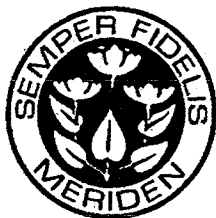


Year 11
Extension1 Mathematics
Assessment Task 1
March 2004



Time Allowed: 75 minutes

Outcomes Assessed:

- P1** - demonstrates confidence in using mathematics to obtain realistic solutions to Problems.
- P5** - understands the concept of a function and the relationship between a function and its graph.
- PE1** - appreciates the role of mathematics in the solution of practical problems.
- PE6** - makes comprehensive use of mathematical language, diagrams and notation for communicating in a wide context.

Instructions:

- Answer all questions.
- Write your name on each page
- Staple **Part A** and **Part B** separately
- Only **Board of Studies** approved calculators may be used.
- Marks might be deducted for careless and untidy work.
- Show all working.

Total Marks: 44

Part A (21 marks)

Q1.

Solve the following simultaneous equations: (leave your answer in exact form)

a) $x - y = 1,$
 $yx = 2$

b) $y = x^2 - 9,$
 $y = 15 - x^2$

(8 marks)

Q2.

Solve the following inequalities :

a) $\frac{3x+2}{x-2} \geq 2$

b) $\frac{x}{x^2-1} \geq 0$

(6 marks)

Q3.

Find the centre and the radius of the circle given by:

$$x^2 - 6x + y^2 + 10y - 15 = 0$$

(3 marks)

Q4.

On the same number plane sketch:

i) $y = |2x-1|$, and $y = |x+1|$

ii) Hence or otherwise, solve $|2x-1| \leq |x+1|$

(4 marks)

START A NEW PAGE

Part B (23 marks)

Q5.

$$\text{Given } f(x) = \begin{cases} x & \text{if } x > 3 \\ x^2 & \text{if } 1 \leq x \leq 3 \\ 5 & \text{if } x < 1 \end{cases}$$

Find

i) $f(3)$

ii) $f(4)$

iii) $f(0)$

(3 marks)

Q6.

For each of the following functions $f(x) = \frac{x}{4-x^2}$ and $g(x) = \frac{1}{\sqrt{x-2}}$

i) State the domain and range.

ii) Determine if the function is odd, even or neither.

iii) Find any vertical or horizontal asymptotes.

iv) Sketch a neat graph for each function separately, showing all important features.

(10 marks)

Q7.

On separate number planes, shade the region that satisfies the following simultaneously:

a) $x^2 + y^2 \leq 9$, $y \geq x^2$, and $y < x + 1$

b) $y < 10^x$, $y \geq \log_{10} x$, and $y \geq 5$

(6 marks)

Q8.

Evaluate the following limits:

a) $\lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3}$

b) $\lim_{x \rightarrow \infty} \frac{x^2 + 2}{3x^2 - 5}$

(4 marks)

Ext. 1 Ass. Task 1.

Part A Q. 1.

a) $x - y = 1 \Rightarrow x = y + 1$ ①

$xy = 2$

$\therefore (y+1)y = 2$ ①

$y^2 + y - 2 = 0$ ①

$(y+2)(y-1) = 0$

$y = -2 \frac{1}{2}$

OR $y = 1 \frac{1}{2}$

$x = -1 \frac{1}{2}$

$x = 2 \frac{1}{2}$

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b) $y = x^2 - 9$ ①

$y = 15 - x^2$ ②

① + ②

$2y = 6$

$\therefore y = 3$ ①

$3 = x^2 - 9$ ①

$x^2 = 12$

$x = \pm 2\sqrt{3}$ ②

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Q. 2.

a) $\frac{3x+2}{x-2} \geq 2$

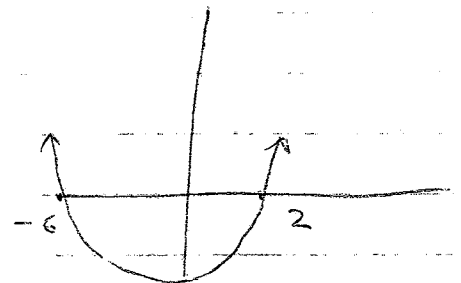
Multiply B.S. by $(x-2)^2$ ①

$(3x+2)(x-2) \geq 2(x-2)^2$

$(x-2)(3x+2 - 2(x-2)) \geq 0$

$(x-2)(x+6) \geq 0$

$x \geq 2$ ① $x \leq -6$ ①



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b) $\frac{x}{x^2-1} \geq 0$

$x(x^2-1)^2$

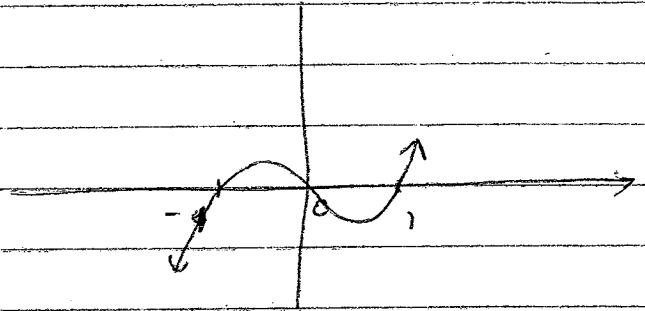
$\therefore x(x^2-1) \geq 0$ (1)

$x(x-1)(x+1) \geq 0$

$x > 1$ OR $-1 < x \leq 0$

(1)

(1)



Q3.

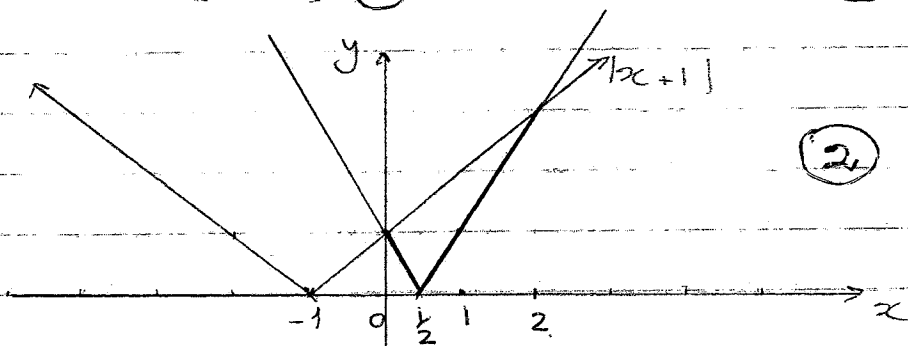
$x^2 - 6x + y^2 + 10y - 15 = 0$

$x^2 - 6x + 9 + y^2 + 10y + 25 = 15 + 9 + 25$ (1)

$(x-3)^2 + (y+5)^2 = 49$

\therefore centre $(3, -5)$ (1) radius = 7 (1)

Q4.



(2)

$y = |2x - 1|$

$|2x + 1| \leq |x + 1|$

$2x - 1 \leq x + 1$

$x \leq 2$ ✓

$\therefore 0 \leq x \leq 2$

$2x - 1$

||2.

Part B.

Q5 $f(3) = 9$
 $f(4) = 4$
 $f(0) = 5$

Q6 $f(x) = \frac{x}{4-x^2}$

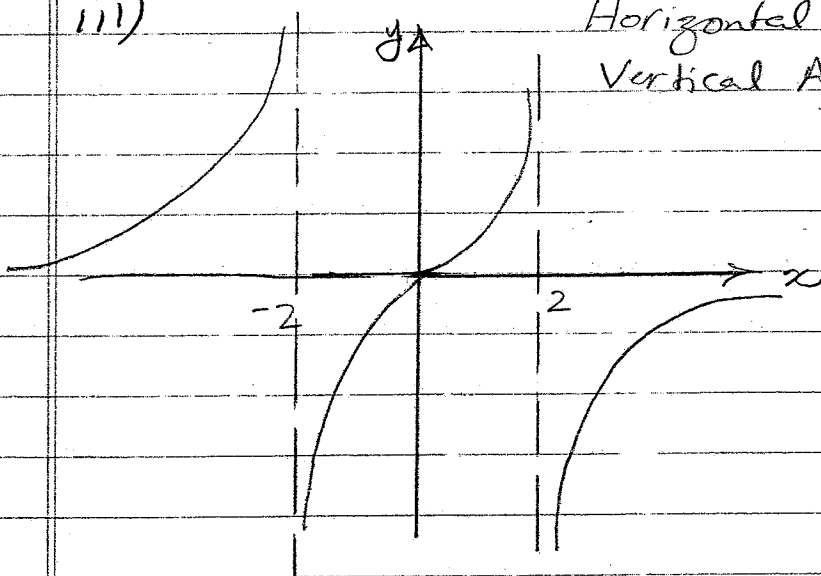
i) Domain $x \neq \pm 2$.
Range: All y .

ii) $f(-x) = \frac{-x}{4-x^2}$

$\therefore = -f(x)$

$\therefore f(x)$ is an odd function.

iii)



Horizontal Asymptotes $y = 0$

Vertical Asymptotes $x = 2$

$x = -2$

$$g(x) = \frac{1}{\sqrt{x-2}}$$

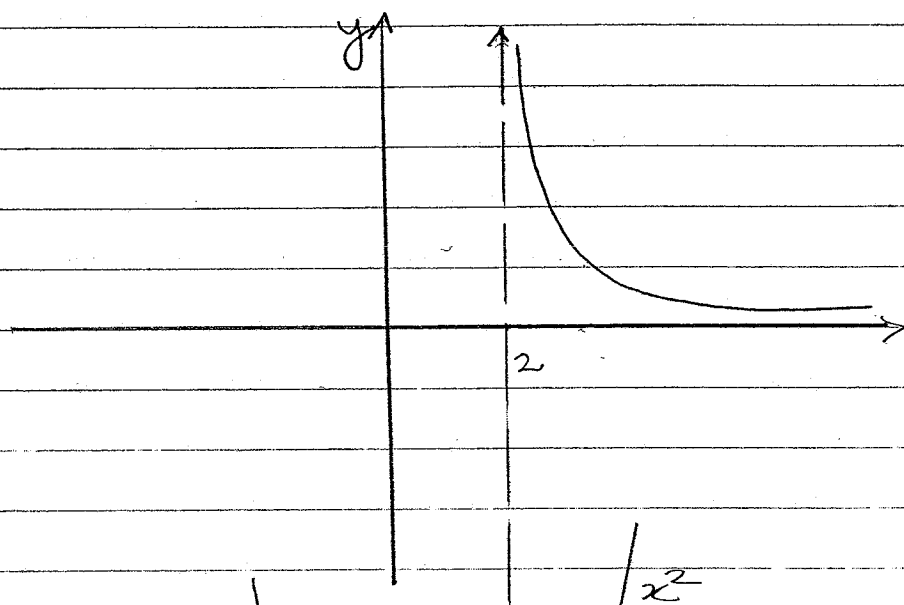
i) Domain $x > 2$.
Range:

ii) $g(x) = \frac{1}{\sqrt{x-2}}$ $g(-x)$ not defined.

$\therefore g(x)$ is neither odd nor even.

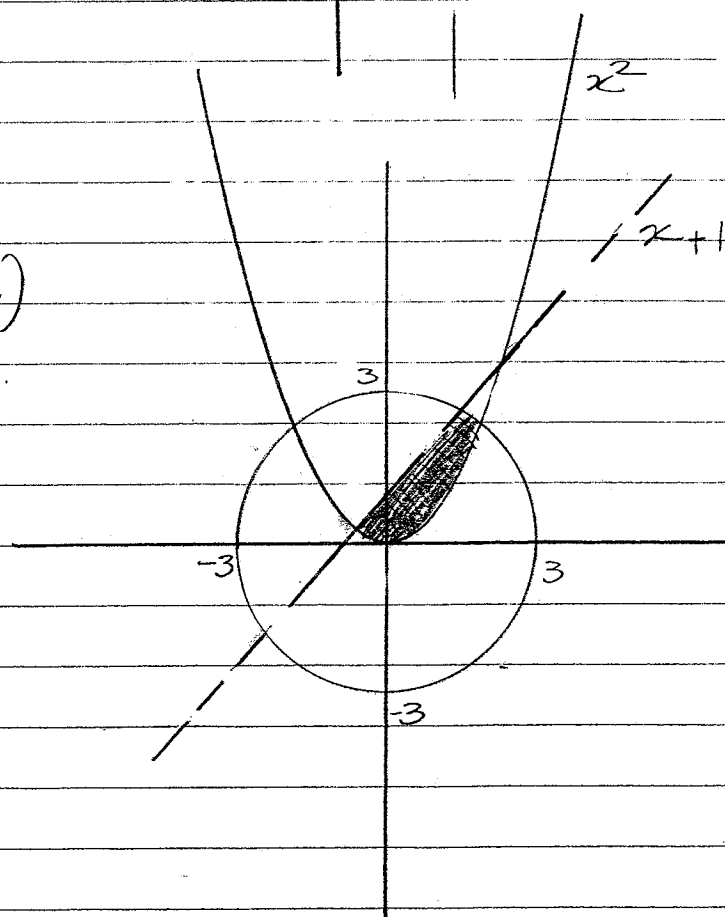
$$\lim_{x \rightarrow \infty} g(x) = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{x-2}}$$
$$= 0$$

$\therefore y=0$ is a horizontal asymptote
vertical asymptote $x=2$

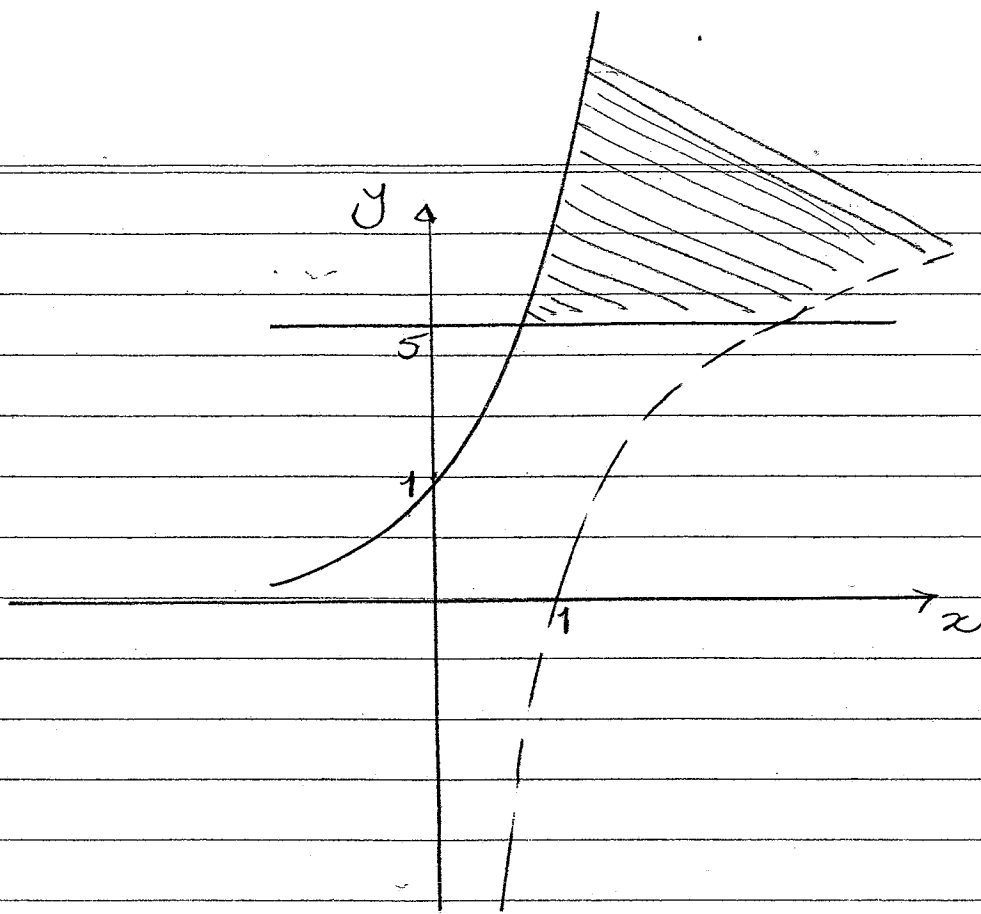


Q7.

a)



b)



Q 8.

$$\begin{aligned}
 a) \quad & \lim_{x \rightarrow 3} \frac{x^2 - 2x - 3}{x - 3} \\
 & = \lim_{x \rightarrow 3} \frac{(x-3)(x+1)}{(x-3)} \\
 & = 4
 \end{aligned}$$

$$\begin{aligned}
 b) \quad & \lim_{x \rightarrow \infty} \frac{x^2 + 2}{3x^2 - 5} \\
 & = \lim_{x \rightarrow \infty} \frac{1 + \frac{2}{x^2}}{3 - \frac{5}{x^2}} \\
 & = \frac{1 + 0}{3 - 0} \\
 & = \frac{1}{3}
 \end{aligned}$$