simplify [2 marks]

$$\frac{9^{k+1} \times 7^{2k-1}}{21^{2k} \times 27}$$

End of Question 3

Question 4 [11 marks]

Start each question on a new page.

(a) Simplify $\left(a-\frac{1}{a}\right)$ if $a=\sqrt{2}+1$ [2 marks]

(b) Solve for x, [3 marks]

$$(0.2)^{x+1}=(0.008)^{x-1}$$

(c) [3 marks]

(i) Show that $\frac{1}{\sqrt{n-1}+\sqrt{n}}=\sqrt{n}-\sqrt{n-1}$, for any positive integer n.

(ii) Hence evaluate

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{8}+\sqrt{9}}+\frac{1}{\sqrt{9}+\sqrt{10}}$$

(d) Solve for x [3 marks]

$$(x^2-12x+36)^{(x^2+x-12)}=1$$

End of Question 4

End of Exam

QUESTION 1

a) $0.4\times0.07\times0.0003$

$=8.4\times10^{-6}$

b) $\sqrt{\frac{7+\pi}{11}}=0.960$

c) $(a^3)^{\frac{1}{5}}=a^{\frac{3}{5}}$

d) Let $4.1818\ldots=x$

$100x=418.18\ldots$

$99x=414$

$x=\frac{414}{99}=\frac{138}{33}=\frac{46}{11}$

e) $\frac{2-\sqrt{10}}{2\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=\frac{2\sqrt{6}-\sqrt{10}\sqrt{6}}{12}$

$=\frac{2\sqrt{6}-\sqrt{60}}{12}=\frac{2\sqrt{6}-2\sqrt{15}}{12}$

$=\frac{\sqrt{6}-\sqrt{15}}{6}$

f) i) $(\sqrt{5}-3)^2$

$=5+9-2(3)(\sqrt{5})$

$14-6\sqrt{5}$

ii) $\frac{\sqrt{15}}{\sqrt{12}}=\frac{\sqrt{15}\times\sqrt{12}}{12}=\frac{\sqrt{180}}{12}=\frac{\sqrt{9}\sqrt{20}}{12}=\frac{\sqrt{9}\sqrt{4}\sqrt{5}}{12}=\frac{\sqrt{5}}{2}$

g) $\left(\frac{25a^4}{b^6}\right)^{-\frac{3}{2}}$

$=\left(\frac{b^6}{25a^4}\right)^{\frac{3}{2}}$

$=\left[\left(\frac{b^6}{25a^4}\right)^3\right]^{\frac{1}{2}}$

$=\sqrt{\frac{b^{18}}{25^3a^{12}}}$

$\Rightarrow\frac{b^9}{125a^6}$

QUESTION 2

a) $\sqrt{45}+\sqrt{80} = \sqrt{9\times5}+\sqrt{16\times5}$

$= \sqrt{9}\times\sqrt{5}+\sqrt{16}\times\sqrt{5}$

$= 3\sqrt{5}+4\sqrt{5} = 7\sqrt{5}$

$\therefore a = 7$

b) i) $4(x+5) = -6(3-x)$

$4x+20 = -18+6x$

$38 = 2x$

$x = \frac{38}{2} = 19$

ii) $\frac{x+2}{4}+\frac{x-3}{5}=1$

multiply both sides by 20

$5(x+2)+4(x-3)=20$

$5x+10+4x-12=20$

$9x = 20+12-10$

$9x = 22$

$x = \frac{22}{9}$

iii) $4+\sqrt{x-2}=7$

$\sqrt{x-2} = 7-4 = 3$

$\sqrt{x-2} = 3$

SQUARE BOTH SIDES

$x-2 = 9$

$x = 9+2 = 11$

c) i) $15b-3ab$

$= 3b(5-a)$

ii) m^2+6m+9

$= (m+3)^2$

iii) $2x^2-11x-21$

$= (2x+3)(x-7)$

Alternatively using the quadratic formula

$a=2,\ b=-11,\ c=-21$

$= \frac{11\pm\sqrt{121+4(2)(21)}}{4}$

$= \frac{11\pm\sqrt{289}}{4} = \frac{11\pm17}{4}$

ROOTS are $-\frac{3}{2}$ AND 7

$\therefore = (2x+3)(x-7)$

iv) p^4+27p

$= p(p^3+27)$

$= p(p+3)(p^2-3p+9)$

v) x^2+7x-y^2-7y

$= x^2-y^2-7x-7y$

$= (x+y)(x-y)+7(x-y)$

$= (x+y)(x-y+7)$

d) i) $\frac{A^2B^3}{C^3}$ $A=\left(\frac{5}{2}\right)^2$ $B=\left(\frac{2}{3}\right)^3$ $C=\left(\frac{5}{3}\right)^3$

$$\frac{\left(\frac{5}{2}\right)^2\cdot\left(\frac{2}{3}\right)^3}{\left(\frac{5}{3}\right)^3} = \frac{\left(\frac{5}{2}\right)^4\cdot\left(\frac{2}{3}\right)^9}{\left(\frac{5}{3}\right)^6}$$

METHOD 1

$$= \frac{\frac{625}{16}\times\frac{512}{19683}}{\frac{15625}{729}}$$

$$= \frac{625}{16}\times\frac{512}{19683}\times\frac{729}{15625} = \frac{233280000}{492075000} = \frac{23328}{4920750} = \frac{2592}{54675} = \frac{288}{6075} = \frac{32}{675}$$

METHOD 2 [EASIER]

$$\frac{\frac{5^4}{2^4}\times\frac{2^9}{3^9}}{\frac{5^6}{3^6}} \Rightarrow \frac{5^4}{2^4}\times\frac{2^9}{3^9}\times\frac{3^6}{5^6}$$

$$= \frac{2^5}{3^3\,5^2} = \frac{32}{675}$$

QUESTION 3

$6x^2 - 19x - 7 = 0$

$6x^2 + 2x - 21x - 7 = 0$

$2x(3x+1) - 7(3x+1) = 0$

$(3x+1)(2x-7) = 0$

$x = -\frac{1}{3}$ or $x = \frac{7}{2}$

b) i) $a(1-b) - b(1-a)$

$a - ab - b + ab$

$= a - b$

ii) $\frac{6m^3n}{5mn^3} \div \frac{3m^5}{mn^2}$

$= \frac{6m^3n}{5mn^3} \times \frac{mn^2}{3m^5}$

$= \frac{6m^4n^3}{15m^6n^3} = \frac{6}{15m^2} = \frac{2}{5m^2}$

iii) $\frac{3}{(x^2+7x+10)} - \frac{1}{x+5}$

$\frac{3}{(x+5)(x+2)} - \frac{1}{x+5}$

$\frac{3}{(x+5)(x+2)} - \frac{(x+2)}{(x+5)(x+2)}$

$= \frac{3-x-2}{(x+5)(x+2)}$

$\Rightarrow \frac{1-x}{(x+5)(x+2)}$

iv) $\frac{8-x^3}{x-2}$

$\Rightarrow \frac{-(x^3-8)}{x-2}$

$= \frac{-(x-2)(x^2+2x+4)}{x-2}$

$= -(x^2+2x+4)$

$= -x^2 - 2x - 4$

v) $\frac{a^2-b^2}{3a^2-3ab} \times \frac{2a^2+2ab}{a^2+2ab+b^2}$

$\frac{(a+b)(a-b) \times (2a)(a+b)}{(3a)(a-b) \times (a+b)^2}$

Simplifying:

$\frac{(a+b)^2 (a-b)(2a)}{3a (a-b)(a+b)^2}$

$= \frac{2a}{3a} = \frac{2}{3}$

QUESTION 3 CONT.

c) $\frac{9^{k+1} \times 7^{2k-1}}{21^{2k} \times 27} = \frac{9^k \cdot 9 \times 7^{2k} \times \frac{1}{7}}{21^{2k} \times 27}$

$= \frac{9^k \times 7^{2k}}{21^{2k} \times 21} = \frac{3^{2k} \times 7^{2k}}{3^{2k} \cdot 7^{2k} \times 21}$

$\Rightarrow \frac{1}{21}$

QUESTION 4

a) $\left(a=\frac{1}{a}\right)$, $a=\sqrt{2}+1$

$$\frac{a^2}{a}-\frac{1}{a}=\frac{a^2-1}{a}$$

$$\frac{(\sqrt{2}+1)^2-1}{(\sqrt{2}+1)}$$

$$\frac{2+1+2\sqrt{2}-1}{(\sqrt{2}+1)}$$

$$=\frac{2(1+\sqrt{2})}{(\sqrt{2}+1)}=2.$$

b) $(0.2)^{x+1}=(0.008)^{x-1}$

$(0.2)^{x+1}=(0.2^3)^{x-1}$

$(0.2)^{x+1}=(0.2)^{3x-3}$

$x+1=3x-3$

$2x=4$

$x=2$

c) $\frac{1}{\sqrt{n-1}+\sqrt{n}}=\sqrt{n}-\sqrt{n-1}$

LTP

LHS $=\frac{1}{\sqrt{n-1}+\sqrt{n}}\times\frac{\sqrt{n-1}-\sqrt{n}}{\sqrt{n-1}-\sqrt{n}}$

$$\frac{\sqrt{n-1}-\sqrt{n}}{[(\sqrt{n-1})+\sqrt{n}][\sqrt{n-1}-\sqrt{n}]}$$

$$\frac{\sqrt{n-1}-\sqrt{n}}{(n-1)-\sqrt{n}\sqrt{n-1}+\sqrt{n}\sqrt{n-1}-n}$$

$$=\frac{\sqrt{n-1}-\sqrt{n}}{n-1-n}=\frac{\sqrt{n-1}-\sqrt{n}}{-1}$$

$$=-(\sqrt{n-1}-\sqrt{n})$$

$$=-(-1)$$

$$=\frac{\sqrt{n}-\sqrt{n-1}}{1}=\sqrt{n}-\sqrt{n-1}$$

= RHS

ii) $n=2 \Rightarrow \frac{1}{\sqrt{1}+\sqrt{2}}=\frac{1}{1+\sqrt{2}}$

$n=3 \Rightarrow \frac{1}{\sqrt{2}+\sqrt{3}}$

$\vdots$

$n=10 \Rightarrow \frac{1}{\sqrt{9}+\sqrt{10}}$

So we have

$(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+$

$\ldots\ (\sqrt{10}-\sqrt{9})$

$=\sqrt{10}-1=2.16$ (2dp.)

d) QUESTION 4 CONT.

$$(x^2-12x+36)^{(x^2+x-12)}=1$$

$$(x-6)^{2(x+4)(x-3)}=1$$

$(x+4)(x-3)\ln[(x-6)^2]=\ln(1)$

$(x+4)(x-3)\ln[(x-6)^2]=0$

$x=-4,\ x=3,\ x=7,\ x=5$