

(1) Write each of the following with positive indices.

(i) gh^{-2}

(ii) $\frac{a^{-1}}{b^{-2}}$

(iii) $\frac{2^{-3}t^5}{3^{-1}v^{-2}}$

(iv) $(5y^{-3}z)^{-1}$

(2) Express in surd form

(i) $a^{\frac{1}{2}}$ (ii) $5^{-1}k^{-4/3}$

(iii) $p^{\frac{1}{5}}q^{-\frac{3}{5}}$ (iv) $a^{\frac{1}{4}}b^{-\frac{2}{3}}$

(3) Express the following without root signs and with positive indices.

(i) $\sqrt[5]{c^2}$

(ii) $\sqrt[3]{8h^{-1}k^2}$

(i) $\frac{\sqrt[4]{g^{-1}h^3}}{j^{-5}}$

(iv) $\frac{\sqrt{(x-3y)^{-2}}}{x^{-3/2}y^3}$

(4) Find the numerical values of:

(i) (a) 9^{-2} (b) 7^0

(c) $\frac{1}{4^{-3}}$ (d) $\frac{5^{-1}}{3^{-2}}$

(e) $27^{\frac{2}{3}}$ (f) $16^{1.25}$

(4) (ii) (a) $(11\frac{1}{9})^{\frac{1}{2}}$

(b) $(6\frac{1}{4})^{-1.5}$

(c) $(1\frac{9}{16})^{-\frac{3}{2}}$

(d) $(0.001)^{-\frac{2}{3}}$

(5) Simplify the following

(i) $y^{\frac{3}{5}} \cdot y^{-\frac{2}{5}} \cdot y^{\frac{4}{5}}$

(ii) $\frac{9b^{-\frac{3}{4}} \times 2b^{\frac{1}{2}}}{6b^{-\frac{1}{4}}}$

(iii) $[6x^{-\frac{1}{2}} \div 2x^{\frac{2}{3}}] \times 3x^{\frac{5}{6}}$

(6) Simplify

(i) $\frac{2^x \cdot 4^{x+1}}{8^{x-2}}$

(ii) $\frac{3^{-n} \cdot 9^2}{3^{3n-2} \cdot 27}$

(iii) $\frac{25^{2n-1} \times 5^{1-n}}{25^{1-n} \times (5^2)^3}$

(iv) $\frac{12^{3x} \cdot 8^{-1}}{9^{-x} \cdot 4^{-2x} \cdot 6^x}$

(7) Solve these equations

(i) $9^x \cdot 27^{x-2} = 3^{-x}$

(ii) $9^x = \sqrt{3}$

(iii) $(0.01)^x = 100 (10^{-x})$

(iv) $(\frac{1}{9})^{2x-1} = 3(27^{-x})$

(8) Solve for x, by first forming a quadratic equation.

(i) $2^{2x} - 3 \cdot 2^x + 2 = 0$

(ii) $49^x - 6 \cdot 7^x = 7$

(9) Simplify, by expanding each of the following:

(i) $x^{\frac{1}{3}}(x^{\frac{2}{3}} + 5x^{-\frac{1}{3}})$

(ii) $(x^{-1} + y^{-1})^2$

(iii) $(x^{\frac{1}{3}} - 1)(x^{\frac{2}{3}} + x^{\frac{1}{3}} + 1)$

(10) Simplify

(i) $x^{-1} - y^{-1}$

(ii) $\frac{1}{x^{-1} - y^{-1}}$

(iii) $\frac{x^{-1} - y^{-1}}{x^{-2} - y^{-2}}$

CHALLENGE QUESTIONS

[1] Show that

$$(xy^{-1} - yx^{-1}) \div (x^2y^{-2} - y^2x^{-2}) = \frac{xy}{x^2 + y^2}$$

[2] If $z = 2 + \sqrt{3}$,

find the values of $z + z^{-1}$, $z^2 + z^{-2}$ in simplest surd form

(1) (i) $\frac{9}{h^2}$ (ii) $\frac{b^2}{a}$ (iii) $\frac{3t^5 v^2}{8}$

(iv) $\frac{y^3}{5z}$

(2) (i) \sqrt{a} (ii) $\frac{1}{5} \sqrt[3]{L^4}$

(iii) $\sqrt[5]{\frac{p}{q^3}}$ (iv) $4\sqrt{a} \cdot \sqrt[3]{\frac{1}{b^2}}$

(3) (i) $c^{\frac{2}{5}}$ (ii) $\frac{2k^{\frac{2}{3}}}{h^{\frac{1}{3}}}$ (iii) $\frac{j^5 h^{\frac{5}{4}}}{g^{\frac{1}{4}}}$

(iv) $\frac{x^{\frac{3}{2}}}{(x-3y)y^3}$

(4) (i) (a) $\frac{1}{81}$ (b) 1

(c) 64 (d) $\frac{9}{5} = 1\frac{4}{5}$

(e) 9 (f) 32

(ii) (a) $3\frac{1}{3}$ (b) $\frac{8}{125}$

(c) $\frac{64}{125}$ (d) 100

(5) (i) y (ii) 3 (iii) $\frac{9}{x^{\frac{1}{3}}}$

(6) (i) 256 (ii) 3^{3-4n} (iii) 5^{2n-3}

(iv) $2^{9x-3} \cdot 3^{4x}$

(7) (i) $x = 1$ (ii) $x = \frac{1}{4}$

(iii) $x = -2$ (iv) $x = 1$

(8) (i) $x = 0$ or 1

(ii) $x = 1$

(9) (i) $x + 5$ (ii) $x^{-2} + y^{-2} - 2x^{-1}y^{-1}$
 $= \frac{1}{x^2} + \frac{1}{y^2} - \frac{2}{xy}$

(iii) $x - 1$

(10) (i) $\frac{y-x}{xy}$ (ii) $\frac{xy}{y-x}$

(iii) $\frac{xy}{x+y}$

CHALLENGE QUESTIONS

[1] L.H.S. = $\left(\frac{x}{y} - \frac{y}{x}\right)$
 $\frac{\left(\frac{x^2}{y^2} - \frac{y^2}{x^2}\right)}{\left(\frac{x^2 - y^2}{yx}\right)} \div \left(\frac{x^4 - y^4}{y^2 x^2}\right)$
 $= \frac{x^2 - y^2}{xy} \times \frac{(xy)^2}{(x^2 + y^2)(x^2 - y^2)}$

Note: Difference of two squares.

$= \frac{xy}{x^2 + y^2} = \text{R.H.S.} \quad \text{Q.E.D.}$

[2] $z + z^{-1} = z + \frac{1}{z}$

$= (2 + \sqrt{3}) + \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$

$= (2 + \sqrt{3}) + \frac{2 - \sqrt{3}}{4 - 3} = (2 + \sqrt{3}) + (2 - \sqrt{3})$

$= 4$

$z^2 + z^{-2} = z^2 + \frac{1}{z^2} = \left(z + \frac{1}{z}\right)^2 - 2$

Check this step!

$= (4)^2 - 2$ (from above)

$= 16 - 2$

$= 14$