



# Trinity Grammar School

Mathematics Department

## 2015

HALF-YEARLY EXAMINATION

PRELIMINARY ASSESSMENT TASK 3

### Year 11

## Mathematics

#### General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Only approved calculators for this course are allowed in this task
- Show all necessary working
- Write your Board of Studies Student Number (Year 12 HSC) or Name (Year 11) and your Class teacher on the question paper and on any answer sheets or writing booklets used to write your responses to the questions submitted
- If you do not attempt a question you must submit an answer sheet or writing booklet for that question clearly indicating N/A and your Student Number or Name.
- Preliminary Assessment Weighting: 30%

Board of Studies Student Number  
(Year 12 only)

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Class Teacher:

Name:  
(Year 11 only)

*Do NOT write solutions on this question paper. Any working on this question paper will NOT be marked.*

Date of examination:

Monday, 20 April 2015

Total marks – 80

#### Section I

10 marks

- Attempt all Questions
- Allow about 15 minutes for this section

#### Section II

70 marks

- Attempt all Questions
- Allow about 1 hour 45 minutes for this section

The School defines malpractice, or cheating, as "dishonest behaviour by a student that gives them an unfair advantage over others". I certify that my attempt at this Assessment task does not involve any malpractice or cheating.

Student signature

Date

#### Section I 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Shade the correct response on the multiple-choice answer sheet for Questions 1 – 10
- Each question is worth 1 mark

1. The solution to  $3^x = \frac{1}{27}$  is :

(A)  $x = 3$

(B)  $x = -3$

(C)  $x = \frac{1}{3}$

(D)  $x = -\frac{1}{3}$

2. How many solutions does the equation  $x^3 + 2x^2 + x = 0$  have?

(A) 0

(B) 1

(C) 2

(D) 3

3.  $(2x - 3y)(4x^2 + 6xy + 9y^2)$  is equivalent to :

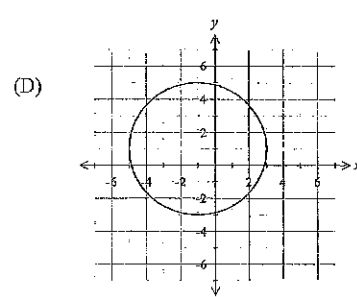
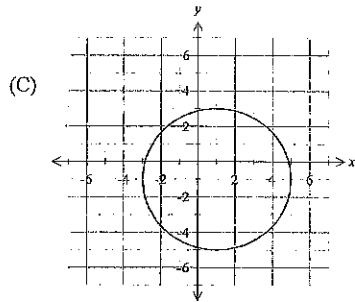
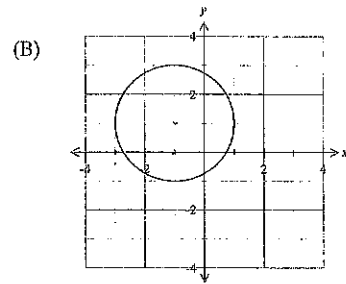
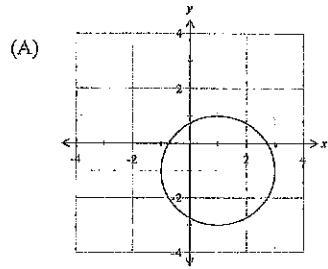
(A)  $6x^3 - 9y^3$

(B)  $6x^3 + 9y^3$

(C)  $8x^3 - 27y^3$

(D)  $8x^3 + 27y^3$

4. Which graph best represents  $(x + 1)^2 + (y - 1)^2 = 4$  ?



5. If  $x^6 = 5000$ , then  $x$  correct to 5 significant figures is

- (A) 4.1351
- (B) 4.1352
- (C) 4.13518
- (D) 4.13519

6. If  $5\sqrt{2} + \sqrt{18} = a\sqrt{2}$  then what is the value of  $a$  ?

- (A)  $a = 6$
- (B)  $a = 8$
- (C)  $a = 12$
- (D)  $a = 14$

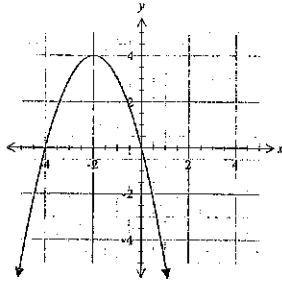
7. The function  $y = |x + 3|$  has x-intercept/s of :

- (A) 3
- (B) -3
- (C) 3 and -3
- (D) 0

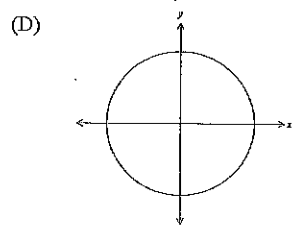
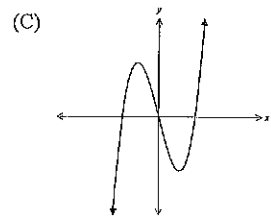
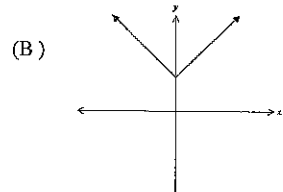
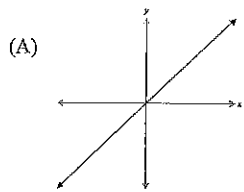
8. The domain of the function  $f(x) = \frac{3}{x + 1}$  is :

- (A) All real values of  $x$
- (B) All real values of  $x$ ,  $x \neq 0$
- (C) All real values of  $x$ ,  $x \neq 1$
- (D) All real values of  $x$ ,  $x \neq -1$

9. If the graph below is a sketch of the function  $y = ax^2 + bx + c$  then which of the following is true for the function?



- (A)  $a = 1$  and  $c = 0$   
 (B)  $a = -1$  and  $c = 0$   
 (C)  $a = 1$  and  $b = 0$   
 (D)  $a = -1$  and  $b = 0$
10. Which of the following is an even function?



End of Section I  
 Section II commences on the next page  
 Page 5 of 12

**Section II 70 marks**

- Attempt Questions 11 – 17
- Allow about 1 hour and 45 minutes for this section
- Answer each question in a SEPARATE writing booklet or answer sheet. Extra writing booklets are available.
- In Questions 11 – 17, your responses should include all relevant mathematical reasoning and/or calculations.

**Question 11 (10 marks)** Use a SEPARATE writing booklet

- a) Evaluate  $42.345 \times 10^{-7} + 0.065 \times 0.03$  and give your answer in scientific notation correct to 3 significant figures. **1**
- b) Expand and simplify  $(2x + 4)^2 - (x - 3)^2$  **2**
- c) Express  $0.12\dot{4}$  as a fraction in its simplest form, showing all working. **2**
- d) Evaluate  $\frac{\sqrt[4]{19.06}}{1.35 \div 7.29}$  to three significant figures. **1**
- e) Sketch the region defined by  $y < 2x$  **2**
- f) If  $a = 2b - c^2$ , find  $c$  when  $b=11$  and  $a=-3$  **1**
- g) Rewrite  $\frac{1}{\sqrt[4]{a^3}}$  in index form. **1**

End of Question 11  
 Question 12 commences on the next page.

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**Question 12 (10 marks)**

Use a SEPARATE writing booklet

a) Evaluate  $-|-3| + |-5| - |-1|$  1

b) Simplify  $\frac{x^2 - 9}{x^2 + 7x + 12}$  2

c) Solve for  $x$ :  $\frac{x+1}{3} = \frac{2x}{5} - 6$  2

d) What is the domain and range of the function  $y = x^3$ ? 2

e) Fully factorise :

(i)  $ab + ad + cb + cd$  1

(ii)  $12x^2 - 27$  2

End of Question 12.

Question 13 commences on the next page.

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**Question 13 (10 marks)**

Use a SEPARATE writing booklet

a) Given  $\frac{7}{3 + 2\sqrt{2}} = a + b\sqrt{2}$ , find the values of  $a$  and  $b$ . 2

b) For the function:

$$f(x) = \begin{cases} |x| + 2 & x < 0 \\ 2 & 0 \leq x < 1 \\ x^2 + 1 & x \geq 1 \end{cases}$$

i) Find the value of  $f(0) - 2f(2) + f(-1)$ . 2

ii) Sketch a graph of the function. 2

iii) State the domain and range of  $y = f(x)$ . 2

c) Solve simultaneously  $2x - 3y = 5$  and  $5x + 2y = -16$ . 2

End of Question 13.

Question 14 commences on the next page.

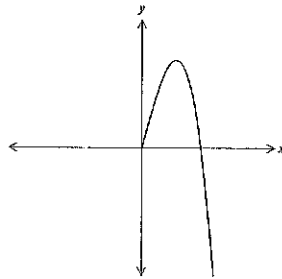
**Question 14 (10 marks)**

Use a SEPARATE writing booklet

a) Solve  $2x^2 - 8x = 13$  by completing the square. Give your answer in exact form. 2

b) Solve  $3x^2 = 8x - 2$ , leaving your answer in fully simplified form. 2

c) Part of the graph of the function  $y = f(x)$  is shown below.



(i) Copy or trace the graph above onto your answer page and complete the graph making  $y = f(x)$  an odd function. 1

(ii) Make another copy of the graph above into your answer book and complete the graph making  $y = f(x)$  neither an odd nor even function. 1

d) If  $f(x) = x^2 + x - 5$  and  $g(x) = 3x + 10$ .

(i) Find the values of  $x$  for which  $f(x) = 15$  1

(ii) Find the values of  $x$  for which  $f(x) = g(x)$ . 2

(iii) Find the values for  $x$  which satisfy  $f(x) < g(x)$ . 1

End of Question 14.

Question 15 commences on the next page.

**Question 15 (10 marks)**

Use a SEPARATE writing booklet

a) Sketch  $y = x^2 + 2x - 7$  showing the vertex and the  $y$  intercept. 3

b) Simplify the expression  $\sqrt{108} + 2\sqrt{27} - \sqrt{48}$ . 2

c) Shade the region satisfying the inequalities  $x^2 + y^2 < 25$  and  $y + 2x \geq 4$  3

d) Give the exact value for  $\frac{\left(\frac{1}{8}\right)^{-\frac{1}{3}} \times 81^{\frac{1}{4}}}{25^{\frac{3}{2}}}$ . Full working must be shown. 2

End of Question 15.

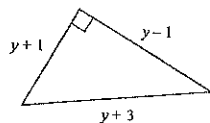
Question 16 commences on the next page.

Question 16 (10 marks)

Use a SEPARATE writing booklet

a) Solve the equation  $9^{2x} = 27^{x+1}$  2

b) Find the value of  $y$  in the triangle below. 2



c) For the function  $y = \frac{4}{x-2} + 3$ ,

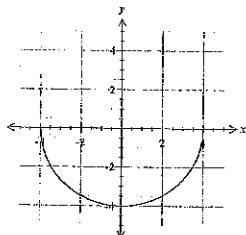
(i) Determine the  $y$  intercept. 1

(ii) Give the equation of any vertical asymptotes. 1

(iii) Find the equation of the horizontal asymptote. 1

(iv) Sketch the graph. 2

d) What is the equation of the graph below? 1



End of Question 16.

Question 17 commences on the next page.

Question 17 (10 marks)

Use a SEPARATE writing booklet

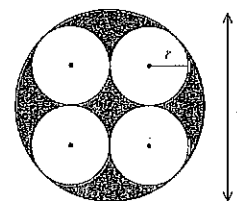
a) If  $a = 3$  and  $b = -2$ , write a pair of simultaneous equations that satisfy this solution. 2

b) Draw a neat sketch of  $y = -3^{-x} - 1$  on your answer page, you must show all relevant information. 3

c) Solve for  $x$ :  $|x + 1| = |2x - 5|$  2

d) If  $6a^5\sqrt{a} = \sqrt{xa^y}$ , find  $x$  and  $y$ . 2

e) Write an algebraic expression in factorised form for the shaded area of the figure below. 1



End of Question 17.

End of Section II  
End of Examination

Trinity 2015 Y11  
Preliminary Mathematics

Simple Answers.

1.  $3^x = \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$

(B)

2.  $x^3 + 2x^2 + x = 0$   
 $x(x^2 + 2x + 1) = 0$   
 $= x(x+1)^2 = 0$

2 solutions.

(C)

3.  $(2x-3y)(4x^2+6xy+9y^2)$

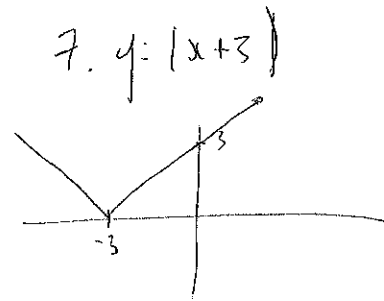
$8x^3 + 12x^2y + 18xy^2$   
 $-12x^2y - 18xy^2 - 27y^3$   
 $= 8x^3 - 27y^3$  (C)

4.  $(x+1)^2 + (y-1)^2 = 4$   
 Circle center  $(-1, 1)$   
 radius 2

(B)

5.  $x^6 = 5000$   
 $= 4.1352$  (5 s.f.)  
 = (B)

6.  $5\sqrt{2} + \sqrt{18} = a\sqrt{2}$   
 $5\sqrt{2} + \sqrt{9}\sqrt{2} = a\sqrt{2}$   
 $5\sqrt{2} + 3\sqrt{2} = a\sqrt{2}$   
 $8\sqrt{2} = a\sqrt{2}$   
 $a = 8$  (B)



(B)

8.  $f(x) = \frac{3}{x+1}$

$x \neq -1$  because then  
 you would be dividing by 0

$\therefore$  (D) all real  $x$ ,  $x \neq -1$

9. Concave down (frowny face)  
 so  $a$  must be negative.

(B)

10.  $f$  is even, same  
 positive (or negative) areas  
 on both sides  $[f(-x) = f(x)]$

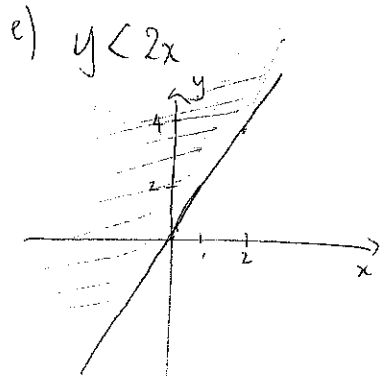
11. a)  $42.345 \times 10^{-7} + [0.065 \times 0.03]$   
 $= 1.95 \times 10^{-3}$  (3 s.f.)

b)  $(2x+4)^2 - (x-3)^2$   
 $4x^2 + 16 + 16x - (x^2 + 9 - 6x)$   
 $4x^2 + 16 + 16x - x^2 - 9 + 6x$   
 $= [4x^2 - x^2] + [16x + 6x] + [16 - 9]$   
 $= 3x^2 + 22x + 7$

c)  $0.1242424...$   
 Let this be  $x$ .  
 $10x = 1.242424...$   
 $100x = 12.424244...$   
 $\therefore 100x - 10x = 123$

$90x = 123$   
 $x = \frac{123}{90} \Rightarrow \left[ \frac{123}{90} \right]$   
 $= \frac{41}{30}$

$$d) \frac{4\sqrt{19.06}}{1.35 \div 7.29} = 11.3 \text{ (3 s.f.)}$$



f.  $a = 2b - c^2$

$$b = 11, a = -3$$

$$-3 = 22 - c^2$$

$$c^2 = 25$$

$$c = \pm 5$$

g.  $\frac{1}{4\sqrt{a^3}} = \frac{1}{a^{3/4}}$

$$= a^{-3/4}$$

12.  $-|-3| + |-5| - |-1|$

$$= -(3) + (5) - (1)$$

$$= -3 + 5 - 1$$

$$= 1$$

b.  $\frac{x^2 - 9}{x^2 + 7x + 12} = \frac{(x+3)(x-3)}{(x+3)(x+4)}$

$$= \frac{x-3}{x+4}$$

c.  $\frac{x+1}{3} = \frac{2x}{5} - 6$

$$\frac{5(x+1)}{15} = \frac{6x}{15} - \frac{90}{15}$$

$$5(x+1) = 6x - 90$$

$$5x + 5 = 6x - 90$$

$$95 = x$$

d. domain:  $y = x^3$

domain: all real  $x$ .

range: all real  $y$

e.  $ab + ad + cb + cd$

$$= (a)(b+d) + (c)(b+d)$$

$$= (a+c)(b+d)$$

ii)  $12x^2 - 27$

$$= (\sqrt{12}x + \sqrt{27})(\sqrt{12}x - \sqrt{27})$$

$$= (2\sqrt{3}x + 3\sqrt{3})(2\sqrt{3}x - 3\sqrt{3})$$

$$3(2x+3)(2x-3)$$

13. a)  $\frac{7}{3+2\sqrt{2}} = a+b\sqrt{2}$

$$\frac{7}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{21 - 14\sqrt{2}}{9 - (4)(2)}$$

$$9 - (4)(2)$$

$$= \frac{21 - 14\sqrt{2}}{1} \quad a=21, b=-14$$

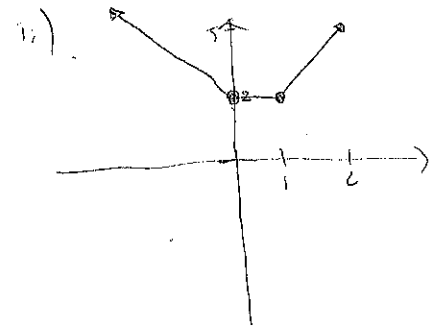
b.  $f(x) = \begin{cases} |x|+2 & x < 0 \\ 2 & 0 \leq x < 1 \\ x^2+1 & x \geq 1 \end{cases}$

i)  $f(0) - 2f(2) + f(-1)$

$$2 - 2(5) + (3)$$

$$= 2 - 10 + 3$$

$$= -5$$



domain: all real  $x$ .

range:  $y \geq 2$

c)  $2x - 3y = 5$

$$5x + 2y = -16$$

$$2x = 5 + 3y$$

$$x = \frac{5+3y}{2}$$

$$5\left(\frac{5+3y}{2}\right) + 2y = -16$$

$$5(5+3y) + 4y = -32$$

$$25 + 15y + 4y = -32$$

$$19y = -57$$

$$y = -3$$

$$2x - 3(-3) = 5$$

$$2x + 9 = 5$$

$$2x = -4$$

$$x = -2$$



14.

a)  $2x^2 - 8x = 13$ .

$$\frac{2x^2 - 8x}{2} = \frac{13}{2}$$

$$x^2 - 4x = \frac{13}{2}$$

$$x^2 - 4x + 4 = \frac{13}{2} + 4$$

$$(x-2)^2 = \frac{21}{2}$$

$$x - 2 = \pm \sqrt{\frac{21}{2}}$$

$$x = \pm \sqrt{\frac{21}{2}} + 2 \quad \checkmark$$

Verify

a=2, b=-8, c=13

$$8 \pm \sqrt{64 - 4(13)(2)}$$

$$\frac{8 \pm \sqrt{168}}{4} = 2 \pm \frac{\sqrt{168}}{16}$$

$$= 2 \pm \sqrt{\frac{21}{2}} \quad \checkmark$$

b.  $3x^2 = 8x - 2$ .

$$3x^2 - 8x + 2 = 0$$

$$(3x-1)(x-2)$$

Using quadratic formula.

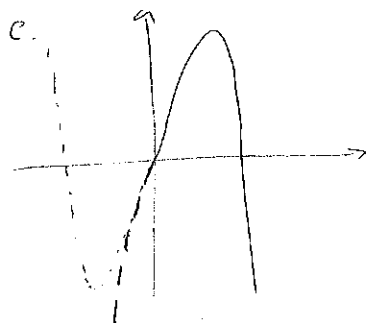
a=3, b=-8, c=2.

$$\frac{8 \pm \sqrt{64 - 4(3)(2)}}{6}$$

$$\frac{8 \pm \sqrt{64 - 24}}{6}$$

$$= \frac{8 \pm \sqrt{40}}{6} = \frac{8 \pm 2\sqrt{10}}{6}$$

$$= \frac{4 \pm \sqrt{10}}{3}$$



see dotted line for (i).

see bold dotted line for (ii)

d.  $f(x) = x^2 + x - 5$   
 $g(x) = 3x + 10$

i)  $x^2 + x - 5 = 15$   
 $x^2 + x - 20 = 0$

$$(x+5)(x-4)$$

$$x = -5 \text{ OR } x = 4$$

ii)  $f(x) = g(x)$

$$x^2 + x - 5 = 3x + 10$$

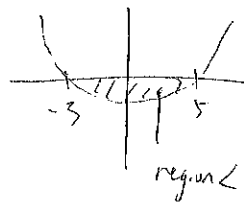
$$x^2 - 2x - 15 = 0$$

$$(x+3)(x-5) = 0$$

$$x = -3 \text{ OR } x = 5$$

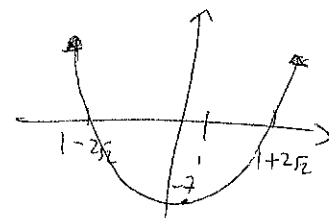
iii)  $f(x) < g(x)$ .

$$x^2 - 2x - 15 < 0$$



$$-3 < x < 5$$

15.  $y = x^2 + 2x - 7$



quadratic formula.

a=1, b=2, c=-7

$$\frac{-2 \pm \sqrt{4 - 4(-7)}}{2}$$

$$= \frac{-2 \pm \sqrt{32}}{2}$$

$$= \frac{-2 \pm 4\sqrt{2}}{2} = 1 \pm 2\sqrt{2}$$

y-intercept = -7

vertex = (1, -4)

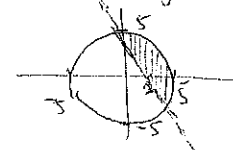
b.  $\sqrt{108} + 2\sqrt{27} - \sqrt{48}$

$$= \sqrt{36} \sqrt{3} + 2\sqrt{9} \sqrt{3} - \sqrt{16} \sqrt{3}$$

$$= 6\sqrt{3} + 6\sqrt{3} - 4\sqrt{3}$$

$$= 8\sqrt{3}$$

c)  $x^2 + y^2 < 25$ ,  $y + 2x \geq 4$



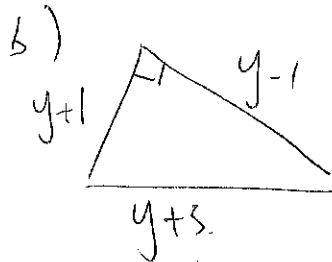
Region satisfying the 2 inequalities

$$\begin{aligned}
 & \text{e) } \frac{\left(\frac{1}{8}\right)^{-\frac{1}{3}} \times 81^{\frac{1}{4}}}{25^{\frac{3}{2}}} \\
 &= \frac{8^{\frac{1}{3}} \times 81^{\frac{1}{4}}}{25^{\frac{3}{2}}} \\
 &= \frac{\sqrt[3]{8} \times \sqrt[4]{81}}{(\sqrt{25})^3} \\
 &= \frac{2 \times 3}{5^3} = \frac{6}{125}
 \end{aligned}$$

$$\begin{aligned}
 & 16. \text{ a) } 9^{2x} = 27^{2+1} \\
 & \quad 9^{2x} = (3^3)^{x+1} \\
 & \quad 9^{2x} = 3^{3(x+1)} \\
 & \quad 3^{2(2x)} = 3^{3(x+1)} \\
 & \quad 2(2x) = 3(x+1) \\
 & \quad 4x = 3x + 3 \\
 & \quad x = 3
 \end{aligned}$$

test case.

$$9^{2(3)} = 27^{(4)}$$



pythagoras theorem

$$(y+1)^2 + (y-1)^2 = (y+3)^2$$

$$y^2 + 1 + 2y + y^2 + 1 - 2y = y^2 + 9 + 6y$$

$$2y^2 + 2 = y^2 + 9 + 6y$$

$$y^2 - 6y - 7 = 0$$

$$(y+1)(y-7)$$

$y = -1$  or  $7$ , but since we cannot have negative lengths,  $y = 7$

$$\begin{aligned}
 & \text{c) } y = \frac{4}{x-2} + 3 \\
 & \text{y-intercept at } x=0.
 \end{aligned}$$

$$y = \frac{4}{-2} + 3$$

$$y = -2 + 3 = 1$$

ii) vertical asymptotes.

$$x-2 \neq 0$$

$$\therefore x \neq 2$$

$x=2$  is a vertical asymptote.

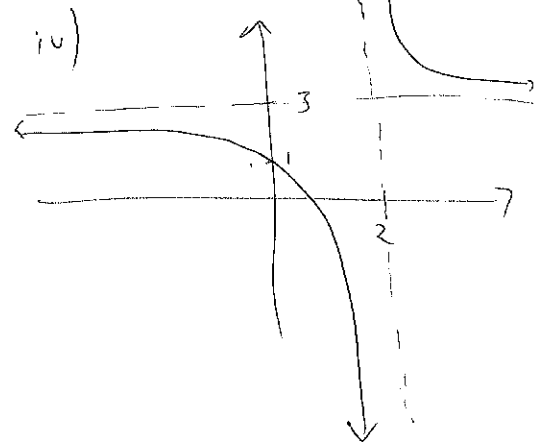
$$\text{iii) } y-3 = \frac{4}{x-2}$$

$$(x-2)(y-3) = 4$$

$$x-2 = \frac{4}{y-3}$$

$$x = \frac{4}{y-3} + 2$$

$y=3$  is a horizontal asymptote.



d). root equation

$$13) \quad x^2 + y^2 = 4^2$$

$$y^2 = 4^2 - x^2$$

$$y = \sqrt{4^2 - x^2} \quad \text{for +ve } y \text{ values}$$

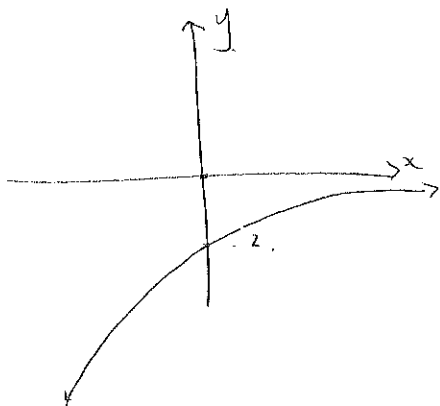
$$y = -\sqrt{4^2 - x^2} \quad \text{for -ve } y \text{ values}$$

17. a)  $a=3, b=-2.$

$$3a + 2b = 5$$

$$b + 4a = 10$$

b)  $y = -3^{-x} - 1.$



c)  $|x+1| = |2x-5|$

$$(x+1)^2 = (2x-5)^2$$

$$x^2 + 1 + 2x = 4x^2 + 25 - 20x$$

$$3x^2 - 22x + 24 = 0$$

$$(3x-4)(x-6)$$

$$x = \frac{4}{3} \text{ or } 6$$

d)  $6a^5 \sqrt{a} = \sqrt{xa^9}.$

$$\sqrt{36a^{10}} \sqrt{a} = \sqrt{xa^9}$$

$$\sqrt{36a^{11}} = \sqrt{xa^9}$$

$$x = 36, y = 11$$

e. diameter of big circle =  $R.$

$$\therefore \text{radius} = \frac{R}{2}.$$

Area = Area of Big Circle - area of 4 small circles

$$A_{\text{Big}} = \pi \left(\frac{R}{2}\right)^2$$

$$A_{\text{small}} = 4\pi r^2$$

Total shaded area

$$= \pi \left(\frac{R}{2}\right)^2 - 4\pi r^2$$

$$= \pi \left(\left(\frac{R}{2}\right)^2 - 4r^2\right)$$