+ SOL'NS

Year 11 Extension 1 Mathematics Name: ____________

Polynomials Test

Wednesday October 25, 2006 **Teacher: HRK HRK/CAB JJA/BMM**

Question 1. ***Marked by CAB***

a) Factorise $3x^3 + 3x^2 - x - 1$ **2**

b) Given that $x = 1$ is a zero of the polynomial $P(x) = x^3 - 11x^2 + 31x - 21$

i. Express $P(x)$ as a product of three linear factors. **2**

ii. Sketch $P(x)$ **1**

iii. Hence solve the inequality $x^3 - 11x^2 + 31x - 21 \leq 0$ **1**

Question 2. ***Marked by BMM***

a) Find the remainder when the polynomial $P(x) = x^3 - 4x$ is divided by $x + 3$

i. Using long division **2**

ii. Using the remainder theorem **1**

b) The polynomial $P(x) = x^3 + ax + 12$ has a factor $(x + 3)$. Find the value of a. **2**

Question 3. ***Marked by JJA***

a) A polynomial is given by $p(x) = x^3 + ax^2 + bx - 18$. Find values for a and b if $(x + 2)$ is a factor of $p(x)$ and if -24 is the remainder when $p(x)$ is divided by $(x - 1)$ **4**

b) Find the cubic polynomial with a double root at $x = 1$ and a root at $x = 7$ that crosses the y axis at 21. **3**

c) The polynomial equation $P(x) = 0$ has a double root at $x = a$ and is monic.

i. By writing $P(x) = (x - a)^2 Q(x)$, where $Q(x)$ is a polynomial, show that $P'(a) = 0$. **2**

ii. Hence or otherwise find the values of a and b if $x = 1$ is a double root of $x^4 + ax^3 + bx^2 - 5x + 1 = 0$

4

Test continues over the page

Question 4. *Marked by HRK*

a) If α, β and γ are the roots of $x^3 - 3x + 1 = 0$ find: **5**

i. $\alpha + \beta + \gamma$

ii. $\alpha\beta + \beta\gamma + \gamma\alpha$

iii. $\alpha\beta\gamma$

iv. $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$

b) When the polynomial $P(x)$ is divided by $(x + 1)(x - 4)$, the quotient is $Q(x)$ and the remainder is $R(x)$.

i. Why is the most general form of $R(x)$ given by $R(x) = ax + b$? **1**

ii. Given that $P(4) = -5$, show that $R(4) = -5$. **4**

iii. Further, when $P(x)$ is divided by $(x + 1)$, the remainder is 5. Find $R(x)$ **3**

c) $P(x)$ is a monic polynomial of the fourth degree.
When $P(x)$ is divided by $x + 1$ and $x - 2$, the remainders are 5 and -4 respectively. Given that $P(x)$ is an even function [ie. one where $P(x) = P(-x)$] **6**

i. Express it in the form $ax^4 + bx^3 + cx^2 + dx + e$

ii. Find all the zeros of $P(x)$

End of test

/6

QUESTION 1

a) Factorise $3x^3+3x^2-x-1$

$\Rightarrow 3x^2(x+1)-(x+1)$

$= (x+1)(3x^2-1)$

or $= (x+1)(\sqrt{3}x-1)(\sqrt{3}x+1)$

b) $P(x) = x^3-11x^2+31x-21$

$x=1$ is a zero

i.

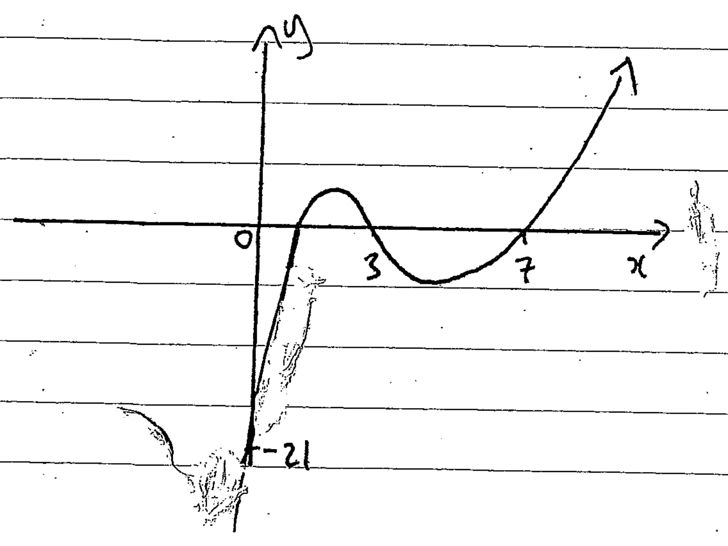
$$\begin{array}{r} x^2-10x+21 \\ (x-1)\,\overline{)\,x^3-11x^2+31x-21} \\ \underline{x^3-x^2} \\ -10x^2+31x \\ \underline{-10x^2+10x} \\ 21x-21 \\ \underline{21x-21} \\ 0 \end{array}$$

$x^2-10x+21$

$= (x-7)(x-3)$

$\therefore P(x) = (x-1)(x-7)(x-3)$

ii.



$(x-1)(x-7)(x-3)=0$

$x = 1, 3, 7$

iii. $x^3-11x^2+31x-21 \leq 0$

$(x-1)(x-7)(x-3) \leq 0$

From graph:

$x \leq 1$ AND $3 \leq x \leq 7$

Q2

(a) $P(x) = x^3 - 4x \quad (x+3)$

(i)
$$
\begin{array}{r|l}
 & x^2 - 3x + 5 \\
\hline
x+3 & x^3 + 0x^2 - 4x + 0 \\
 & x^3 + 3x^2 \\
\hline
 & -3x^2 - 4x \\
 & -3x^2 + 9x \\
\hline
 & 5x + 0 \\
 & 5x + 15 \\
\hline
 & -15
\end{array}
$$

(ii) $P(-3) = (-3)^3 - 4(-3)$

$= -27 + 12$

$= -15$

(b) $P(-3) = (-3)^3 - 3a + 12 = 0$

$-27 - 3a + 12 = 0$

$-15 - 3a = 0$

$-15 = 3a$

$a = -5$

Factor Theorem
Remainder Theorem

3.) a) $p(x) = x^3 + ax^2 + bx - 18$

$p(-2) = -8 + 4a - 2b - 18$

$0 = 4a - 2b - 26$ ✓

$\boxed{2a - b = 13}$ ①

Do not use long division!!

$P(1) = 1 + a + b - 18$

$-24 = a + b - 17$ ✓

$\boxed{a + b = -7}$ ②

$b = -7 - a$

$2a - (-7 - a) = 13$

$3a + 7 = 13$

$3a = 6$

$a = 2$ ✓ (4)

$a + b = -7$ ②

$2 + b = -7$

$b = -9$ ✓

b) $P(x) = k(x-1)^2(x-7)$ ✓ ✓

$x = 0, \quad P(0) = 21$

$21 = k(-1)^2(-7)$ (3)

$21 = -7k$

$k = -3$ ✓

$P(x) = -3(x-1)^2(x-7)$

Cannot do:

$P(x) \neq (x-1)^2(x-7) + 21$

$(x-1)$ and $(x-7)$ would not be factors if this were the case.

c) $P(x) = (x-a)^2 Q(x)$

Product Rule!

$P'(x) = 2(x-a)Q(x) + (x-a)^2 Q'(x)$ ✓

(2)

$P'(a) = 2 \times 0 \times Q(x) + 0 \times Q'(x)$ ✓

$= 0$

$P(x) = x^4 + ax^3 + bx^2 - 5x + 1$

$P(1) = 1 + a + b - 5 + 1$

$0 = a + b - 3$ ✓

$a + b = 3$ (1)

$P'(x) = 4x^3 + 3ax^2 + 2bx - 5$

$P'(1) = 4 + 3a + 2b - 5$

$\boxed{0 = 3a + 2b - 1}$ (2) ✓

$\boxed{a = 3 - b}$ (1)

$\Rightarrow 0 = 3(3-b) + 2b - 1$ (4)

$0 = 9 - 3b + 2b - 1$

$= 9 - b - 1$

$= 8 - b$

$b = 8$ ✓

$0 = a + 8 - 3$ ✓

$a = -5$

Common mistake: ~~finding~~ finding $P(-1)$ instead of $P(1)$ and $P'(-1)$ instead of $P'(1)$.

* 1ST WRITE

4 a) $x^3+0x^2-3x+1=0$

(5) i $\alpha+\beta+\gamma+\delta=0$ ✓

ii $\alpha\beta+\beta\gamma+\gamma\alpha=-3$ ✓

iii $\alpha\beta\gamma=-1$ ✓

iv $\dfrac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma} = \dfrac{-3}{-1}=3$ ✓

b) i deg $R(x)$ < deg (quotient)

(1) deg (Quotient) = 2

✓ $\therefore$ deg $R(x) < 2$

$\therefore R(x)=ax+b$ is most general form.

(4) ii ✓ $P(x)=(x+1)(x-4)\,Q(x)+ax+b$

✓ $P(4)=(4+1)(4-4)\,Q(4)+4a+b$

$=4a+b$

now

✓ $P(4)=-(5)$

✓ $\therefore\ 4a+b=-5$

ie $R(x)=-5$

iii $P(-1)=5$ ie $0-a+b=5$ (1)

(3) and from (ii) $P(4)=-5$ ✓

ie $4a+b=-5$ (2)

solve (1), (2) ✓

(1) − (2) $5a=-10$

$a=-2$

$\therefore\ b=3$ ✓

hence $R(x)=ax+b$

$=-2x+3$

c) $P(x)=ax^4+bx^3+cx^2+dx+e$

(6) Monic $\therefore a=1$ ✓

$P(-1)=5$ ✓

$P(2)=-4$ ✓

EVEN $\therefore P(x)=P(-x)$ $\therefore b=d=0$

ie $P(x)=x^4+\ \ x^2+e=0$

$P(-1)=(-1)^4+c(-1)^2+e=5$

$1+c+e=5$

$c+e=4$ (1)

$P(2)=2^4+c(2)^2+e=-4$

$16+4c+e=-4$

$4c+e=-20$ (2)

(2) − (1)

$3c=-24$ ✓

$c=-8$

$\therefore e=12$

$\therefore P(x)=x^4-8x^2+12$ ✓

$v^2-8v+12=0$

$(v-6)(v-2)=0$

$x^2=6,\ 2$

$\therefore x=\pm\sqrt{6},\ \pm\sqrt{2}$ ✓