ST SPYRIDON COLLEGE

2015
Year 11
Preliminary Assessment Task 1
Class Test
Monday 30th March

Mathematics Extension 1

Weighting: 25%

Working time: 80 minutes (plus 5 minutes reading time)

Total marks: 45

Topics examined:
Basic arithmetic and algebra
Other inequalities
Real functions

Outcomes assessed: P4, P5, PE3 and PE6

General instructions:
- Write using blue or black pen on the writing paper provided
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Questions are of equal value
- Full marks may not be awarded for careless or badly arranged work
- Questions are not necessarily arranged in order of difficulty
- Diagrams are not necessarily drawn to scale
- Begin each question on a new page

Total marks (45)
Attempt Questions 1 – 3
The questions are not of equal value
Answer each question on a SEPARATE page.

Question 1 (17 marks) — Marks

a. Simplify:

(i) $\dfrac{x^2+xy-12y^2}{x^2-9y^2}$ — 2

(ii) $f(g(x))$ if $f(x)=\dfrac{1-x}{2+x}$ and $g(x)=\dfrac{1}{x-3}$ — 2

b. Solve the inequality below:

$x^2-3x>10$ — 2

c. (i) Solve the inequality $\dfrac{5x}{x-4}\geq 0$ — 3

(ii) Hence, find the domain of $f(x)=\sqrt{\dfrac{5x}{x-4}}$ — 2

d. Solve the following:

(i) $|2x-5|\leq 7$ — 2

(ii) $|6-x|=2x+15$ — 2

Question 2 (15 marks) Begin writing on a new page. **Marks**

a. Consider $f(x)=\frac{2}{x}-x$. Find the exact value(s) of a for which $f(a+2)=2a$. **3**

b. Draw neat sketches showing the main features of each of the following functions;
Write down the **domain** and the **range** of the function next to each sketch.

(i) $y=4-(x-1)^2$ **3**

(ii) $y=\sqrt{4-(x-1)^2}$ **3**

(iii) $y=4-\frac{1}{x-1}$ **3**

c. (i) Draw a neat sketch of the graph $y=x^2-4x+3$ **1**

(ii) Find the equation of the curve $y=x^2-4x+3$ when it is:
shifted to the left by 2 units and down by 3 units. **2**

Identify the coordinates of the **vertex** of the new curve..

Question 3 (15 marks) Begin writing on a new page. **Marks**

a. Consider the function $f(x)=\begin{cases} -\sqrt{9-x^2} & -3\le x<0 \\ x-3 & 0\le x<3 \\ (x-4)^2 & x\ge 3 \end{cases}$

(i) Draw a neat sketch of $y=f(x)$ **4**

(ii) Write down the range of $y=f(x)$ **1**

b. Given that $x=\frac{2+\sqrt{3}}{2-\sqrt{3}}$,

(i) Show that $x+\frac{1}{x}=14$ **without** using a calculator. **2**

(ii) Hence, find the value of $x^2+\frac{1}{x^2}$ **1**

c. (i) On the same diagram, draw a neat sketch of the graphs, $y=|2x|$ and $y=|x+3|$. **3**
Show all intercept and points of intersection.

(ii) On your diagram, shade the region where $y\le|2x|$ and $y\ge|x+3|$ both hold. **2**

(iii) Hence, or otherwise use your graph to solve the inequality $\left|\frac{x+3}{2x}\right|\le 1$ **2**

END OF EXAMINATION

Question 1:

(a) (i) $\left.\begin{matrix} x & -12 \\ + & 1 \end{matrix}\right\} -3, 4.$

$$= \frac{(x-3y)(x+4y)}{(x-3y)(x+3y)}$$

$$= \frac{x+4y}{x+3y}$$

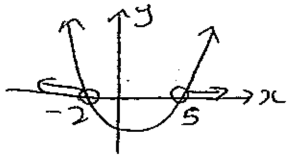
(ii) $f(g(x)) = \dfrac{1-\frac{1}{x-3}}{2+\frac{1}{x-3}} \times \dfrac{x-3}{x-3}$

$$= \frac{x-3-1}{2(x-3)+1}$$

$$= \frac{x-4}{2x-5}$$

(b) $x^2-3x-10>0$

$(x-5)(x+2)>0$

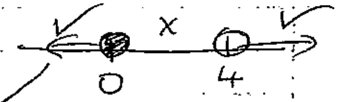


$\therefore x<-2,\ x>5$

(c) (i) $\dfrac{5x}{x-4} \geq 0$

$x-4 \neq 0$

$x \neq 4$



consider $\dfrac{5x}{x-4} = 0$

$5x = 0$

$x = 0$

Test $x=-1$ ✓

$x=1$ ×

$x=5$ ✓

$\therefore x \leq 0,\ x>4$

(ii) $f(x) = \sqrt{\dfrac{3x}{x-4}}$

D: $f(x) \geq 0$

$\dfrac{5x}{x-4} \geq 0$

$\therefore$ D: $x \leq 0,\ x>4$

(d) (i) $|2x-5| \leq 7$

$-7 \leq 2x-5 \leq 7$

$-2 \leq 2x \leq 12$

$-1 \leq x \leq 6.$

(ii) $|6-x| = 2x+15$

$6-x = 2x+15$

$3x = -9$

$x = -3$

$6-x = -2x-15$

$x = -21$

not a solution.

$\therefore x = -3.$

Question 2

(a) $f(a+2) = 2a$

$$\frac{2}{a+2} - (a+2) = 2a$$

$$2-(a+2)^2 = 2a(a+2)$$

$$2-a^2-4a-4 = 2a^2+4a$$

$$3a^2+8a+2=0.$$

$$a = \frac{-8 \pm \sqrt{8^2-4\times3\times2}}{2\times3}$$

$$= \frac{-8\pm\sqrt{40}}{6} = \frac{-8\pm2\sqrt{10}}{6}$$

$$= \frac{-4\pm\sqrt{10}}{3}.$$

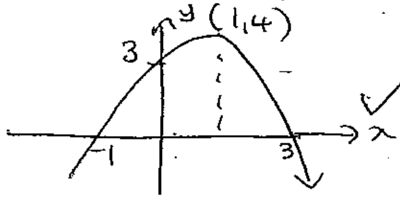
(b) (i) $y = 4-(x-1)^2$

vertex $(1,4)$

x-axis: $y=0$: $(x-1)^2 = 4$

$x-1 = 2, -2$

$x = 3, -1.$



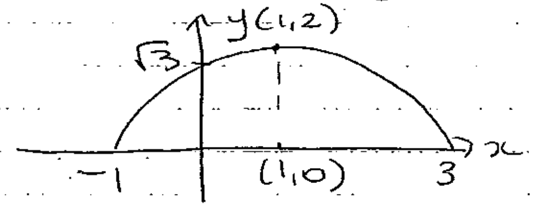
D: all real x

R: $y \leq 4$.

(ii) $y = \sqrt{4-(x-1)^2}$

centre $(1,0)$.
$r=2$

x-axis: $y=0$: $\quad 0 = 4-(x-1)^2$
$x = 3, -1$.
y-axis: $x=0$ $\quad y=\sqrt{3}$.

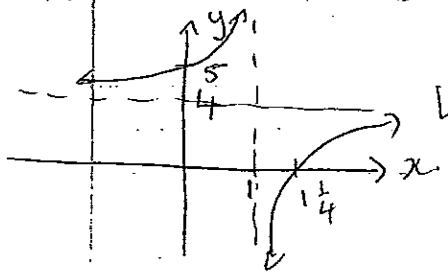


D: $-1 \le x \le 3$ ✓
R: $0 \le y \le 2$ ✓

(iii) $y = 4 - \frac{1}{x-1}$

asymptotes: $y=4$, $x=1$.
x-axis: $y=0$: $\quad 4 = \frac{1}{x-1}$

$4x-4=1$
$4x=5$
$x = 5/4$

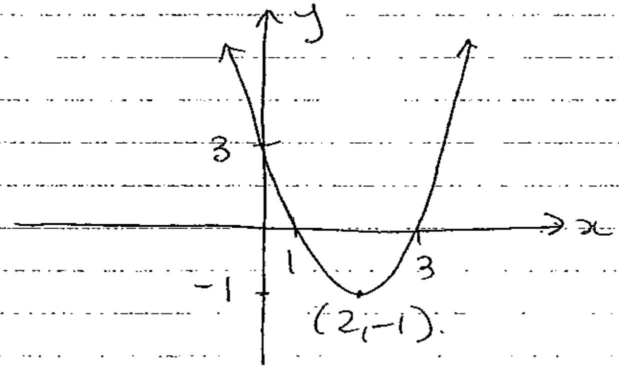


y-axis: $x=0$
$y = 4 - -1$
$y=5$

D: all reals $x \neq 1$ ✓
R: all reals $y \neq 4$ ✓

(c)(i) $y = x^2-4x+3$
$= x^2-4x+2^2-1$
$y = (x-2)^2-1$
vertex $(2,-1)$

x-int: $0 = (x-3)(x-1)$
$x = 3, 1$
y-int: $y=3$.



(ii) $y = (x-2+2)^2 - 1 - 3$
$y = x^2-4$ ✓
vertex $(0,-4)$. ✓

$y = x^2-1 \quad (1)$

Question 3:

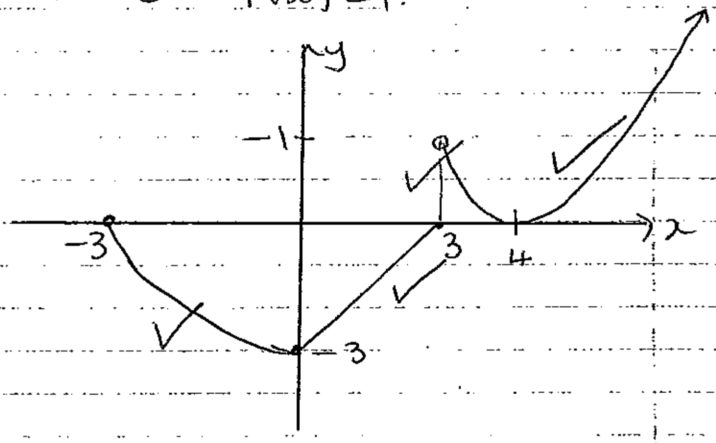
(a) $x=3 \quad f(x)=0$
$x=0 \quad f(x)=-3$

$x=0 \quad f(x)=-3$
$x=3 \quad f(x)=0$

$x=3 \quad f(x)=1$

$y=x-3$
$x=0: y=-3$
$y=0: x=3$

(i)



(ii) R: $y \geq -3$

(b) $x+\frac{1}{x}$

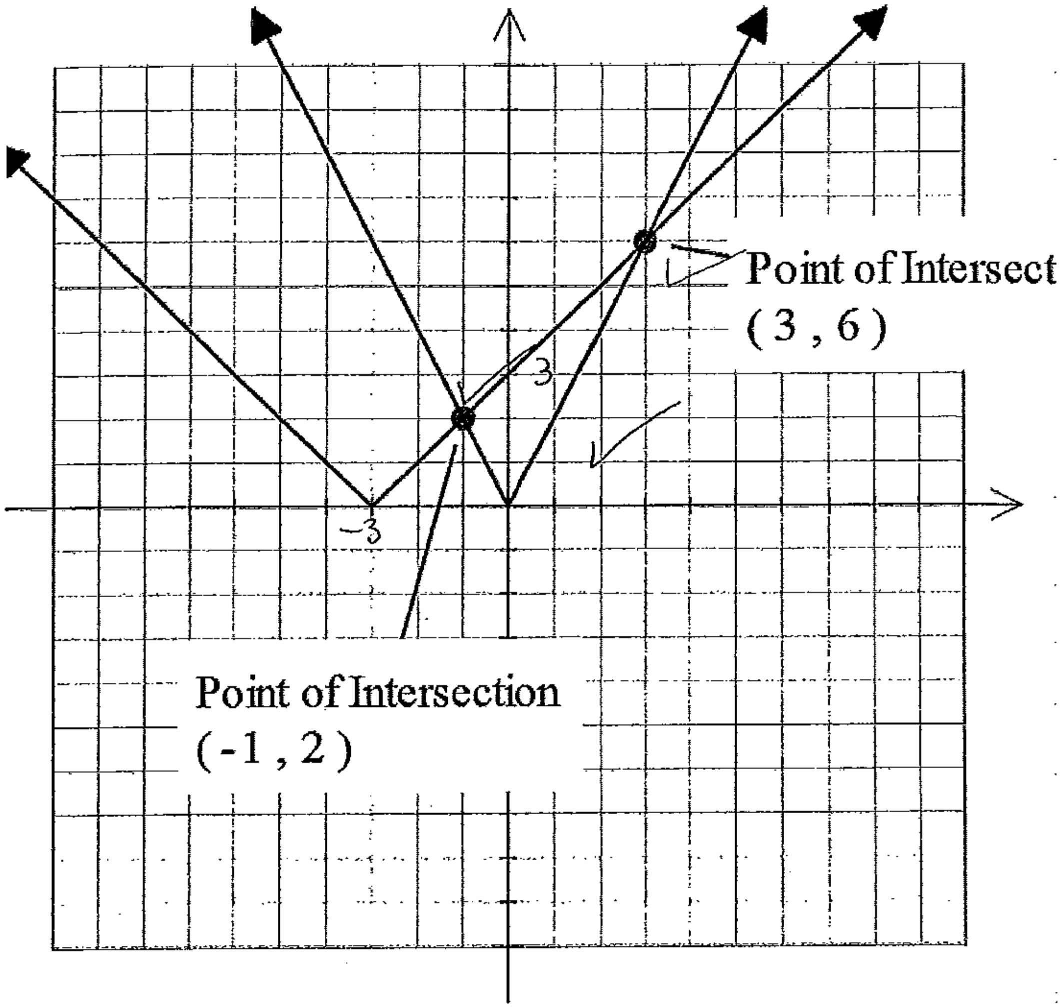
$$= \frac{2+\sqrt{3}}{2-\sqrt{3}} + \frac{2-\sqrt{3}}{2+\sqrt{3}}$$

$$= \frac{(2+\sqrt{3})^2+(2-\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})}$$

$$= \frac{4+4\sqrt{3}+3+4-4\sqrt{3}+3}{4-3}$$

$$= 6+8 = 14.$$

(c) (i)



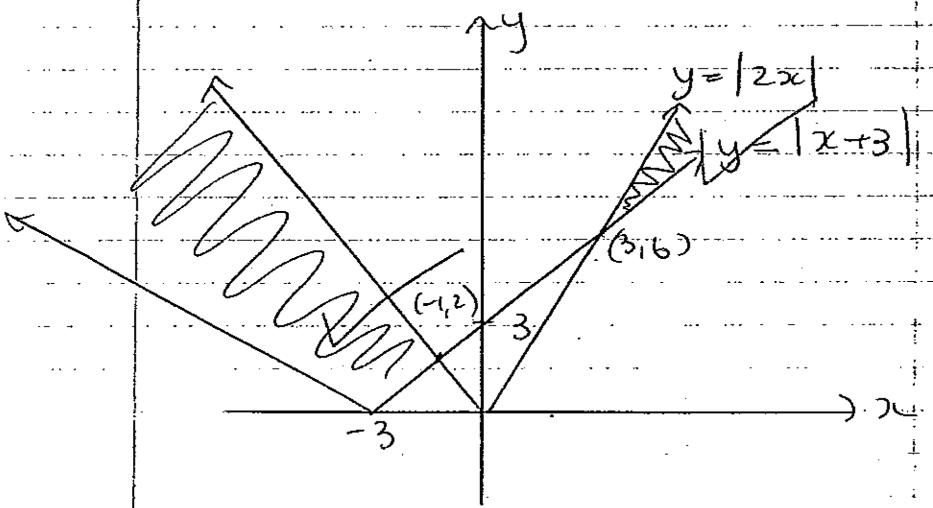
(ii) $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x}$

$= \left(x + \frac{1}{x}\right)^2 - 2$

$= 14^2 - 2$

$= 194$

(c) (ii)



both hold when: $x \leq -1$, $x \geq 3$.

(iii) $\left|\frac{x+3}{2x}\right| \leq 1$

$|x+3| \leq |2x|$

ie. $y \geq |x+3|$, $y \leq |2x|$

$\therefore x \leq -1$, $x \geq 3$