Measurement error

There is always some amount of measurement error involved in reading scales, especially when the quantity being measured doesn't finish exactly on a scale line. This type of measurement error is known as a calibration error. Parallax error (human error) is another way error can be introduced into a measurement. Try to avoid looking at a scale from the side and use a ruler to help you line up scale markings to avoid parallax error. An error that cannot be avoided is the error due to the limitations of the measuring device itself.

Remember

When a scale is used correctly, the maximum possible error in the measurement is half (or 0·5) of the measurement unit used. It could be smaller or larger. Some people record the maximum error as ±0·5.

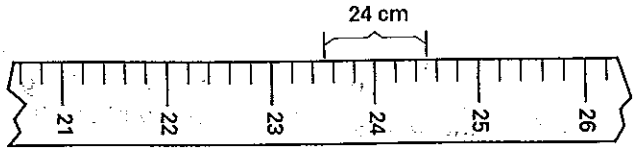
Example 10

When David fell off a ladder in a building accident he landed in some broken glass. The hospital's casualty staff recorded that he had a 24 cm long cut on his back, **correct to the nearest centimetre.**

a What was the shortest length the cut could have been?

b Within what range of values did the actual length of the cut lie?

Solution 10



24 cm to the nearest 1 cm. The maximum error possible is half of 1 cm, or 0·5 cm.

The diagram shows that lengths greater than 23·5 cm but less than 24·5 cm are recorded as 24 cm correct to the nearest centimetre.

a The shortest the cut could have been was 23·5 cm.

b The cut was at least 23·5 cm but shorter than 24·5 cm.

Did you know?

The measurement 'inch' (2·5 cm) comes from the Roman word meaning 'the length from the knuckle of the thumb to the thumb nail'. However, basing measurement units on body sizes didn't work that well as people are different sizes.

Worksheet 4.5

1 The largest living thing in the world is 'General Sherman', a giant redwood tree in California. Its height is 83 m correct to the nearest m.

a What is the smallest the height of the tree could be?

b Within what range is the actual height of the tree?

c What is the maximum possible measurement error involved in recording General Sherman's height as 83 metres?

2 The radius of the Earth is often given as 6400 km, correct to the nearest 100 km.

a What is the smallest the radius could actually be?

b Within what range is the actual radius of the Earth?

c What is the maximum possible error when the radius of the Earth is given as 6400 km, correct to the nearest 100 km?

3 Complete this table.

	Measurement	Shortest it could be	Actual range	Maximum possible error
a	6 km to the nearest km	i	ii	iii
b	43 cm to the nearest cm	i	ii	iii
c	8.1 m to 1 decimal place	i	ii	iii
d	6·38 m to the nearest cm	i	ii	iii

Percentage error in measurement

$$\text{Percentage error} = \frac{\text{error}}{\text{actual size}} \times 100$$

Example 11

David's height is 178.3 cm but he always writes it as 178 cm. Calculate the percentage error involved.

Solution 11

The error $= 178{\cdot}3 - 178$

$= 0{\cdot}3$ cm

$$\text{Percentage error} = \frac{0{\cdot}3}{178{\cdot}3} \times 100$$

$= 0.168\%$ correct to 3 decimal places

Example 12

Noel's weight is 87 kg, correct to the nearest kilogram.

a What is the smallest weight he could be?

b What is the maximum possible error in the measurement of his weight?

c Calculate the percentage error in this measurement, correct to 3 decimal places.

Solution 12

a The smallest weight = 86·5 kg

b The maximum possible error = 0·5 kg

c The percentage error $= \frac{0{\cdot}5}{87} \times 100$

$= 0{\cdot}575\%$

Worksheet 4·6

1 Bob always has trouble reading the scale on his tape measure.

a Bob measured a 60 cm piece of timber as 60·5 cm.

i What was the size of the error in Bob's measurement?

ii Calculate the percentage error in his measurement, correct to 1 decimal place.

b Bob measured a 5 cm piece of plastic as 5·5 cm.

i What was the size of the error in Bob's measurement of the piece of plastic?

ii Calculate the percentage error in this measurement.

2 Before the metric system of measurements was introduced in Australia, feet and inches were commonly used to measure lengths. An inch is actually 25·4 mm but it is usually recorded as 25 mm.

a What is the error in the recorded measurement of 25 mm?

b Calculate the percentage error involved in this approximation.

3 When 28·75 is written correct to the nearest whole number:

a What is the error in the approximation?

b What is the percentage error in this approximation?

4 Sister Marion made the following entries in the hospital stocktake register of the contents of the supply cupboard. By completing the missing columns calculate the percentage error in each entry.

	Number in stack	Recorded to the nearest	Maximum possible error	Maximum percentage error
Roll of bandages	450	50		
Safety pins	1500	100		
Hot water bottles	40	10		
Bed pans	90	10		

5 The length of a match is 4 cm, correct to the nearest centimetre.
   a What is the smallest length the match could actually be?
   b Within what range of values is the actual length of the match?
   c Calculate the percentage error involved in this measurement.

6 A more accurate measurement of the length of a match is 4·3 cm, correct to 1 decimal place.
   a What is the smallest length this match could be?
   b Calculate the percentage error in this measurement.

7 After Dr Hamilton recorded Robyn's weight in kg she used this calculation to determine the percentage error in the measurement.

$$\text{Percentage error} = \frac{0{\cdot}5}{56} \times 100$$

   a What is the maximum possible error in this measurement?
   b What is the smallest value Robyn's weight could be?

Significant figures

Rounding-off large or small numbers

A calculator will often give answers with many decimal places. To give a justifiable degree of accuracy, answers are often written correct to a given number of decimal places.

For example: 13·142 135 correct to 2 decimal places is 13·14 and
15·1871 correct to 2 decimal places is 15·19

Note: the 8 goes up to a 9 because the digit in the 3rd decimal place is greater than 5.

Significant figures are another way to round off measurements appropriately.

Jon said 'the distance from the Earth to the Sun is 149 496 783 km'. Unless Jon was very sure of his facts a more reasonable distance would be:

149 496 783 km = 149 000 000 km correct to 3 significant figures or
= 150 000 000 km correct to 2 significant figures

Rule
When reading from left to right the first significant figure is the first non-zero digit.

Example 13
a Write 138·41 correct to 2 significant figures.
b Express 0·002 613 correct to 2 significant figures.

Solution 13
a 138·41 ≈ 140 correct to 2 significant figures
b 0·002 613 ≈ 0·0026 correct to 2 significant figures