Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics

Preliminary HSC Course

Yearly Exam

September, 2016

Time allowed: 2 hours

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided

Section 1	Multiple Choice Questions 1-10 10 Marks	
Section II	Questions 11-18 72 Marks	
Total	82 marks	

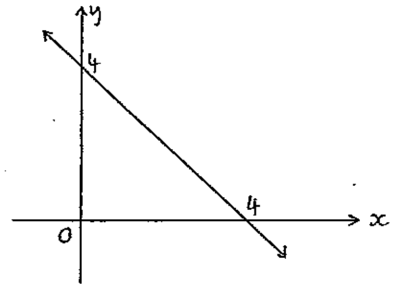
SECTION 1 -- MULTIPLE CHOICE (10 marks)

QUESTION 1

What is the gradient of a line parallel to the line $2x + 3y - 1 = 0$?

A. 2 B. -2 C. $\frac{3}{2}$ D. $-\frac{2}{3}$

QUESTION 2



The equation of the line above is: A. $x - y + 4 = 0$ B. $x + y - 4 = 0$

C. $x + y + 4 = 0$ D. $x - y - 4 = 0$

QUESTION 3

A function is given by $f(x) = \sqrt{9 - x^2}$. What is its natural domain?

A. $x < 3$ B. $x \leq 3$ C. $-3 \leq x \leq 3$ D. $-9 \leq x \leq 9$

QUESTION 4

The function in Question 3 above is:

A. even B. odd C. neither D. cannot be determined

QUESTION 5

What is the minimum value of $x^2 - 4x + 6$? =

A. 2 B. 4 C. 6 D. 8

QUESTION 6

If $a^b = 5$, what is the value of $2a^{3b}$?

A. 30 B. 250 C. 500 D. 1000

QUESTION 7

If $3^{x-4} = 9^{2x}$, then $x = ?$

A. $\frac{3}{4}$ B. $\frac{4}{3}$ C. $-\frac{3}{4}$ D. $-\frac{4}{3}$

QUESTION 8

If $2x^2 - 12x + 11$ is expressed in the form $2(x-b)^2 + c$, what is the value of c?

A. -25 B. -7 C. 2 D. 29

QUESTION 9

$\frac{\sin(180°-\theta)}{\cos(90°-\theta)}$ simplifies to:

A. 1 B. 2 C. $\tan\theta$ D. $\cot\theta$

QUESTION 10

If $a > b$, which of the following is always true?

A. $a^2 > b^2$ B. $\frac{1}{a} < \frac{1}{b}$ C. $-a > -b$ D. $2^a > 2^b$

END OF SECTION 1

SECTION 2

QUESTION 11 (9 marks)

a) Evaluate $13.6\sin 42°15'$ correct to 2 significant figures. 1

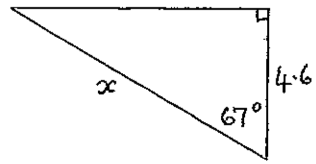
b) Expand and simplify $(2\sqrt{3}-1)(\sqrt{3}+4)$ 2

c) Write the exact value of cosec $60°$. 1

d) Simplify $\frac{x-3}{x^2-4x+3}$ 1

e) Find the value of x, correct to 1 decimal place. 2

Not to Scale



f) Solve $(x+1)^2 = 5$, giving answers correct to 1 decimal place. 2

QUESTION 12 (9 marks) Start a new page.

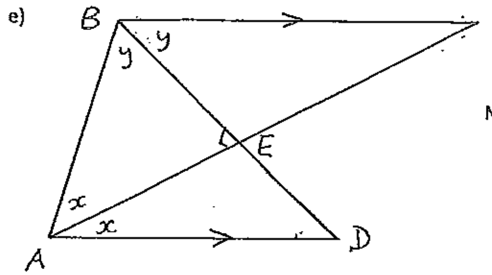
a) Solve $|3x-6| < 12$ 2

b) Find θ to the nearest degree if $\cos\theta = 0.4$ and $0° \le \theta \le 360°$. 1

c) Fully simplify $\frac{a+b}{\frac{1}{a}+\frac{1}{b}}$ 2

d) Find derivatives of: i) $y = 3x^2 - 4 + 7x$ 1

ii) $f(x) = \frac{4}{x^2}$ 1

e) 

Not to Scale

$AD \parallel BC$. AC and BD intersect at E. $\angle BAD$ and $\angle ABC$ are bisected as shown.

Prove that $\angle BEA = 90°$. 2

QUESTION 13 (9 marks) Start a new page.

a) Factorise $x^3 - 27$. 1

b) Simplify $\sin\theta(1 + cot^2\theta)\tan\theta$. 2

c) Differentiate i) $(x^2 + 5)^4$ 1

ii) $x\sqrt{x}$ 1

d) Find the gradient of the curve $y = \frac{2x}{x+3}$ when $x = -2$. 2

e) The solutions of a quadratic equation are $x = \frac{1\mp\sqrt{5}}{2}$. Write a quadratic equation with these solutions. 2

QUESTION 14 (9marks) Start a new page.

a) Find the coordinates of the vertex of the parabola $y = (x + 3)^2 + 4$. 1

b) Given $f(x) = x^2 + \frac{x}{2}$, evaluate $f(2) + f'(2)$. 2

c) Solve for θ, given $0° \leq \theta \leq 360°$:

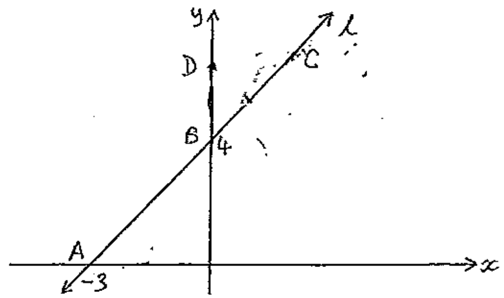
i) $(\sin\theta + 1)(\cos\theta - 1) = 0$ 2

ii) $3tan^2\theta - 1 = 0$ 2

d) If $\cos\theta = -\frac{2}{3}$ and $\sin\theta > 0$, find the exact value of $\tan\theta$. 1

e) Fully factorise $x^2 + 8x + 16 - y^2$. 1

QUESTION 15 (9 marks) Start a new page.



Not to Scale

In the diagram above, line l cuts the x axis at A(-3,0) and the y axis at B(0,4). D has coordinates (0,6) and point C is on l.

a) Find the gradient of line l. 1

b) Show that line l has equation $4x - 3y + 12 = 0$. 1

c) B is the midpoint of AC. Find the coordinates of C. 1

d) Find the perpendicular distance from D to the line l. 1

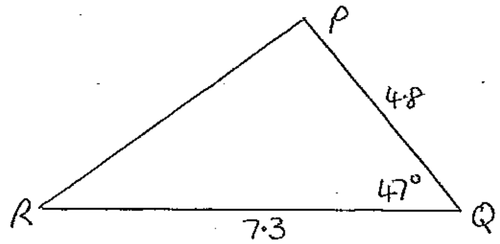
e) Find the area of ΔBDC. 2

f) Find the equation of the perpendicular bisector of AB. Leave your answer in general form. 3

QUESTION 16 (9 marks) Start a new page.

a) Find $\frac{d}{dr}(\frac{4}{3}\pi r^3)$ 1

b) Find the point(s) on the curve $y = x^3 - 3x^2 + 3x$ where the tangent is horizontal. 2

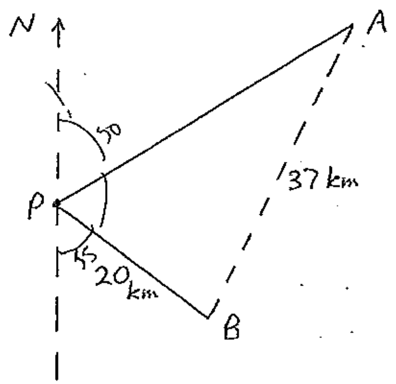
c) 

Not to Scale

i) Find the area of ΔPQR. 1

ii) Find the length of RP, correct to 1 decimal place. 2

d)



Not to Scale

Ship A leaves port P and sails on a compass bearing of $N50°E$. Ship B also leaves port P and sails 20 km on a compass bearing of $S55°E$. The two ships are now 37 km apart.

i) Find ~~$\angle APB$.~~ 1

ii) Find $\angle PAB$ to the nearest minute. 2

QUESTION 17 (9marks) Start a new page.

a) The interior angle of a regular polygon is 165°. How many sides does the polygon have? 1

b) Find the centre and radius of the circle $x^2 + y^2 - 4y - 1 = 0$. 2

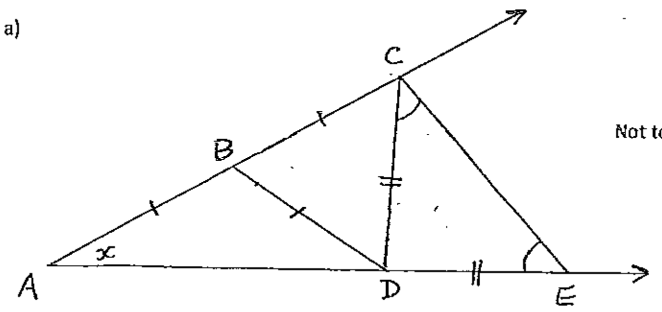
c) i) On the same axes, neatly sketch the functions $y = \frac{1}{x+2}$ and $y - x = 2$. Use a ruler, label any asymptotes and all x and y intercepts. 2

ii) Find the points of intersection of the two graphs. Show working. 2

iii) Find the equation of the normal to the curve $y = \frac{1}{x+2}$ at the point where it crosses the y axis. 2

QUESTION 18 (9 marks) Start a new page.

a)



Not to Scale

Rays AC and AE enclose isosceles triangles ABD, BCD and CDE as shown above.

i) If $\angle A = x$, find $\angle CBD$ in terms of x, giving reasons. 2

ii) Hence, find the size of $\angle DEC$. Reasons are not required. 1

b) Simplify $\lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$. Show full working. 2

c) Prove that $\sec\theta - \sin\theta\tan\theta = \cos\theta$. 2

d) Differentiate $y = x^2(x^3 - 1)^4$. Leave your answer in fully factored form. 2

END OF TEST

SYDNEY TECH.

PRELIMINARY HSC COURSE

YEARLY EXAM 2016 SEPTEMBER.

SAMPLE SOLUTIONS

SECTION 1 : MULTIPLE CHOICE

1. $2x+3y-1=0$

$3y=-2x+1$

$y=-\frac{2}{3}x+\frac{1}{3}$.

$m=-\frac{2}{3} \Rightarrow D$

2. Points are $(0,4)$ and $(4,0)$

$\frac{0-4}{4-0}=\frac{-4}{4}=-1$.

taking $(0,4)$ as a reference point

$y-4=-1(x-0)$

$y-4=-x$.

$y+x-4=0$

$x+y-4=0 \Rightarrow B$

3. $f(x)=\sqrt{9-x^2}$.

ie $9-x^2\geq 0$

$x^2\leq 9$.

$-3\leq x\leq 3. \Rightarrow C$.

4. $f(-x)=\sqrt{9-(-x)^2}$

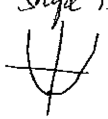
$=\sqrt{9-(x)^2}$

So even function $\Rightarrow A$

5. x^2-4x+6.

turning point of a parabola

$=\frac{-b}{2a}$, as a is positive the shape is concave up $\rightarrow$



ie $\frac{-b}{2a}=\frac{4}{2}=2$

$y=2^2-8+6 \Rightarrow 4-8+6=2$.

$\Rightarrow A$

6. $a^b=5$

$2a^{3b}=2(a^b)^3$

$=2(5^3)=2\times125$

$=250 \Rightarrow B$

7. $3^{x-4}=9^{2x}$

$3^{x-4}=3^{2(2x)}$

$x-4=2(2x)$

$x-4=4x$

$3x=-4$

$x=-\frac{4}{3}. \Rightarrow D$.

8. $2x^2-12x+11$

in the form $2(x-b)^2+C$.

$2(x^2-6x+3^2)+11-18=0$

$2(x-3)^2-7=0. \Rightarrow B$.

9. $\sin(180-\theta)=\sin\theta$

$\cos(90-\theta)=\sin\theta$

So $\Rightarrow \frac{\sin\theta}{\sin\theta}=1 \Rightarrow A$.

10. If $a>b$. TAKE 3 cases

a is positive, b is positive ①

a is positive, b is negative ②

a is negative, b is negative ③

in this case ①, take $a=10$, $b=5$; case 2, $a=10$, $b=-5$

Case 3, $a=-5$, $b=-10$.

①	②	③
$a^2>b^2$ ✓	$a^2>b^2$ ✓	$a^2>b^2$ X
$\frac{1}{a}<\frac{1}{b}$ ✓	$\frac{1}{a}<\frac{1}{b}$ X	
$-a>-b$ X	$-a>-b$ X	
$2^a>2^b$ ✓	$2^a>2^b$ ✓	

Thus the only possible is $2^a>2^b$

$\Rightarrow D$.

SECTION 2.

11. a) $13.6 \sin 42°15' = 9.1$ (2 s.f)

b) $(2\sqrt{3}-1)(\sqrt{3}+4)$

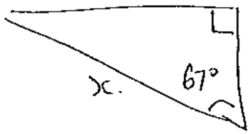
$= (2\sqrt{3}\sqrt{3}) + 4(2\sqrt{3}) - \sqrt{3} - 4$

$= 2(3) + 8\sqrt{3} - \sqrt{3} - 4$

$= 2 + 7\sqrt{3}$

c) $\text{Cosec } 60° = \dfrac{1}{\sin 60°} = \dfrac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \dfrac{2}{\sqrt{3}}$

d) $\dfrac{x-3}{x^2-4x+3} = \dfrac{x-3}{(x-3)(x-1)} = \dfrac{1}{x-1}$

e) 

$\dfrac{4.6}{x} = \cos 67°$

$x = 11.8$ (1 d.p)

f) $(x+1)^2 = 5$

$x^2 + 1 + 2x = 5$

$x^2 + 2x - 4 = 0$

using quadratic formula.

$a = 1, \quad b = 2, \quad c = -4$

$\dfrac{-2 \pm \sqrt{20}}{2} = \dfrac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}.$

$= 1.2, -3.2$ (1 d.p)

12

a) $|3x - 6| < 12$

$-12 < 3x - 6 < 12.$

$-6 < 3x < 18$

$-2 < x < 6$

b). $\cos\theta = 0.4.$

$0° \leq \theta \leq 360°$

S	A
T	C.

Only on 1st and 4th quadrant.

$= 66°25', 360 - 66°25'$

$= 66°25', 293°34'$

c). $\dfrac{a+b}{\left(\frac{1}{a}+\frac{1}{b}\right)} = \dfrac{a+b}{\left(\frac{a+b}{ab}\right)} = ab$

d) i) $y = 3x^2 - 4 + 7x$

$\dfrac{dy}{dx} = 6x + 7$

ii) $f(x) = \dfrac{4}{x^2} = 4x^{-2}.$

$f'(x) = -8x^{-3} = \dfrac{-8}{x^3}.$

e. let $\angle CBE$ be $y°$

then $\angle ABD$ is $y°$

$\therefore \angle ABC = 2y°$

Similarly let $\angle BAC$ be $x°$

then $\angle CAD$ is also $x°$

$\therefore \angle BAD = 2x°$

$2x° + 2y° = 180°$ (co-interior)

$2(x° + y°) = 180°$

$x + y = 90°$

So in ΔBAE

it is composed of $\angle y, \angle x$ and $\angle BEA$.

$\therefore y + x + \angle BEA = 180°$

$(90°) + \angle BEA = 180°$

$\therefore \angle BEA = 90°$

(5)

13.

a) $x^3 - 27$.

Factored Form

$= (x-3)(x^2+3x+9)$

b) $\sin\theta(1+\cot^2\theta)\tan\theta$

$= \sin\theta\operatorname{cosec}^2\theta\tan\theta$

$= \sin\theta\left(\frac{1}{\sin^2\theta}\right)\left(\frac{\sin\theta}{\cos\theta}\right)$

$= \frac{1}{\cos\theta} = \sec\theta$

c). i) $\frac{d}{dx}(x^2+5)^4$

$= 4(x^2+5)^3 \times 2x.$

$= 8x(x^2+5)^3.$

ii) $x\sqrt{x} = x(x^{\frac{1}{2}})$

$= x^{\frac{3}{2}}.$

So $\frac{d}{dx}\left(x^{\frac{3}{2}}\right)$

$= \frac{3}{2}x^{\frac{1}{2}}. = \frac{3\sqrt{x}}{2}.$

d). $y = \frac{2x}{x+3}.$

Using quotient rule.

$u = 2x \quad v = x+3$

$\frac{du}{dx} = 2 \quad \frac{dv}{dx} = 1.$

$\frac{vu' - uv'}{v^2}$

$\frac{2(x+3) - (2x)}{(x+3)^2}$

$= \frac{2x+6-2x}{(x+3)^2}$

$= \frac{6}{(x+3)^2}$

So when $x = -2$.

gradient $= \frac{6}{(-2+3)^2} = 6.$

e) 2 ways to do this.

1. By inspection.

$-b = 1 \rightarrow b = -1.$

$b^2 - 4ac = 5$

$2a = 2. \rightarrow a = 1$

$1 - 4(1)(c) = 5$

(6)

$1 - 4c = 5$

$-4c = 4$

$c = -1.$

$\therefore$ equation is.

$x^2 - x - 1 = 0$

Method 2. recombine.

$\left(x - \left(\frac{1+\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{1-\sqrt{5}}{2}\right)\right) = 0$

$x^2 + \left(\frac{1+\sqrt{5}}{2}\right)\left(\frac{1-\sqrt{5}}{2}\right) - x\left(\frac{1-\sqrt{5}}{2}\right) - x\left(\frac{1+\sqrt{5}}{2}\right)$

$= x^2 + \frac{(1+\sqrt{5})(1-\sqrt{5})}{4} - \frac{x+x\sqrt{5}}{2} - \frac{x-x\sqrt{5}}{2}.$

$= x^2 + \left(\frac{-4}{4}\right) - \frac{2x}{2}$

$= x^2 - x - 1 = 0$ {Same as method 2}

14.

a). $y = (x+3)^2 + 4.$

Vertex. $= (-3, 4)$ $(h = -3,\ k = 4).$

Confirm by expanding.

$x^2 + 9 + 6x + 4.$ $\rightarrow$ $x = \frac{-b}{2a} = \frac{-6}{2} = -3 \rightarrow x$

$x^2 + 6x + 13$ $\quad 9 - 18 + 13 = 4. \rightarrow y.$

b). $f(x) = x^2 + \frac{x}{2}$.

$f(2) = 4 + \frac{2}{2} \Rightarrow 5$

$f'(x) = 2x + \frac{1}{2}$.

$f'(2) = 4 + \frac{1}{2} \Rightarrow 4\frac{1}{2}$.

$f(2) + f'(2) = 5 + 4.5 = 9.5$

c) with the restriction of

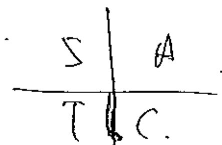
$0 \leq \theta \leq 360°$

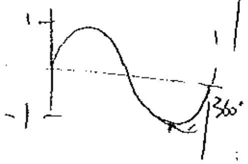
i) $(\sin\theta + 1)(\cos\theta - 1) = 0$

we have $\sin\theta = -1$ ①

OR $\cos\theta = 1$. ②

Case 1.





So $\theta = 270°$

Case 2. $\cos\theta = 1$.

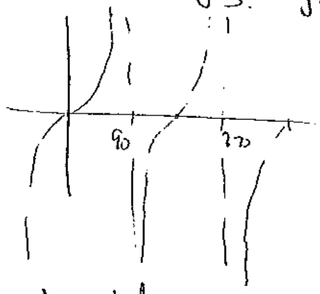
only at $0°$, 360

So solutions are $0°, 270°, 360°$

ii) $3\tan^2\theta - 1 = 0$.

$\tan^2\theta = \frac{1}{3}$.

$\tan\theta = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$.



4 solutions.

$\Rightarrow 330°, 150°$

$30°, 210°$.

d) $\cos\theta = -\frac{2}{3}$.

$\cos^2\theta + \sin^2\theta = 1$.

$\frac{4}{9} + \sin^2\theta = 1$.

$\sin^2\theta = \frac{5}{9}$.

$\therefore \sin\theta = \frac{\sqrt{5}}{3}$ (since > 0)

thus $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \frac{\left(\frac{\sqrt{5}}{3}\right)}{\left(\frac{-2}{3}\right)} = \frac{\sqrt{5}}{-2}$.

$x^2 + 8x + 16 - y^2$

$= (x+4)^2 - y^2$

Sum of two squares.

$= (x+4-y)(x+4+y)$

15. Graphing and linear equations.

a) Taking A and B as reference points (as they both lie on ℓ)

$A(-3,0)$ $\quad \frac{4-0}{0-(-3)} = \frac{4}{3}$.

$B(0,4)$

b). $y - y_1 = m(x - x_1)$ {point gradient form}

$y - 4 = \frac{4}{3}(x)$

$3y - 12 = 4x$.

$4x - 3y + 12 = 0$

c). let C be (x,y).

$\frac{x-3}{2} = 0 \Rightarrow x = 3$

$\frac{y-0}{2} = 4 \Rightarrow y = 8$

So $C = (3, 8)$

d) Perpendicular Distance Formula.

$D(0,6)$.

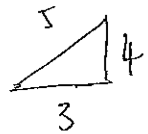
$d = \frac{|4(0) - 3(6) + 12|}{\sqrt{16+9}}$

$= \frac{|-18+12|}{5}$

$= \frac{6}{5}$

e) area $= \frac{1}{2}bh$.

for base



base = 5

area $= \frac{1}{2}(5)\left(\frac{6}{5}\right) = 3u^2$.

f) Perpendicular Bisector of AB has the conditions.

1. gradient of $-3/4$ (negative reciprocal of ℓ)
2. hits the midpoint of AB [Bisector]

The midpoint of AB is $\frac{-3+0}{2}, \frac{0+4}{2}$

$= -\frac{3}{2}, 2$.

point gradient formula.

$y-2 = -\frac{3}{4}\left(x+\frac{3}{2}\right)$

$4y-8 = -3\left(x+\frac{3}{2}\right)$

$4y-8 = -3x - \frac{9}{2}$

$3x+4y-\frac{7}{2} = 0$.

16. a) $\frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{4(3)}{3}\pi r^2$

$= 4\pi r^2$

b) $y = x^3 - 3x^2 + 3x$

$\frac{dy}{dx} = 3x^2 - 6x + 3$

ie the values of x where

$3x^2 - 6x + 3 = 0$.

$\cancel{3}(x^2 - 2x + 1) = 0$

$x^2 - 2x + 1$

$(x-1)^2 = 0$

So when $x = 1$, the tangent is horizontal.

c). Area of ΔPQR.

$\frac{1}{2}ab \sin C$.

$= \frac{1}{2}(4.8)(7.3)\sin 47°$

$\approx 12.8u^2$.

ii) length of RP.

using Cosine rule

$RP^2 = 4.8^2 + 7.3^2 - 2(4.8)(7.3)\cos 47°$

$RP^2 = 28.53$

$RP \approx 5.3$ (1dp.)

d). i) $\angle APB = 180 - 50 - 55 = 75°$

ii) Sine rule.

$\frac{37}{\sin(75)} = \frac{20}{\sin(\angle PAB)}$.

$\angle PAB = 31° 28'$ (nearest minute)

17. $\frac{(n-2) \times 180°}{n} = 165$

$165n = 180n - 360°$

$15n = 360°$

$n = 24$ sides

b). $x^2 + y^2 - 4y - 1 = 0$

$x^2 + (y-2)^2 - 4 - 1 = 0$.

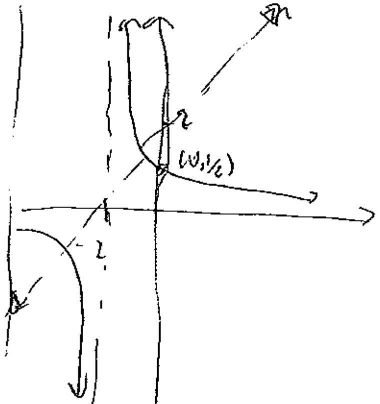
$x^2 + (y-2)^2 = 5$

centre of $(0, 2)$

radius of $\sqrt{5}$ units.

c) i) $y = \frac{1}{x+2}$.

$y - x = 2$.



Intersection Point.

$2+x = \frac{1}{x+2}$.

$(x+2)^2 = 1$

$x^2 + 4 + 4x - 1 = 0$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -3, -1$

$\therefore y = -1, 1$

intercepts are

$(-3, -1)$ AND $(-1, 1)$

iii) Find tangent at $(0, \frac{1}{2})$ first.

$y = (x+2)^{-1}$

$\frac{dy}{dx} = -(x+2)^{-2}$

The point where it crosses the y-axis is when $x = 0$

ie $y = \frac{1}{0+2} = \frac{1}{2}$

so $(0, \frac{1}{2})$

$M_{Tangent} = -(2)^{-2} = -\frac{1}{2^2} = -\frac{1}{4}$

$\therefore M_{normal} = 4$, as both are negative reciprocals of each other.

ie the equation is

$y - \frac{1}{2} = 4(x)$

11

$2y - 1 = 8x$

$2y - 8x - 1 = 0$

18.

i) $\angle A = x$

$\therefore \angle BDA = \cancel{x}$

$\therefore \angle ABD = 180 - 2x$ (angle sum of triangle)

$\therefore \angle CBD = 2x$ (angle sum of straight line)

ii) $\angle DEC = 45°$

OPTIONAL REASONING.

$\angle DBC = 2x$

thus $\angle BDC = \angle BCD = \frac{180 - 2x}{2}$

$= 90 - x$

therefore $\angle CDA = 90 - x + x = 90°$

$\therefore \angle CDE = 90°$ (straight line)

$\therefore \angle DCE = \angle DEC = 45°$

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b) $\lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$

expanding

$3(x^2 + h^2 + 2xh) - 3x^2$

$\lim_{h \to 0} \frac{= \cancel{3x^2} + 3h^2 + 6xh - \cancel{3x^2}}{h}$

$\lim_{h \to 0} \frac{3h^2 + 6xh}{h}$

$\lim_{h \to 0} 3h + 6x$

$= 3(0) + 6x$

$= 6x$

c) RTP $\sec\theta - \sin\theta \tan\theta = \cos\theta$

$\frac{1}{\cos\theta} - \sin\theta \left(\frac{\sin\theta}{\cos\theta}\right)$

$= \frac{1}{\cos\theta} - \frac{\sin^2\theta}{\cos\theta}$

$= \frac{(1 - \sin^2\theta)}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} = \cos\theta$

d) Differentiate

$y = x^2(x^3 - 1)^4$

product rule.

$u = x^2$

$v = (x^3 - 1)^4$

$\frac{du}{dx} = 2x$

$\frac{dv}{dx} = 4(x^3 - 1)^3(3x^2)$

$v\frac{du}{dx} + u\frac{dv}{dx}$

$= x^2\left[4(x^3-1)^3(3x^2)\right] + 2x(x^3-1)^4$

$= (4x^2)(3x^2)(x^3-1)^3 + 2x(x^3-1)^4$

$= 12x^4(x^3-1)^3 + 2x(x^3-1$

$= 2x(x^3-1)^3\left[6x^3 + (x^3-1)\right]$

$= 2x(x^3-1)^3\left[7x^3 - 1\right]$