2014
Preliminary Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Board approved calculators may be used.
- Write using black or blue pen
- All necessary working should be shown in Question 11 – 14
- Write your student number and/or name at the top of every page

Total marks – 70

Section I - Pages 3 – 6

10 marks

Attempt Questions 1 - 10

Allow about 15 minutes for this section

Section II - Pages 7 – 10

60 marks

Attempt Questions 11 – 14

Allow about 1 hours 45 minutes for this section

This paper MUST NOT be removed from the examination room

STUDENT NUMBER/NAME:.....

Marks

Section I

10 marks
Attempt Question 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 - 10.

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1. What is the value of $\frac{3.7+6.2}{10.6+4.1}$ correct to 2 decimal places? **1**

   A. 0.67

   B. 4.12

   C. 5.03

   D. 8.38

2. What is $\frac{2}{3-\sqrt{2}}$ as a fraction with a rational denominator? **1**

   A. $6-2\sqrt{2}$

   B. $6+2\sqrt{2}$

   C. $\frac{6-2\sqrt{2}}{7}$

   D. $\frac{6+2\sqrt{2}}{7}$

3. What is the value of $f(-1)$ if $f(x)=x^3-4x$? **1**

   A. $f(-1)=-3$

   B. $f(-1)=-5$

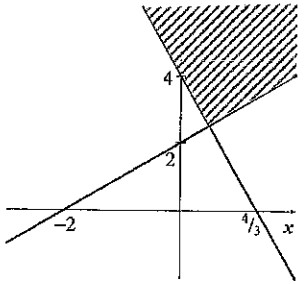
   C. $f(-1)=3$

   D. $f(-1)=5$

STUDENT NUMBER/NAME:.....

Marks

4. Which pair of inequalities describe the shaded region? 1



NOT TO SCALE

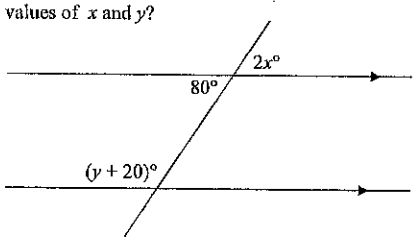
A. $y \leq 4-3x$
$y \leq x+2$

B. $y \leq 4-3x$
$y \geq x+2$

C. $y \geq 4-3x$
$y \leq x+2$

D. $y \geq 4-3x$
$y \geq x+2$

5. What are the values of x and y? 1



A. $x = 40°,\ y = 60°$

B. $x = 40°,\ y = 80°$

C. $x = 80°,\ y = 60°$

D. $x = 80°,\ y = 80°$

STUDENT NUMBER/NAME:.....

Marks

6. Which of the following is true for the equation $6x^2 + x - 2 = 0$ 1

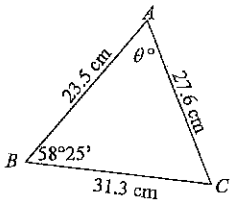
A. no real roots

B. one real root

C. two rational distinct roots

D. two irrational distinct roots.

7. Which of the following is **NOT** a correct expression involving θ in triangle ABC? 1



Not to scale

A. $31.3^2 = 27.6^2 + 23.5^2 - 2 \times 27.6 \times 23.5 \cos\theta$

B. $\cos\theta = \dfrac{23.5^2 + 27.6^2 - 31.3^2}{2 \times 23.5 \times 27.6}$

C. $\dfrac{31.3}{\sin\theta} = \dfrac{27.6}{\sin 58°25'}$

D. $\dfrac{\sin\theta}{31.3} = \dfrac{\sin 58°25'}{23.5}$

8. The sum of the interior angles of a regular polygon is 2340°. 1
What is the size of each interior angle?

A. 130°

B. 146°15'

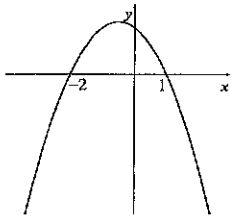
C. 156°

D. 157°30'

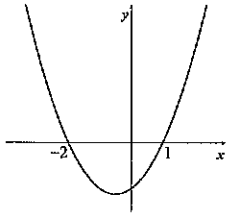
Marks

9. Which graph best represents $y = x^2 + x - 2$? 1

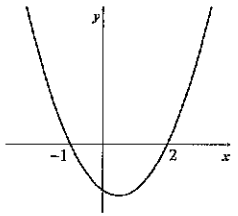
A.



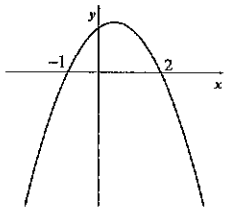
B.



C.



D.



10. What is a solution to the equation $\sin\left(\frac{\theta}{2}+30°\right) = \cos\theta$? 1

A. $\theta = 40°$

B. $\theta = 60°$

C. $\theta = 80°$

D. $\theta = 100°$

Section II

60 marks
Attempt Question 11 – 14
Allow about 1 hours 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Marks

Question 11 (15 marks). Use a SEPARATE writing booklet.

(a) Simplify $\frac{3x^2}{x^2-9} \times \frac{x-3}{4x}$. 2

(b) Solve $|x-4| = 1-2x$. 2

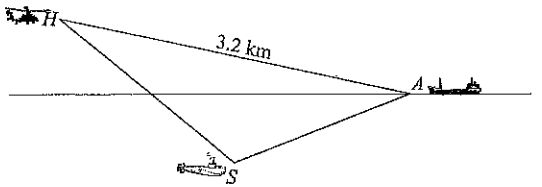
(c) Factorise $2x^2 + 9x - 5$. 2

(d) Solve $2\cos x + 1 = 0$ for $0° \le x \le 360°$. 2

(e) Sketch $y = \sin 2x + 1$ for $0° \le x \le 360°$. 2

(f) Prove that $\frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta} \equiv \tan^2\theta + \cos^2\theta$ 2

(g) From a helicopter (H) an aircraft carrier (A) is at a distance of 3.2 km and at an angle of depression of 15° and a submarine (S) is at an angle of depression of 58°. The angle of depression from the aircraft carrier to the submarine is 36°. Copy the diagram into your answer booklet and show all information.



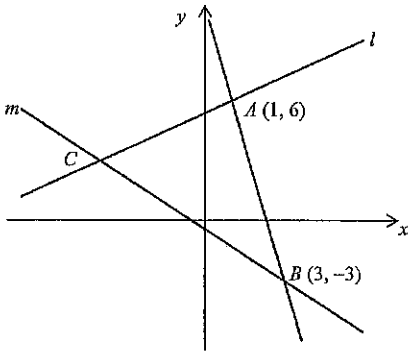
Find, correct to the nearest 100 m, the distance of the submarine (S) from the aircraft carrier (A). 3

STUDENT NUMBER/NAME:.....

Marks

Question 12 (15 marks). Use a SEPARATE writing booklet.

(a)



The point A (1, 6) lies on the line l given by the equation $3x-5y+27=0$ and the point B (3, –3) lies on the line m given by the equation $6x+7y+3=0$.

(i) Show that C, which is the point of intersection of the lines l and m, has coordinates (–4, 3). **1**

(ii) Find the gradient of the line joining A and B. **1**

(iii) Find the acute angle the line AB makes with the x axis. Give your answer correct to the nearest minute. **1**

(iv) Find the midpoint of the interval AB. **1**

(v) Find the equation of the line AB. **2**

(vi) Find the perpendicular distance of the point A from the line m. **2**

(vii) Find the area of triangle ABC. **2**

(b) The quadratic equation $2x^2-5x-3=0$ has roots α and β. Find:

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\frac{1}{\alpha}+\frac{1}{\beta}$ **1**

(iv) $\alpha^2+\beta^2$ **2**

STUDENT NUMBER/NAME:.....

Marks

Question 13 (15 marks). Use a SEPARATE writing booklet.

(a) Find $\lim_{x\to 1}\frac{x^2+3x-4}{x-1}$. **2**

(b) Differentiate with respect to x.

(i) $\sqrt{x^2+1}$ **2**

(ii) $\frac{2x}{3x-2}$ **2**

(c) Find the equation of the tangent to the curve $y=x^3$ at the point where $x=-2$. **3**

(d) Find the domain and range of the function $y=\frac{1}{\sqrt{x+4}}$ **2**

(e) Determine if the function $f(x)=\frac{3x}{x^2-1}$ is an odd or even function. **2**

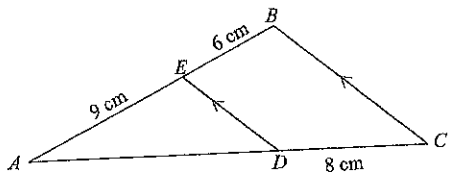
(f) Find the equation of the locus of a point $P(x,y)$ which moves so that it is equidistant from the points $A(2,-4)$ and $B(-1,3)$. **2**

STUDENT NUMBER/NAME:.....

Marks

Question 14 (15 marks). Use a SEPARATE writing booklet.

(a) In the diagram $BC \parallel ED$, $BE = 6$ cm, $DC = 8$ cm and $AE = 9$ cm.



(i) Prove that ΔABC is similar to ΔAED. **2**

(ii) Hence, or otherwise, calculate the length of AD. **1**

(b) If $3x^2+4 \equiv a(x-2)^2+b(x-2)+c$, find the values of a, b and c. **3**

(c) Solve the equation $\left(x^2+x\right)+\dfrac{12}{\left(x^2+x\right)}-8=0$ **3**

(d) Consider the parabola $x^2-4x-12=8y$

(i) Find the coordinates of the vertex **2**

(ii) Find the equation of the directrix. **1**

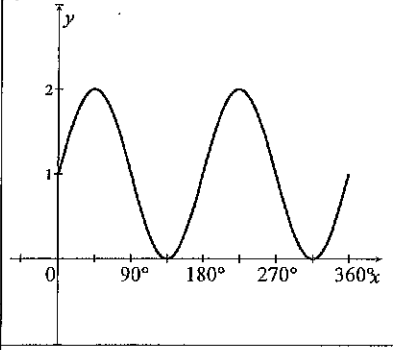
(e) (i) Write down the discriminant of $2x^2+(k-1)x+2$, where k is a constant. **1**

(ii) Hence, or otherwise, find the values of k for which the parabola $y=2x^2-x+7$ intersects the line $y=5-kx$ at more than one point. **2**

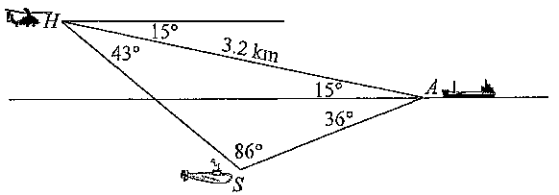
End of Paper

Question	Working	Solution
1)	$\dfrac{3.7+6.2}{10.6+4.1}=0.67$	A
2)	$\dfrac{2}{3-\sqrt{2}}=\dfrac{2}{3-\sqrt{2}}\times\dfrac{3+\sqrt{2}}{3+\sqrt{2}}$ $=\dfrac{2\left(3+\sqrt{2}\right)}{9-2}$ $=\dfrac{6+2\sqrt{2}}{7}$	D
3)	$f(x)=x^3-4x$ $f(-1)=(-1)^3-4(-1)$ $=-1+4$ $=3$	C
4)	$y \geq 4-3x$ $y \geq x+2$	D
5)	$2x=80$ $x=40°$ $y+20=100$ $y=80°$	B
6)	$\Delta=1-4\times6\times(-2)$ $=49$ 2 rational, distinct roots.	C
7)	$\dfrac{\sin\theta}{31.3}=\dfrac{\sin 58°25'}{23.5}$	D
8)	$(n-2)\times180=2340$ $n-2=13$ $n=15$ $2340\div15=156°$	C
9)	$y=x^2+x-2$ $=(x+2)(x-1)$ Concave up, Zeros –2 & 1	B
10)	$\sin\left(\dfrac{\theta}{2}+30°\right)=\cos\theta$ $\sin\left(\dfrac{\theta}{2}+30°\right)=\sin(90-\theta)$ $\dfrac{\theta}{2}+30=90-\theta$ $\dfrac{3\theta}{2}=60$ $3\theta=120$ $\theta=40°$	A

Question 11.

	Solution	Marks
(a)	$\frac{3x^2}{x^2-9}\times\frac{x-3}{4x}=\frac{3x}{(x-3)(x+3)}\times\frac{x-3}{4}$ $=\frac{3x}{4(x+3)}$	1 for correct factorisation of denominator 1 for correct answer
(b)	$x-4=1-2x$ $\quad$ $x-4=2x-1$ $3x=5$ $\quad$ $x=-3$ $x=\frac{5}{3}$ OR $\lvert-3-4\rvert=1-2(-3)$ $\lvert\frac{5}{3}-4\rvert=1-2(\frac{5}{3})$ $\quad$ $\lvert-7\rvert=1+6$ $\lvert-2\frac{1}{3}\rvert\neq-2\frac{1}{3}$ $\quad$ $=7$ Not a solution $\quad$ Solution $x=-3$	1 for 2 solutions 1 for correctly rejecting one solution with test shown
(c)	$2x^2+9x-5=(2x-1)(x+5)$	2 for correct solution
(d)	$2\cos x+1=0$ $2\cos x=-1$ $\cos x=-\frac{1}{2}$ $x=\cos^{-1}\left(-\frac{1}{2}\right)$ $x=120°,240°$	1 for correct basic angle 1 for angles in correct quadrants
(e)	$y=\sin 2x+1$  	1 for correct period & sin curve. 1 for correct x and y intercepts
(f)	$\frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta}=\tan^2\theta+\cos^2\theta$ $LHS=\frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta}$ $=\frac{1}{\cos^2\theta}-\sin^2\theta$ $=\sec^2\theta-\sin^2\theta$ $=\tan^2\theta+1-\sin^2\theta$ $=\tan^2\theta+\cos^2\theta$ $=RHS$	1 for simplifying 1 correctly shown

Question 11 continued.

	Solution	Marks
(g)	 $\frac{AS}{\sin 43}=\frac{3.2}{\sin 86}$ $AS=\frac{3.2}{\sin 86}\times\sin 43$ $=2.1877$ $=2.2$ km	1 mark for attempt at sin rule 1 mark for value of AS 1 correct rounding

Question 12

		Solution	Marks
(a)	(i)	$3x-5y=-27$ $6x+7y=-3$ $-6x+10y=54$ $17y=51$ $y=3$ $3x-15=-27$ $3x=-12$ $x=-4$ $C(-4,3)$	1
	(ii)	$m=\frac{6-(-3)}{1-3}$ $=-\frac{9}{2}$	1
	(iii)	$\tan\theta=\frac{9}{2}$ $\theta=\tan^{-1}\left(\frac{9}{2}\right)$ $=77°28'$	1
	(iv)	$MP=\left(\frac{1+3}{2},\frac{6+-3}{2}\right)$ $=\left(2,1\frac{1}{2}\right)$	1
	(v)	$y-6=\frac{-9}{2}(x-1)$ $\quad$ $y+3=-\frac{9}{2}(x-3)$ $2y-12=-9x+9$ OR $2y+6=-9x+27$ $9x+2y-21=0$ $\quad$ $9x+2y-21=0$	1 for a correct method 1

Question 12 continued.

	Solution		Marks
(a)	(vi)	$d=\dfrac{\lvert 6(1)+7(6)+3\rvert}{\sqrt{6^2+7^2}}$	1
		$=\dfrac{51}{\sqrt{85}}$	1
	(vii)	$AB=\sqrt{(1-3)^2+(6--3)^2}$	
		$=\sqrt{4+81}$	
		$=\sqrt{85}$	1
		$\text{Area}=\dfrac{1}{2}\times\sqrt{85}\times\dfrac{51}{\sqrt{85}}$	
		$=\dfrac{51}{2}$	
		$=25\dfrac{1}{2}\ \text{u}^2$	1
(b)	(i)	$\alpha+\beta=\dfrac{5}{2}$	1
	(ii)	$\alpha\beta=-\dfrac{3}{2}$	1
	(iii)	$\dfrac{1}{\alpha}+\dfrac{1}{\beta}=\dfrac{\alpha+\beta}{\alpha\beta}$	
		$=\dfrac{5}{2}\div-\dfrac{3}{2}$	
		$=-\dfrac{5}{3}$	1
	(iv)	$(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$	
		$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$	1
		$=\left(\dfrac{5}{2}\right)^2-2\left(-\dfrac{3}{2}\right)$	
		$=\dfrac{37}{4}=9\dfrac{1}{4}$	1

Question 13

	Solution		Marks
(a)	$\lim\limits_{x\to 1}\dfrac{x^2+3x-4}{x-1}=\lim\limits_{x\to 1}\dfrac{(x+4)\cancel{(x-1)}}{\cancel{x-1}}$		1 for correct factorisation
	$=5$		1
(b)	(i)	$\dfrac{d}{dx}\left(\sqrt{x^2+1}\right)=\left(x^2+1\right)^{\frac{1}{2}}$	
		$=\dfrac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}\times 2x$	1
		$=x\left(x^2+1\right)^{-\frac{1}{2}}$	1
		$=\dfrac{x}{\sqrt{x^2+1}}$	
	(ii)	$\dfrac{d}{dx}\left(\dfrac{2x}{3x-2}\right)=\dfrac{(3x-2).2-2x.3}{(3x-2)^2}$	1
		$=\dfrac{6x-4-6x}{(3x-2)^2}$	
		$=\dfrac{-4}{(3x-2)^2}$	1
(c)	$y=x^3$		1 – point
	$(-2,-8)$		
	$\dfrac{dy}{dx}=3x^2$		
	At $x=-2$		
	$\dfrac{dy}{dx}=12$		1 – gradient
	$y-(-8)=12(x-(-2))$		
	$y+8=12x+24$		
	$y=12x+16$		1 – equation
(d)	Domain – $x+4>0$		
	$x>-4$		1 – domain
	Range – $y>0$		1 – range
(e)	$f(x)=\dfrac{3x}{x^2-1}$		
	$f(a)=\dfrac{3a}{a^2-1}$		
	$f(-a)=\dfrac{3(-a)}{(-a)^2-1}$		
	$=\dfrac{-3a}{a^2-1}$		1 – working
	$=-f(a)$		
	$\therefore$ odd function		1 – odd

Question 13 continued.

	Solution	Marks
(f)	$PA = PB$	
	$\sqrt{(x-2)^2+(y+4)^2} = \sqrt{(x+1)^2+(y-3)^2}$	1
	$x^2-4x+4+y^2+8y+16 = x^2+2x+1+y^2-6y+9$	
	$20-10 = 6x-14y$	
	$3x-7y-5=0$	1

Question 14

		Solution	Marks
(a)	(i)	In ΔABC and ΔAED	
		$\angle ABC = \angle AED$ (corresponding angles $BC \parallel ED$)	1
		$\angle ACB = \angle ADE$ (corresponding angles $BC \parallel ED$)	
		$\angle BAC = \angle EAD$ (common angle)	
		$\Delta ABC \equiv \Delta AED$ (equiangular)	1
	(ii)	$\frac{AD}{DC} = \frac{AE}{EB}$ ($\parallel$ lines cut intercepts in same ratio)	
		$\frac{AD}{8} = \frac{9}{6}$	
		$AD = 12$ cm	1
(b)		$3x^2+4 \equiv a(x-2)^2+b(x-2)+c$	
		$a=3$ (by inspection)	1
		$3(2)^2+4=c$ (Let $x=2$)	
		$c=16$	1
		$3+4=a-b+c$ (Let $x=1$)	
		$7=3-b+16$	
		$b=12$	1

(c)		$x^2+x+\frac{12}{x^2+x}-8=0$	
		Let $m=x^2+x$	
		$m+\frac{12}{m}-8=0$	
		$m^2+12-8m=0$	
		$m^2-8m+12=0$	
		$(m-6)(m-2)=0$	
		$m=2,6$	1
		$x^2+x=2$	
		$x^2+x-2=0$	
		$(x+2)(x-1)=0$	
		$x=-2,1$	1
		$x^2+x=6$	
		$x^2+x-6=0$	
		$(x+3)(x-2)=0$	
		$x=2,-3$	1
		$\therefore\ x=-3,-2,1,2$	
(d)	(i)	$x^2-4x-12=8y$	
		$x^2-4x+4=8y+12+4$	
		$(x-2)^2=8y+16$	
		$(x-2)^2=4(2)(y+2)$	1
		Vertex $(2,-2)$	1
	(ii)	Directrix $y=-4$	1
(e)	(i)	$2x^2+(k-1)x+2$	
		$\Delta=(k-1)^2-4\times2\times2$	
		$=k^2-2k+1-16$	
		$=k^2-2k-15$	1
	(ii)	$2x^2-x+7=5-kx$	
		$2x^2+kx-x+2=0$	
		$2x^2+(k-1)x+2=0$	1
		For real different roots $\Delta>0$	
		$k^2-2k-15>0$	
		$(k-5)(k+3)>0$	
		$k<-3$, or $k>5$	1