Student:_____

Teacher:_____

Preliminary Course Task 2 Half Yearly

Mathematics
2017

General Instructions

- Working time – 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown on every question
- Write your name & teacher's name on top of every section

Multiple Choice   (10 marks)

1. What is the value of $\dfrac{(1.49)^2 - 1.98}{\sqrt{11.62 + 8.34 \times 2.72}}$ correct to three significant figures?

   (A)   0.040

   (B)   0.041

   (C)   0.0409

   (D)   0.0410

2. What is the solution to the equation $\dfrac{x+4}{3} = \dfrac{x}{2} - 2$?

   (A) $x = 2$

   (B) $x = 5$

   (C) $x = 6$

   (D) $x = 20$

3. If $2x^2 - 6x - 3 = 0$, then $x =$

   (A) $\dfrac{3 \pm \sqrt{3}}{2}$

   (B) $\dfrac{3 \pm \sqrt{15}}{2}$

   (C) $\dfrac{-3 \pm \sqrt{3}}{2}$

   (D) $\dfrac{-3 \pm \sqrt{15}}{2}$

4. The formula $H = 5m(Y - X)$ is used to calculate the heat (H) required to raise the temperature of a steel rod, of mass m, from a temperature of X to a temperature of Y. Rearrange the formula to make X the subject.

   (A) $X = \dfrac{5mY - H}{5m}$

   (B) $X = \dfrac{H - 5m}{Y}$

   (C) $X = \dfrac{H - 5mY}{5m}$
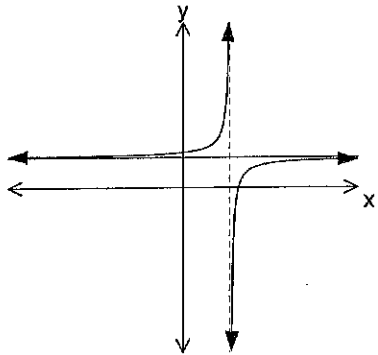
   (D) $X = \dfrac{5m - H}{Y}$

5. If $f(x) = 2x^2 - 3x - 5$ then $\dfrac{f(a)}{2a + 2}$ equals

(A) $\quad \dfrac{2a - 5}{2}$

(B) $\quad \dfrac{2x - 5}{2}$

(C) $\quad 2a - 5$

(D) $\quad x - 5$

6. Which of the following expressions does NOT have $m+1$ as a factor?

(A) $\quad m^2 - 1$

(B) $\quad m^2 + 1$

(C) $\quad m^2 + m$

(D) $\quad m^2 + 2m + 1$

7.

The graph could be represented by the equation:

(A) $\quad y = -\dfrac{1}{x-3} - 2$

(B) $\quad y = -\dfrac{1}{x-3} + 2$

(C) $\quad y = -\dfrac{1}{x+3} + 2$

(D) $\quad y = -\dfrac{1}{x+3} - 2$

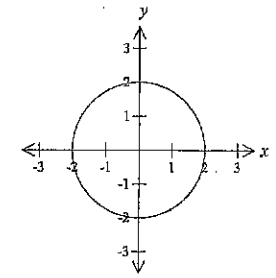8. The minimum value of $y = x^2 - 7x + 10$ is:
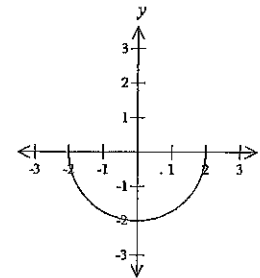
(A) $\quad 2$

(B) $\quad 3\dfrac{1}{2}$

(C) $\quad -2\dfrac{1}{4}$

(D) $\quad 2\dfrac{1}{4}$

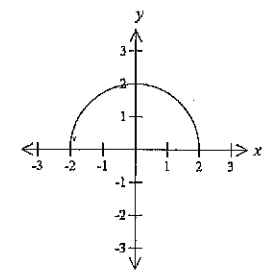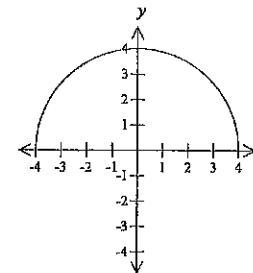9. Which of the following is true for the function $f(x) = 3x^2 - x$?

(A) Even function

(B) Odd function

(C) Neither odd or even

(D) Zero function

10. Which graph best represents $y = \sqrt{4 - x^2}$?

(A)

(B)

(C)

(D)

Question 11   (Start a new page – 18 marks)

a.   Simplify $\dfrac{x^2-1}{x-3} \times \dfrac{x^2-3x}{2x-2}$ as a single fraction in simplest form.   **2**

b.   Solve $3x = x^2$ **2**

c.   Completely factorise the following

   (i) $6x^2 - 3xy - 4xz + 2yz$.   **2**

   (ii) $x^4 - 1$ **2**

   (iii) $8x^3 - 125y^3$ **2**

d.   Expand and simplify: $(2x-3y)^2 - 5x(x-2y)$.   **2**

e.   Solve for x and y : $\begin{array}{l} x+3y=2 \\ 2x-y=11 \end{array}$ **3**

f.   Solve the equation by completing the square: **3**

$x^2 - 10x = 11$

Question 12   (Start a new page – 21 marks)

a.   Simplify

   (i) $\sqrt{32}$ **1**

   (ii) $\sqrt{3}+\sqrt{27}-\sqrt{18}$ **2**

b.   Express $0.1\overset{..}{2}\overset{.}{5}$ as a fraction in simplest form, showing all working.   **2**

c.   Expand and simplify: $(2\sqrt{7}+\sqrt{11})(2\sqrt{7}-\sqrt{11})$ **2**

d.   Express $\dfrac{3\sqrt{2}+1}{2\sqrt{3}}$ with a rational denominator.   **2**

e.   Find the value of p and q such that: $\dfrac{\sqrt{5}}{\sqrt{5}-2}=p+q\sqrt{5}$ **3**

f.   Sketch the following function, stating their domain and range: **3**

$f(x)=\dfrac{3}{x-4}$

g.   A function is defined as $y=15-7x-2x^2$

   (i)   Determine the x intercepts.   **2**

   (ii)   Determine the vertex.   **1**

   (iii)   Sketch the function, showing the important features.   **1**

   (iv)   Hence solve $15-7x-2x^2 \geq 0$ **1**

   (v)   State the range of the function.   **1**

Question 13   (Start a new page – 22 marks)

a.   Solve for x:

    (i) $1 < 1 + x < 2$ **1**

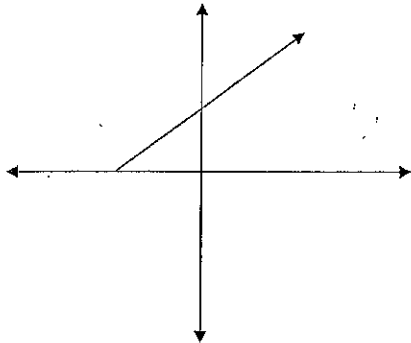   (ii) $|x+1| = 3x+2$ **2**

b.   A function is defined by the rule $p(x) = \begin{cases} x+1, & \text{if } x \geq 1 \\ -1, & \text{if } -2 < x < 1 \\ |x|, & \text{if } x \leq -2 \end{cases}$

Find:

    (i) $p(0) + p(-2)$ **2**

    (ii)   Sketch the function.   **3**

c.   Copy and complete the graph below, given that it is not a function.   **1**

d.   Show the region of the number plane where the following hold simultaneously: **3**
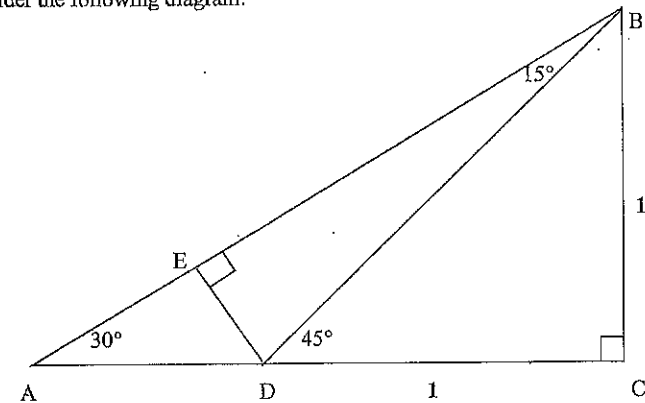
    $(x-2)^2 + y^2 \leq 4$ and $y < 2 - x$

e.   Show that the function $f(x) = 8x^3 - 7x$ is odd.   **1**

f.   Solve for θ over the domain $0° \leq \theta \leq 360°$:

    (i) $\sin 40° = \cos(90-\theta)°$.   **1**

    (ii) $\sin\theta = -\dfrac{1}{2}$ **2**

g.   Consider the following diagram:

    (i)   On about one third of a page, copy the above diagram.

    (ii)   Find the length of AC in exact form.   **1**

    (iii)   Hence show that $ED = \dfrac{\sqrt{3}-1}{2}$ **2**

    (v)   Hence, using exact values, show that $\sin 15° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$ **3**

END OF EXAM

Sample Solutions.

1. $\dfrac{(1.49)^2 - 1.98}{\sqrt{11.62 + 8.34 \times 2.72}}$

$= \dfrac{0.2401}{5.857}$

$= 0.0410 \ (3 \text{ sig fig})$ Ⓓ

2. $\dfrac{x+4}{3} = \dfrac{x}{2} - 2$

$\dfrac{2(x+4)}{6} = \dfrac{3x}{6} - \dfrac{12}{6}$

$\dfrac{2(x+4)}{6} = \dfrac{3x - 12}{6}$

$2(x+4) = 3x - 12$

$2x + 8 = 3x - 12$

$x = 8 + 12 = 20$

Ⓓ

3. $2x^2 - 6x - 3 = 0.$

$a = 2, \ b = -6, \ c = -3.$

$\dfrac{6 \pm \sqrt{36 - 4(-3)(2)}}{4} = \dfrac{6 \pm \sqrt{12}}{4} = \dfrac{6 \pm 2\sqrt{3}}{4} = \dfrac{3 \pm \sqrt{3}}{2}$ Ⓐ

4. $H = 5m(Y - X)$

$\dfrac{H}{5m} = Y - X$

$Y - \dfrac{H}{5m} = X$

$X = \dfrac{5mY - H}{5m}$ Ⓐ

CHECK BY REARRANGING.

$5mX = 5mY - H$

$X = Y - \dfrac{H}{5m}$

$\dfrac{H}{5m} = Y - X$

$H = 5m(Y - X)$ ✓

5. $f(x) = 2x^2 - 3x - 5.$

$\dfrac{f(a)}{2a+2} = \dfrac{2a^2 - 3a - 5}{2(a+1)}$

$= \dfrac{(2a - 5)(a+1)}{2(a+1)}$

$= \dfrac{2a - 5}{2}$ Ⓐ

6 $m^2 - 1$

$= (m+1)(m-1)$ ✗

$m^2 + 1$ ✓

$m^2 + m = m(m+1)$ ✗

$m^2 + 2m + 1 = (m+1)^2$

$\Rightarrow$ Ⓑ

7. horizontal asymptote at positive y and positive x.

graph is shifted up 2 units

$\therefore \Rightarrow y = -\dfrac{1}{x-3} + 2.$

Check horizontal asymptote.

$y - 2 = -\dfrac{1}{x - 3}$

$\dfrac{1}{y - 2} = -(x - 3)$

$\hookrightarrow$ Asymptote at $y = 2$!

Ⓑ

8 $y = x^2 - 7x + 10$

$\dfrac{dy}{dx} = 2x - 7 = 0$ $\left(\dfrac{-b}{2a} = \dfrac{7}{2} \right)$

$2x = 7$

$x = \dfrac{7}{2} = 3\tfrac{1}{2} \Rightarrow$ Ⓑ

9. $f(x) = 3x^2 - x.$

$= 3x(x - 1)$

root at 0 and 1

Graph is neither. because the axis of symmetry / rotation is not centred at the axis $\Rightarrow$ Ⓒ

10. $y = \sqrt{4 - x^2}$

Range: $y \geqslant 0.$

Domain $4 - x^2 \geqslant 0$

$x^2 \leqslant 4$

$x \leqslant \pm 2.$

$\Rightarrow$ Ⓓ

11.

a) $\dfrac{x^2-1}{x-3} \times \dfrac{x^2-3x}{2x-2}$

$= \dfrac{(x+1)(x-1) \times (x-3)}{(x-3)(2(x-1))}$

$= \dfrac{x(x+1)}{2} \Rightarrow \dfrac{x^2+x}{2}$ expanded.

b) $3x = x^2$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$ OR $x = 3$.

c) i) $6x^2 - 3xy - 4xz + 2yz$

$3x(2x-y) - 2z(2x-y)$

$= (3x - 2z)(2x - y)$.

ii) $x^4 - 1$

$= (x^2)^2 - 1$

$= (x^2+1)(x^2-1)$

$= (x^2+1)(x+1)(x-1)$

iii) $8x^3 - 125y^3$

$= (2x)^3 - (5y)^3$

$= (2x-5y)(4x^2 + 25y^2 + 10x\text{...})$

d) $(2x-3y)^2 - 5x(x-2y)$

$4x^2 + 9y^2 - 12xy - 5x^2 + 10xy$

$= 9y^2 - x^2 - 2xy$.

e) $x + 3y = 2$

$2x - y = 11$

$x = 2 - 3y$

$2(2-3y) - y = 11$

$4 - 6y - y = 11$

$-7y = 7$

$\boxed{y = -1}$

$x + 3(-1) = 2$

$\boxed{x = 5}$

f) $x^2 - 10x - 11 = 0$.

$(x-5)^2 - 25 - 11 = 0$

$(x-5)^2 = 36$

$x - 5 = \pm 6$

$x = -1$ OR $x = 11$

Double check by factorising

$(x+1)(x-11)$

$= x^2 - 11 - 10x$ ✓

12. a) i) $\sqrt{32} = \sqrt{16 \times 2}$

$= \sqrt{16}\sqrt{2}$

$= 4\sqrt{2}$.

ii) $3 + \sqrt{27} - \sqrt{18}$

$\sqrt{3} + \sqrt{9}\sqrt{3} - \sqrt{9}\sqrt{2}$

$\sqrt{3} + 3\sqrt{3} - 3\sqrt{2}$

b. $0.1\dot{2}\dot{5} \Rightarrow 0.125252 5...$

let this be x.

$10x = 1.2525....$

$1000x = 125.2525...$

$1000x - 10x = 124$

$990x = 124 \rightarrow x = \dfrac{62}{495}$

c) $(2\sqrt{7} + \sqrt{11})(2\sqrt{7} - \sqrt{11})$

$= (2\sqrt{7})^2 - (\sqrt{11})^2$

Difference of 2 squares

$= 4(7) - 11$

$= 28 - 11 = 17$

d) $\dfrac{3\sqrt{2}+1}{2\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$ Rationalise.

$= \dfrac{3\sqrt{6} + \sqrt{3}}{6}$

e) $\dfrac{\sqrt{5}}{\sqrt{5} - 2} = p + q\sqrt{5}$.

$\dfrac{\sqrt{5}(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$

$= \dfrac{5 + 2\sqrt{5}}{5 - 4} = 5 + 2\sqrt{5}$

$p = 5, \ q = 2$.

f. let $y = f(x)$

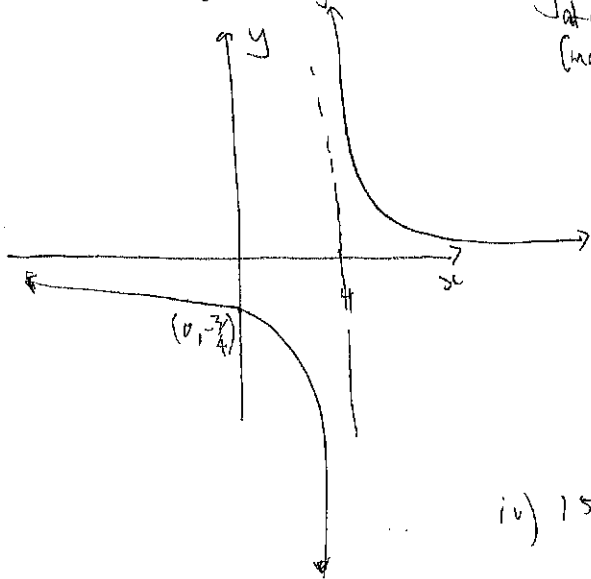$y = \frac{3}{x-4}$ Vertical asymptote at $x=4$.

$\frac{x-4}{3} = \frac{1}{y}$

$x - 4 = \frac{3}{y}$

$x = \frac{3}{y} + 4$ horizontal asymptote at $y=0$

domain: $x \in \mathbb{R} ; x \neq 0$

Range: $y \in \mathbb{R}, y \neq 0$

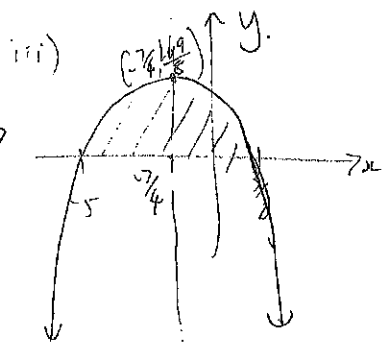$(0, \frac{-3}{4})$

g) $y = 15 - 7x - 2x^2$

$(3 - 2x)(5 + x)$

Roots

$x = -5, \quad x = \frac{3}{2}.$

ii) Vertex

$a = -2$
$b = -7$ $\frac{b}{2a} = \frac{7}{-4}$
$c = 15$

$15 - 7\left(\frac{7}{4}\right) - 2\left(\frac{49}{16}\right)$

y-coordinate at vertex (maximum) $= \frac{169}{8}$

iii)

$\left(-\frac{7}{4}, \frac{169}{8}\right)$
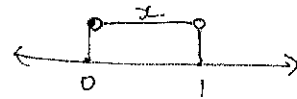
iv) $15 - 7x - 2x^2 \geq 0$

$-5 \leq x \leq \frac{3}{2}.$

v. $y \leq \frac{169}{8}$

13.

a. i) $1 < 1 + x < 2$

$0 < x < 1$

ii) $|x+1| = 3x + 2.$

$(x+1)^2 = (3x+2)^2$

$x^2 + 1 + 2x = 9x^2 + 4 + 12x.$

$8x^2 + 10x + 3$

$(2x + 1)(4x + 3)$

$x = -\frac{1}{2}$ OR $x = \frac{3}{4}.$

Substitue back to find correct answer

$\left|-\frac{1}{2} + 1\right| = 3\left(-\frac{1}{2}\right) + 2.$ ✓

$\left|-\frac{3}{4} + 1\right| = 3\left(-\frac{3}{4}\right) + 2$ ✗

So the only solution is $x = -\frac{1}{2}$

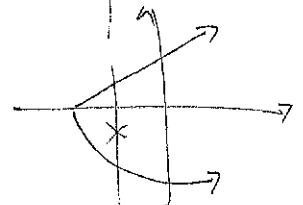b. $p(x) = \begin{cases} x+1 & ; x \geq 1 \\ -1 & : -2 < x < 1 \\ |x| & x \leq -2. \end{cases}$

$p(0) = -1$

$P(-2) = |-2| = 2.$

$P(0) + P(-2) = 1$

$p(x)$
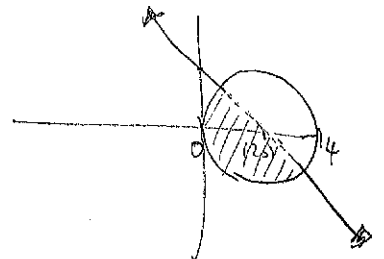
c).

Any line drawn that does not pass the straight line test will suffice.
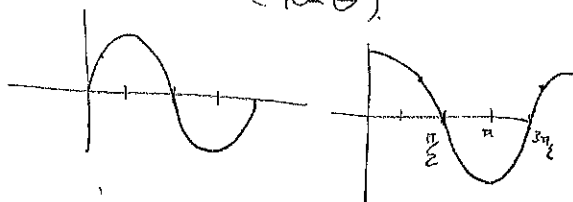
d).

e) $f(x) = 8x^3 - 7x$.

$f(-x) = 8(-x)^3 - 7(-x)$

$= -8x^3 + 7x$

$= -(8x^3 - 7x)$

$= -f(x)$

$\therefore$ ODD FUNCTION.

f. $\sin\theta = \cos(90 - \theta)$

$\theta = 40, 140$

ii) $\sin\theta = -\frac{1}{2}$

$\theta = 210°, 330°$

g) i) Check it yourself!

ii) $\frac{1}{AC} = \tan 30°$

$\frac{1}{AC} = \frac{1}{\sqrt{3}}$

$AC = \sqrt{3}$ units

$AD = \sqrt{3} - 1$

$\frac{EO}{\sqrt{3}-1} = \sin 30$

$EO = \frac{1}{2}(\sqrt{3}-1)$

$= \frac{\sqrt{3}-1}{2}$

v. $\sin 15 = \frac{EO}{BD}$

$\frac{\frac{\sqrt{3}-1}{2}}{\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$