Sydney Technical High School

Name: ....

Maths Class: ....

Year 11

Mathematics

Preliminary Course Final Exam

September 2017

Time allowed: 120 minutes

General Instructions:	
- Marks for each question are indicated on the question. - Approved calculators may be used - All necessary working should be shown - Full marks may not be awarded for careless work or illegible writing *Begin each question on a new page* - Write using black or blue pen - All answers are to be in the writing booklet provided - A reference sheet is provided at the rear of this Question Booklet, and may be removed at any time.	Section 1 Multiple Choice Questions 1-8 8 Marks Section II Questions 9-16 80 Marks Total = 88 marks

SECTION 1 (10 marks)

Choose the letter corresponding to the correct answer and fill in the Answer sheet provided at the front of your answer booklet.

DO NOT REMOVE THIS SHEET

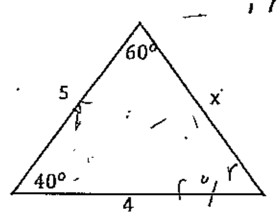
1 Which of the following is NOT always a true statement?

A. The diagonals of a rhombus bisect at right angles
B. The opposite angles of a rhombus are equal
C. The diagonals of a parallelogram bisect at right angles
D. The opposite angles of a parallelogram are equal

2 The quadratic equation $2x^2 - 4x + 5 = 0$ has:

A. No real roots
B. 1 real root
C. 2 equal roots
D. 2 distinct Real roots

3 Which statement below is true for the diagram shown?



A. $\cos 60^\circ = \frac{5^2 + 4^2 - x^2}{2\times5\times4}$

B. $\frac{4}{\sin 60^\circ} = \frac{x}{\sin 100^\circ}$

C. $x^2 = 25 + 16 - 2\times5\times4\cos 60^\circ$

D. $\frac{5}{\sin 80^\circ} = \frac{x}{\sin 40^\circ}$

4 Find $\lim\limits_{x\to\infty} \frac{3x^2-2x+3}{2x^2-5}$

A. $-\frac{3}{5}$
B. $\frac{2}{3}$
C. $\frac{3}{2}$
D. 1

5	If $= \frac{2\sqrt{3}+3}{\sqrt{3}-2} = x + y\sqrt{3}$, then A. $x = 12$ and $y = 7$ B. $x = -12$ and $y = 7$ C. $x = 12$ and $y = -7$ D. $x = -12$ and $y = -7$	
6	If $y = \frac{1}{(5x-1)^2}$ then $\frac{dy}{dx} =$ A. $\frac{-10}{(5x-1)^3}$ B. $\frac{-10}{(5x-1)}$ C. $\frac{-2}{(5x-1)^3}$ D. $\frac{-2}{(5x-1)}$	
7	If $\cos\theta = \frac{k}{5}$ for an acute angle θ, then $\tan\theta =$ A. $\frac{\sqrt{25-k^2}}{k}$ B. $\frac{\sqrt{25-k^2}}{5}$ C. $\frac{5}{\sqrt{25-k^2}}$ D. $\frac{k}{\sqrt{25-k^2}}$	
8	If $5^{2x-1} = \frac{1}{125}$ then $x =$ A. 13 B. -12 C. -2 D. -1	

SECTION 2

Complete all answers in your answer booklet provided

QUESTION 9: (10 Marks)

Marks

(a) Expand and simplify: $(x+3)(x^2-3x+9)$ — 1

(b) Solve the equation: $|3x-4| = 5$ — 2

(c) What is the size of one of the exterior angles of a regular pentagon? — 1

(d) (i) What are the Domain and Range of the function $f(x) = \sqrt{16 - x^2}$? — 2

(ii) Sketch $y = f(x)$ — 2

(e) Find the equation of the tangent to the curve $y = \frac{1}{4}x^3 - 4$ at the point P $(2, -3)$ — 2

QUESTION 10: (10 Marks) *Start a new page*

Marks

(a) Find the derivatives of:

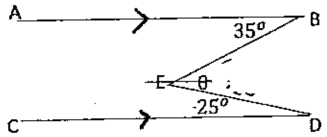
(i) $y = x^3 + 3x - 1$ — 1

(ii) $y = (3x-5)^4$ — 1

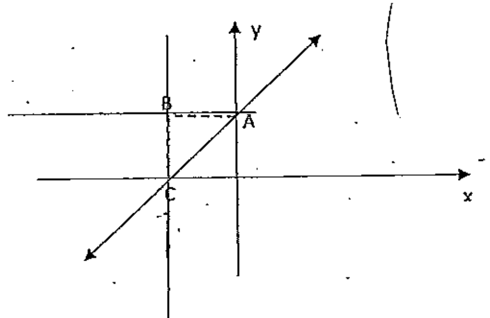
(iii) $y = \frac{2}{x}$ — 1

(iv) $2\sqrt{x}$ — 1

(b) Find the size of θ in the following., given AB ∥ CD, (no reasons necessary — 1



(c) In the diagram below, the line AC is given as $3x - 2y + 6 = 0$



B has the same x-coordinate as C and the same y-coordinate as A

(i) Find the point B. — 2

(ii) Find the equation of the line through B perpendicular to line AC — 3

QUESTION 11: (10 marks) *Start a new page*

Marks

(a) For the function defined by:

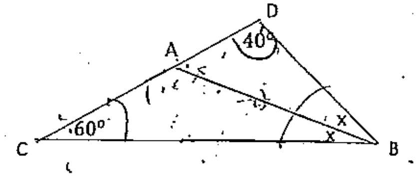
$$f(x) = \begin{cases} 2x, & x \geq 1 \\ 2-2x, & x < 1 \end{cases}$$

(i) Sketch $y = f(x)$ — 3

(ii) Find the value of $f(-1) + f(1) + f(3)$ — 1

(b) Solve simultaneously $\begin{cases} 4x - y = 19 \\ x + 2y + 2 = 0 \end{cases}$ — 2

(c) In the diagram below, AB bisects $\angle DBC$, $\angle ACB = 60^o$ and $\angle CDB = 40^o$ — 4



Copy the diagram into your answer booklet

Setting out a formal proof, prove that ΔCBA ||| ΔCDB

QUESTION 12: (10 marks) *Start a new page*

Marks

(a) Find the equation of the normal to the curve $y = 2x^3 - 4x^2$ at the point (1, -2) — 3

(b) α and β are the roots of the quadratic equation $2x^2 - 3x + 5 = 0$

(DO NOT ATTEMPT TO FIND THESE ROOTS)

Find the value of:

(i) $\alpha + \beta$ — 1

(ii) $\alpha\beta$ — 1

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ — 1

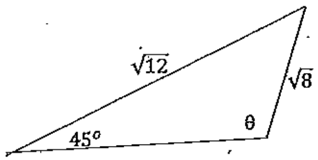
(iv) $\alpha^2 + \beta^2$ — 2

(v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ — 2

QUESTION 13: (10 Marks) *Start a new page*

Marks

(a) 3



In the diagram above, find the value of θ, if $90^o < \theta < 180^o$

(b) (i) On the same diagram shade the region corresponding to the simultaneous solution of: 3

$$(x-3)^2 + y^2 \le 4 \quad \text{and} \quad x + y \ge 3$$

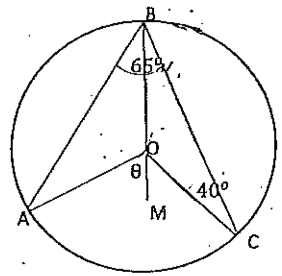
(ii) The point P lies somewhere in the shaded region described in part (i). 1
At what point in the region above is P furthest from the origin? Give the co-ordinates of this point.

(c) If the roots of the quadratic equation $kx^2 + (k-1)x + (2k+1) = 0$ are such that one root is the reciprocal of the other, find the value of k. 3

QUESTION 14: (10 Marks) *Start a new page*

Marks

(a) For the figure below, O is the centre of the circle, $\angle BCO = 40^o$
$\angle ABC = 65^o$
BO is produced to M.

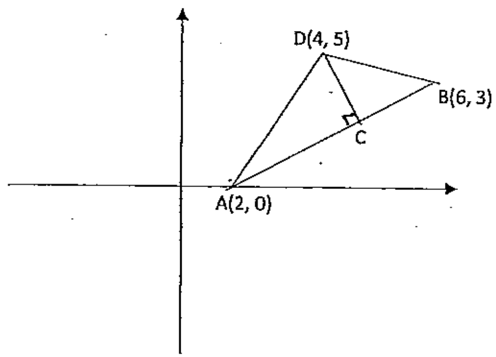


(i) Find the size of $\angle ABM$ 2

(ii) Find the size of $\angle AOM$ 2

You must provide reasons for each line of your proofs.

(b) The point A is (2,0) while B is (6, 3) and D (4, 5) as shown.



(i) Find the length of AB 1

(ii) Find the equation of the line AB in general form 2

(iii) Find the shortest distance of the point D from AB (ie CD) 2

(iv) Find the area of ΔABD 1

QUESTION 15: (10 Marks) *Start a new page*

Marks

(a) If $f(x) = 3x^2$, find $\frac{f(x+h)-f(x)}{h}$ 3

(b) Prove that $\frac{\tan^2 x}{\sec x+1} = \sec x - 1$ 2

(c) Solve $4\sin^2\theta - 3 = 0$ for $0^o \leq \theta \leq 360^o$ 3

(d) If $f(x) = x^{\frac{3}{2}}$ find the value of $f'(4)$ 2

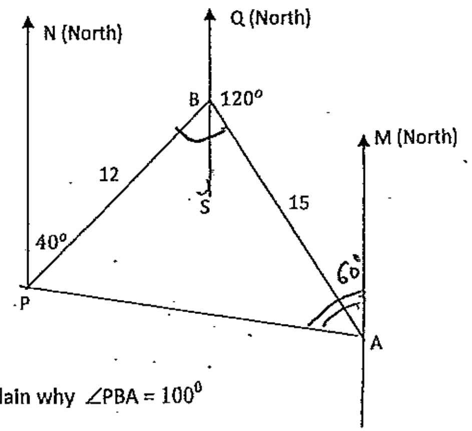
QUESTION 16: (10 Marks) *Start a new page*

(a) Find $\frac{dy}{dx}$ if:

(i) $y = \sqrt{x^3+3}$ 2

(ii) $y = \frac{x}{x+1}$ 2

(b) The diagram below shows the course of a ship, which sails from a port P on a bearing of 040^0 for 12 km before changing course to a bearing of 120^0 and travelling a further 15 km to a destination A.



(i) Explain why $\angle PBA = 100^0$ 1

(ii) Find the distance of A from P to the nearest km. 2

(iii) Find the bearing of P from A to the nearest degree. 3

Mathematics Solutions

Multiple Choice

Q1. (C)

Q2. $\Delta = 16 - 4(2)(5) < 0$ $\therefore$ (A)

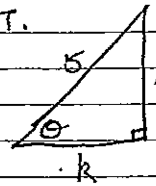
Q3. (D)

Q4. (C)

Q5. $\dfrac{(2\sqrt{3}+3)(\sqrt{3}+2)}{3-4} = \dfrac{6+6+7\sqrt{3}}{-1} = -12-7\sqrt{3}$ $\therefore$ (D)

Q6. $\dfrac{d}{dx}(5x-1)^{-2} = -2(5)(5x-1)^{-3} = \dfrac{-10}{(5x-1)^3}$ (A)

Q7.



$\leftarrow$ By Pythagoras $= \sqrt{25-k^2}$

$\therefore \tan\theta = \dfrac{\sqrt{25-k^2}}{k}$ (A)

Q8. $5^{2x-1} = 5^{-3}$

$\therefore 2x-1 = -3$

$\therefore 2x = -2$

$\therefore x = -1.$ (D)

Question 9:

(a) x^3+27

(b) $3x=9$ or $3x=-1$

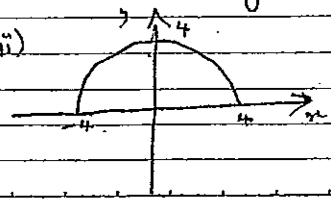
$\therefore x=3$ or $x=-\frac{1}{3}$

(c) Sum $= 360^\circ$ $\therefore$ Each angle $\frac{360}{5} = 72^\circ$

(d) (i) D: $-4 \le x \le 4$

R: $0 \le y \le 4$

(ii)



(e) $\dfrac{dy}{dx} = \dfrac{3}{4}x^2$

At $(2,-2)$ $m_T = 3$

$\therefore$ Equation is

$y+2 = 3(x-2)$

$y = 3x-8$

Question 10:

(a) (i) $3x^2+3$ (ii) $12(3x-5)^3$

(iii) $-\frac{2}{x^2}$ (iv) $x^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{x}}$.

(b) $\theta = 60^\circ$

(c) (i) A is $(0,3)$ C is $(-2,0)$ $\therefore$ B is $(-2,3)$.

(ii) $m_{AC} = \frac{3}{2}$ $\therefore$ m of perp $= -\frac{2}{3}$

$\therefore$ Equation is

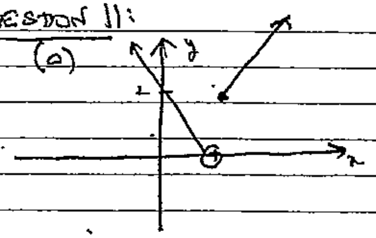
$y-3 = -\frac{2}{3}(x+2)$

$3y-9 = -2x-4$

$2x+3y-5=0.$

Question 11:

(a)



(ii) $f(-1)+f(1)+f(3)$

$= 4+2+6$

$= 12$

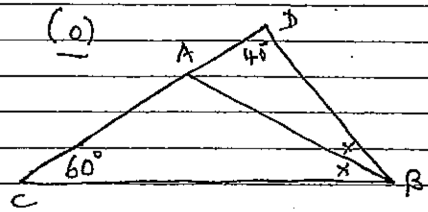
(b) $8x-2y=38$ (1)

$x+2y+2=0$ (2)

(1)+(2) $9x+2=38$

$\therefore \begin{cases} x=4 \\ y=-3 \end{cases}$

(c)



In $\triangle CDB$, $2x = 80^\circ$ (angle sum)

$\therefore x = 40^\circ$

In $\triangle CBA$ and $\triangle CDB$

$\angle ACB = \angle BCD$ (same angle)

$\angle CBA = \angle CDB = 40^\circ$.

$\therefore \triangle CBA \,|||\, \triangle CDB$ (equiangular)

QUESTION 12:

(a) $\frac{dy}{dx} = 6x^2 - 8x$

at $(1,-2)$ $m_T = -2$

$m_N = \frac{1}{2}$

EQUATION:

$y + 2 = \frac{1}{2}(x-1)$

$2y + 4 = x - 1$

$2y = x - 5$

(b) (i) $\alpha + \beta = \frac{3}{2}$ (ii) $\alpha\beta = \frac{5}{2}$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{3/2}{5/2}$

$= \frac{3}{5}$

(iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \frac{9}{4} - 5$

$= -\frac{11}{4}$

(v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= -\frac{11}{4} / \frac{5}{2}$

$= -\frac{11}{10}$

QUESTION 13:

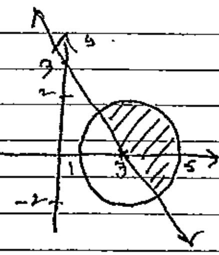
(a) By sine rule, $\frac{\sin\theta}{\sqrt{12}} = \frac{\sin 45}{\sqrt{8}}$

$\therefore \sin\theta = \frac{\frac{1}{\sqrt{2}} \cdot \sqrt{12}}{\sqrt{8}}$

$= \frac{\sqrt{3}}{2}$

$\therefore \theta = 60°$

(b) (i)



(ii) P is $(5,0)$

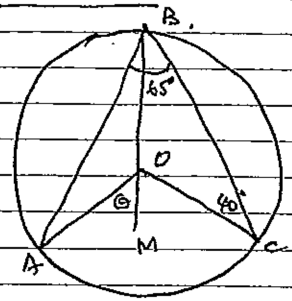
(c) Let the roots be α and $\frac{1}{\alpha}$.

PRODUCT $= 1 = \frac{2k+1}{k}$

$\therefore k = -1$

QUESTION 14:

(a)



(i) In $\triangle BOC$, OB and OC are radii.

$\therefore \angle MBC = 40°$ (isosceles $\triangle BMC$)

$\therefore \angle ABM = 25°$ (angle sum)

(ii) $\triangle AOB$ is isosceles (equal radii)

$\therefore \angle BAO = 25°$ (base angles are equal)

$\therefore \theta = 50°$ (exterior angle of $\triangle ABO$)

(b)

(i) $AB = \sqrt{(6-2)^2 + 3^2}$

$= 5$

(ii) $m_{AB} = \frac{3}{4}$ EQUATION AB: $y = \frac{3}{4}(x-2)$

$4y = 3x - 6$

ie $3x - 4y - 6 = 0$

(iii) $p = \left|\frac{3(4) - 4(5) - 6}{5}\right|$

$= \frac{14}{5}$

(iv) Area $(ABD) = \frac{1}{2}(AB)(CD)$

$= \frac{1}{2} \times \frac{14}{5} \times 5$

$= 7$ units^2.

QUESTION 15:

(a) $\frac{f(x+h) - f(x)}{h} = \frac{3(x+h)^2 - 3x^2}{h}$

$= \frac{6xh + 3h^2}{h}$

$= 6x + 3h$

(b) $\frac{\tan^3 x(\sec x - 1)}{\sec^2 x - 1} = \frac{\tan^3 x(\sec x - 1)}{\tan^2 x}$

$= \sec x - 1$

(c) $\sin\theta = \pm\frac{\sqrt{3}}{2}$

$\therefore \theta = 60°, 120°, 240°, 300°$

(d) $f'(x) = \frac{3}{2}x^{\frac{1}{2}} \Rightarrow f'(4) = \frac{3}{2}(2)$

$= 3$

Question 16:

(a) (i) $\frac{dy}{dx} = \frac{1}{2}(x^3+3)^{-1/2} \cdot 3x^2$

$= \frac{3x^2}{2\sqrt{x^3+3}}$

(ii) $\frac{dy}{dx} = \frac{(x+1) \cdot 1 - x \cdot 1}{(x+1)^2}$

$= \frac{1}{(x+1)^2}$

(b) (i) $\angle PQS = 40^\circ$ (alternate angles $NP \parallel QS$)

and $\angle SQA = 60^\circ$ (straight $\angle QBS$)

$\therefore \angle PBA = 100^\circ$

(ii) In $\triangle PBA$,

$PA^2 = 12^2 + 15^2 - 2 \times 12 \times 15 \cos 100^\circ$

$PA \doteq 21$ km

(iii) $\angle BAm = 60^\circ$ (co-interior angles $QS \parallel mA$)

Let $\angle BAP = \theta$

$\frac{\sin\theta}{12} = \frac{\sin 100^\circ}{21}$

$\therefore \theta = 34.24^\circ$

$\therefore \angle PAm = 94.24^\circ$

$\therefore$ Bearing is $(360 - 94.24)^\circ$

$= 265.76^\circ$

$= 266^\circ$