

1. Sketch the following relations, showing main features, and stating the domain & range.

(a) $xy = 1$	(b) $y = x $	(c) $y = x^2 - 4$
(d) $x^2 + y^2 = 36$	(e) $y = 2^x$	(f) $y = x^3 - 4x^2$
(g) $y = -\sqrt{49 - x^2}$	(h) $y = -\sqrt{3 - 2x}$	(i) $y = 1 + \frac{3}{x+4}$
(j) $(x - 1)^2 + (y + 2)^2 = 1$	(k) $y = 1 + x + 1 $	(l) $y = x^3 - 4x^2 + 3x$

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2. Determine the equation of the axis of symmetry and the coordinates of the vertex of the parabola $y = 3x^2 - 6x + 1$. Hence sketch the parabola and write down its range.

3. For the domain $-1 \leq x \leq 3$, determine the maximum value and the minimum value of each of the functions defined by $f(x) = 3x^2$, $g(x) = 4x - x^2$.
Find all values of x in the interval $-1 \leq x \leq 3$ for which $f(x) = g(x)$.
Sketch the curves $y = f(x)$ and $y = g(x)$, for $-1 \leq x \leq 3$, on the same diagram. Indicate clearly the points where maximum and minimum values are attained and the points of intersection of the two curves.

4. A function is defined by the following rule:

$$f(x) = \begin{cases} 0 & \text{if } x \leq -2 \\ -1 & \text{if } -2 < x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

Find: (a) $f(-2) + f(-1) + f(0)$; (b) $f(a^2)$.

5. The function $f(x)$ is defined by the rule $f(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 2x & \text{if } x > 0 \end{cases}$
Sketch the function $f(x)$, from $x = -2$ to $x = 2$.

6. If $f(x) = ax^2 + bx + c$ find a simplified expression for $f(x) - f(-x)$.

7. (a) What is the equation of the circle whose centre is at the origin and which passes through the point $(5, -7)$?

(b) A circle, of radius 5 units, has its centre at the point $(-3, 4)$. What are the coordinates of the two points at which the circle cuts the y axis?

(c) Find the equation of the parabola with vertex $(-1, -4)$ which passes through the origin and whose axis is parallel to the y -axis.

8. The parabola $y = ax^2 - c$, and the circle $x^2 + y^2 = 16$ meet on both the x and y axes. If a and c are both positive, what are their values?

9. (i) Draw the graphs of $y = |x|$ and $y = x + 4$ on the same set of axes.

- (ii) Find the coordinates of the point of intersection of these two graphs.

10. State the natural (ie. the largest possible) domain of the function given by

$$y = \sqrt{1+x} - \sqrt{1-x}.$$

11. Find the range of the function f given by $f(x) = \frac{1}{1+x^2}$ over the domain of all real numbers.

12. Consider $f(x) = \frac{1-x^2}{x^4}$, $x \neq 0$.

- (i) State the zeros of this function, ie. the values of x where $f(x) = 0$.

- (ii) Describe how this function behaves near $x = 0$.

- (iii) Describe how this function behaves for large x .

(iv) Give a rough sketch of this function.

13. Find the maximum value of $2x(1 - x)$.

14. The function $f(x)$ is defined by the rule $f(x) = 9x(x - 2)^2$ in the domain $-1 \leq x \leq 3$. Draw a sketch of the graph of $y = f(x)$, showing clearly the intercepts with the x and y axes, and the values at the endpoints of the domain.

15. (a) Sketch the curve $y = x^3 + x^2 - x - 1$ over the domain $-1 \leq x \leq 2$.

(b) Given that one of the turning points of this curve occurs when $x = \frac{1}{3}$, find the range of this function over the stated domain.

16. Sketch graphs for

(a) $y = \frac{x^2 + x}{x}$

(b) $y = \frac{x^2 + x}{x + 1}$

(c) $y = \frac{x^3 + 2x^2 + x}{x + 1}$

17. $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x + 1}$. For what values of x does

(a) $g(x^2) = [g(x)]^2$?

(b) $f(x^2) = [f(x)]^2$?

18. Find $f(x)$ if

(a) $f(x + 1) = x + 4$

(b) $f(x + 1) = 3x - 2$

(c) $f(x+1) = 1 - x^2$

(d) $f(2a) = a^2 - 2a - 1$

19. $f(x) = 3x - 4$ and $g(x) = x^2 - 1$, show that

(a) $f(a) + f(b) = f(a + b)$

(b) $g(a) + g(b) \neq g(a + b)$

20. Write down the range of the function $f(x) = \frac{1}{x^2 + 4x + 7}$.

21. Sketch the graph of the function $y = \sqrt{16 - (x - 3)^2} + 1$. State the domain and range of the function.
22. If $f(x) = 3x + 5$ and $g(x) = \frac{x - 5}{3}$, find a simplified expression for $f[g(x)] + g[f(x)]$.
23. The parabola $y = ax^2 + bx + c$ has its vertex at $(2, 1)$ and passes through the point $(0, 0)$. Find a , b and c .

24. Given that $Q(x) = ax^2 + bx + c$ for all x , and that $Q(0) = 4$, $Q(1) = 23$, $Q(-1) = 1$, determine the constants a , b , c ,

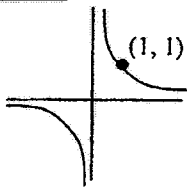
25. Sketch $y = \sqrt{(x+2)^2}$

26. (a) By first sketching $y = x^2 - 1$, draw a sketch of $y = \frac{1}{x^2 - 1}$.

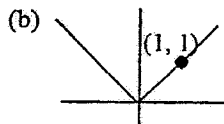
- (b) Hence write down the domain and range of the function $y = \frac{1}{x^2 - 1}$.

ANSWERS:

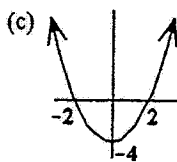
1. (a)



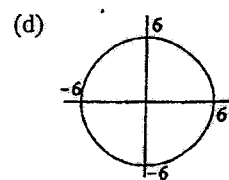
D: $x \neq 0$
R: $y \neq 0$



D: all real x
R: $y \geq 0$



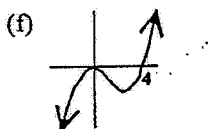
D: all real x
R: $y \geq -4$



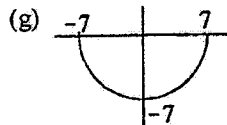
D: $-6 \leq x \leq 6$
R: $-6 \leq y \leq 6$



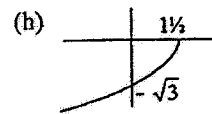
D: all real x
R: $y > 0$



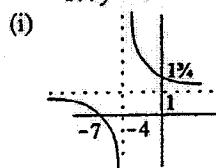
D: all real x
R: all real y



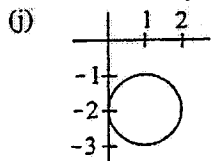
D: $-7 \leq x \leq 7$
R: $-7 \leq y \leq 0$



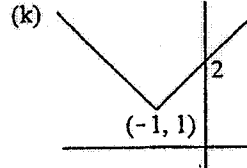
D: $x \leq 1\frac{1}{2}$
R: $y \leq 0$



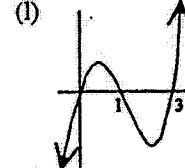
D: $x \neq 4$
R: $y \neq 1$



D: $0 \leq x \leq 2$
R: $-3 \leq y \leq -1$

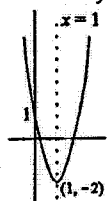


D: all real x
R: $y \geq 1$



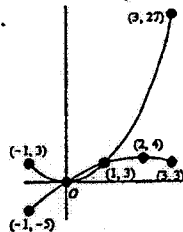
D: all real x
R: all real y

2.



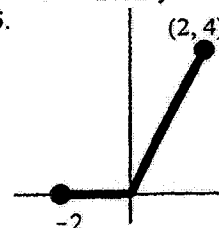
Axis of Sym: $x = 1$
Range: $y \geq -2$

3.



4. (a) -1 (b) a^2

5.



6. $2bx$

7.

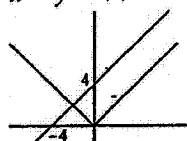
(a) $x^2 + y^2 = 74$

(b) $(0, 0), (0, 8)$

(c) $y = 4(x + 1)^2 - 4$

8. $a = \frac{1}{4}, c = 4$

9. (i)



(ii) $(-2, 2)$

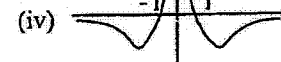
10. $-1 \leq x \leq 1$

11. $0 < y \leq 1$

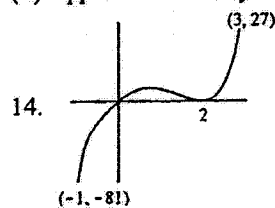
12. (i) $x = \pm 1$

(ii) approaches infinity

(iii) approaches 0 from below

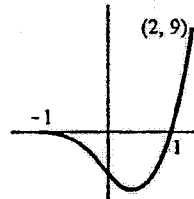


13. $\frac{1}{2}$



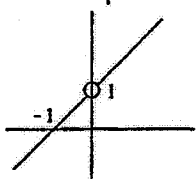
14.

15. (a)

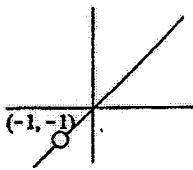


(b) $y \geq -\frac{32}{27}$

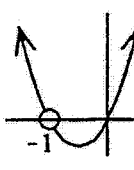
16. (a)



(b)



(c)



17. (a) $x = 0$ (b) $x \geq 0$

18. (a) $f(x) = x + 3$

(b) $f(x) = 3x - 5$

(c) $f(x) = 2x - x^2$

(d) $f(x) = \frac{1}{4}x^2 - x - 1$