Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics Extension 1

Preliminary Course

Assessment 1

May, 2017

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- BOSTES reference sheet is located at the end of the exam.

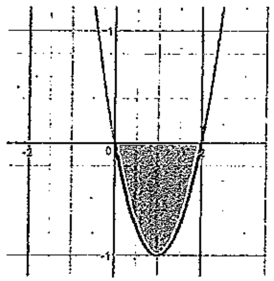
Section I	Multiple Choice Questions 1-5 5 Marks
Section II	Questions 6-11 60 Marks

Section 1 - Multiple Choice – (5 marks)

Answer on the sheet provided

Allow approximately 10 minutes for this section

1. The diagram shows the graph of the function $y = x^2 - 2x$



Which pair of inequalities specify the shaded region?

A. $y \le x^2 - 2x$ and $y \le 0$

B. $y \le x^2 - 2x$ and $y \ge 0$

C. $y \ge x^2 - 2x$ and $y \le 0$

D. $y \ge x^2 - 2x$ and $y \ge 0$

2. The graph with equation $y = x^2$, is translated 2 units down and 3 units to the right. Which equation represents the resulting graph?

A. $y = (x-3)^2 + 2$

B. $y = (x+3)^2 + 2$

C. $y = (x+3)^2 - 2$

D. $y = (x-3)^2 - 2$

3. What is the domain of the function $f(x) = \sqrt{x+1} - \sqrt{x+2}$

   A. $x \leq -2$

   B. $x \geq -1$

   C. $-1 \leq x < -2$

   D. $x \leq -2$ or $x \geq -1$

4. What is $8^3 \times 6^{1/2} \div 32^{3/2}$ in simplest form?

   A. $4\sqrt{3}$

   B. $2\sqrt{3}$

   C. $3\sqrt{2}$

   D. $4\sqrt{2}$

5. What are the solutions of $3x^2 - 7x - 1 = 0$?

   A. $x = \frac{-7\pm\sqrt{61}}{6}$

   B. $x = \frac{-7\pm\sqrt{37}}{6}$

   C. $x = \frac{7\pm\sqrt{61}}{6}$

   D. $x = \frac{7\pm\sqrt{37}}{6}$

End of section 1

Section II

Answer questions in booklet provided.

Start each question on a new page

Allow approximately 80 minutes for this section

<u>Question 6: (10 marks)</u> **Marks**

a) Write $(1+\sqrt{7})^2$ in the form $a + b\sqrt{7}$ 2

b) Solve for x, $16^{4-x} = \frac{1}{8^x}$ 2

c) Factorise fully, $81 - x^4$ 2

d) What is the centre and radius of $x^2 + y^2 + 6x + 8y - 11 = 0$? 2

e) Write down the equations of horizontal and vertical asymptotes for 2

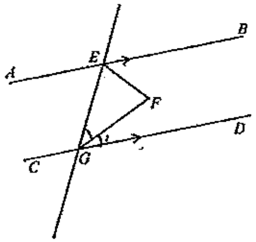
$y = \frac{x^2+1}{x^2-1}$

(Start each new question on a new page)

Question 7: (10 marks) **Marks**

a) In the diagram below, AB ∥ CD. EF bisects $\angle BEG$ and GF bisects $\angle EGD$. 3

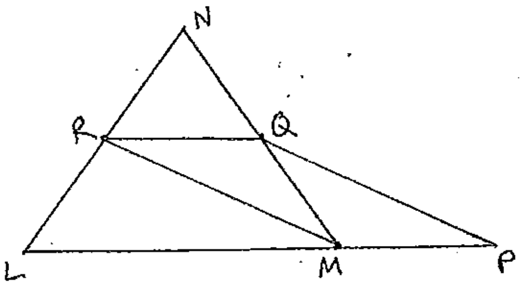
What is the size of $\angle EFG$? Give clear reasons.



b) Solve simultaneously $x + 2y = 5$ and $2xy - x^2 = 3$ 3

c) The side LM of ΔLNM is produced to P so that MP=$\frac{1}{2}$LM. If Q and R are the midpoints of MN and LN respectively,

i) Copy the diagram onto your answer sheet.



ii) Prove that ΔRNQ is similar to ΔLNM. 2

iii) Prove that PQRM is a parallelogram. 2

(Start each new question on a new page)

Question 8: (10 marks) **Marks**

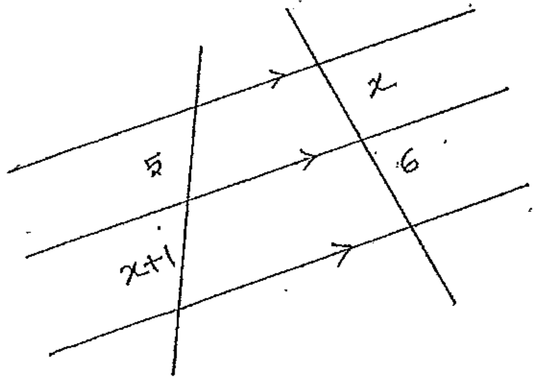
a) Solve $3x - 2\sqrt{x} - 8 = 0$ 3

b) A function is defined by $f(x) = \begin{cases} 2 - x^3, x \leq -5 \\ 2x + 1, -5 < x < 0 \\ x^2 - 9, x > 0 \end{cases}$ 2

Find the value of $2f(-5) + [f(1)]^2 + f(-1)$

c) Solve $2x - 1 \geq \frac{6}{x}$ 3

d) Find the value of x in: 2

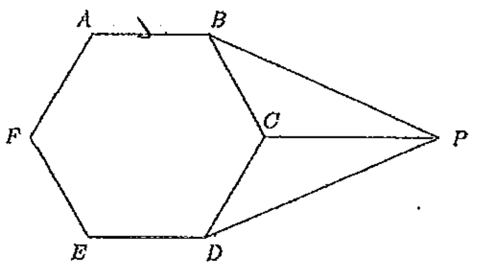


(Start each new question on a new page)

Question 9: (10 marks) Marks

a) Fully factorise $x^6 - 26x^3 - 27$ 2

b) ABCDEF is a regular hexagon, and CP || AB.



i) Find the size of $\angle BCP$, giving reasons. 2

ii) Prove that $\Delta BCP \equiv \Delta DCP$. 2

c) i) On the same set of axes, sketch the graphs of 2

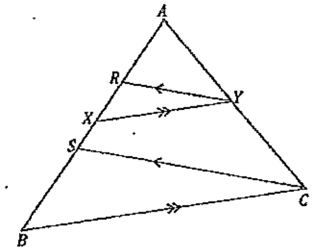
$y = |2x+5|$ and $y = x+4$

ii) Hence or otherwise, solve $|2x+5| \leq x+4$ 2

(Start each new question on a new page)

Question 11: (10 marks) Marks

a) Given that in ΔABC, XY||BC and RY||SC 3



Prove $\frac{AX}{XB} = \frac{AR}{RS}$

b) i)Factorise $x^3 - 6x^2 + 9x$ 2

ii) Hence solve $x^3 - 6x^2 + 9x \leq 0$ 2

c) Express the following as a single fraction with rational denominator 3

when $x = 3\sqrt{3} + 2$.

$\frac{1}{x-1} + \frac{1}{x+1} - \frac{2}{x^2-1}$

End of Assessment Task

1. C $y \geq x^2 - 2x$ and $y \leq 0$.

2. D $y = (x-3)^2 - 2$

3. B $x \geq -1$

4. A $4\sqrt{3}$

5. C $x = \frac{7 \pm \sqrt{37}}{6}$

6. a) $(1+\sqrt{7})^2 = 1 + 2\sqrt{7} + 7 = 8 + 2\sqrt{7}$

$a = 8, \ b = 2$

b) $16^{4-x} = \frac{1}{8^x}$

$2^{16-4x} = 2^{-3x}$

$16 - 4x = -3x$

$-x = -16$

$\therefore x = 16$

c) $81 - x^4$

$= 9^2 - (x^2)^2$

$= (9 - x^2)(9 + x^2)$

$= (3^2 - x^2)(3^2 + x^2)$

$= (3-x)(3+x)(9+x^2)$

d) $x^2 + y^2 + 6x + 8y - 11 = 0$

$x^2 + 6x + 9 + y^2 + 8y + 16 = 11 + 9 + 16$

$(x+3)^2 + (y+4)^2 = 36$ c: $(-3, -4)$ r: 6 units

e) $y = \frac{x^2+1}{(x+1)(x+1)}$

vertical asymptote: $x = \pm 1$

horizontal asymptote: $y = 1$

7. a) Let $\angle BEF = \alpha$ and $\angle FGD = \beta$

$\therefore \angle FEG = \alpha$ (EF bisects $\angle BEG$) Similarly, $\angle FGE = \beta$

$2\alpha + 2\beta = 180°$ (cointerior angles, $AB \parallel CD$)

$\alpha + \beta = 90°$

$\angle EFG + \alpha + \beta = 180°$ (angle sum of $\triangle EFG$)

$\therefore \angle EFG = 90°$

b) $x + 2y = 5 \Rightarrow y = \frac{5-x}{2}$

$2xy - x^2 = 3$

$2x\frac{(5-x)}{2} - x^2 = 3$

$2x^2 - 5x + 3 = 0$

$(2x-3)(x-1) = 0$

$\therefore x = \frac{3}{2}, 1$

when $x = \frac{3}{2}$ $y = \frac{7}{4}$

when $x = 1$ $y = \frac{5-1}{2} = 2$

$\therefore y = \frac{7}{4}, 2$

c) ii) In $\triangle RNQ$ and $\triangle LNM$

$\angle LNM$ is common

$\frac{RN}{LN} = \frac{1}{2}$ (given)

$\frac{QN}{MN} = \frac{1}{2}$ (given) $\therefore \frac{RN}{LN} = \frac{QN}{MN}$

$\therefore \triangle RNQ \parallel\!| \triangle LNM$ (Since two pairs of sides in equal ratio and included angles are equal)

iii) $\frac{RQ}{LM} = \frac{1}{2}$ (matching sides in similar triangles)

$\frac{MP}{LM} = \frac{1}{2}$ (given) $\therefore RQ = MP$

$\angle NQR = \angle QML$ (matching angles in similar triangles)

$\therefore RQ \parallel MP$ (Since $\angle NQR = \angle QML$, corresponding angles are equal)

$\therefore PQRM$ is a parallelogram (One pair of opposite sides are equal and parallel)

8\. a) $3x - 2\sqrt{x} - 8 = 0$

let $\sqrt{x} = a$

$3a^2 - 2a - 8 = 0.$

$(3a+4)(a-2) = 0$

$a = -4/3, 2$

$\therefore \sqrt{x} = -4/3, 2$

Since $\sqrt{x} > 0.$

$x = 4$ only solution

b) $f(-5) = 2 - (-5)^3$

$= 127$

$f(1) = 1^2 - 9 = -8$

$f(-1) = 2(-1) + 1 = -1$

$2f(-5) + [f(1)]^2 + f(-1)$

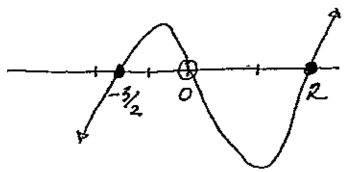
$= 2 \times 127 + 64 - 1 = \underline{317}$

c) $2x - 1 \geq \frac{6}{x}, \quad x \neq 0$

$x^2(2x-1) \geq 6x$

$2x^3 - x^2 - 6x \geq 0$

$x(2x+3)(x-2) \geq 0$



$\therefore -\frac{3}{2} \leq x < 0$ or $x \geq 2$

d) $\frac{5}{x+1} = \frac{x}{6}$

$30 = x^2 + x$

$x^2 + x - 30 = 0$

$(x+6)(x-5) = 0$

$\therefore x = -6, 5$

Since $x > 0$

$x = 5$ only

9\. a) $x^6 - 26x^3 - 27$

let $x^3 = a$

$a^2 - 26a - 27$

$= (a-27)(a+1)$

$= (x^3 - 3^3)(x^3 + 1^3)$

$= (x-3)(x^2+3x+9)(x+1)(x^2-x+1)$

b) i) $\angle ABC = \frac{(6-2) \times 180}{6} = 120°$ (angle in hexagon)

$\therefore \angle BCP = 120°$ (Alternate angles, $CP \parallel AB$)

ii) In $\triangle BCP$ and $\triangle DCP$

$\angle BCP = 120°$ (Proved in part (i))

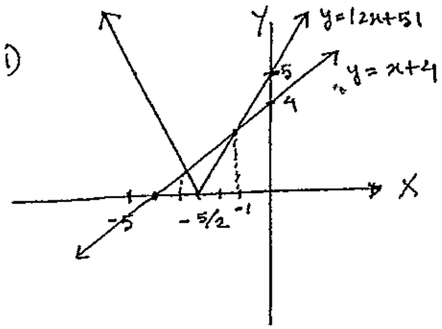
$\angle DCP = 120°$ (Alternate angles, $ED \parallel CP$)

$BC = DC$ (sides of a regular hexagon)

CP is common

$\therefore \triangle BCP \equiv \triangle DCP$ (SAS)

c) i)



$2x + 5 = x + 4$

$x = -1$

or

$2x + 5 = -x - 4$

$x = -3$

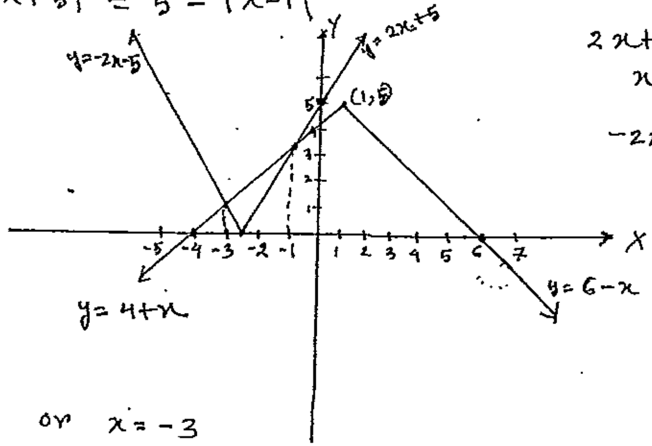
ii) $-3 \leq x \leq -1$

10. a) domain: All real x

range: $\{y \in R,\ y \leq 5\}$

b) $|2x+5| + |x-1| = 5$

$|2x+5| = 5 - |x-1|$



$2x+5 = 4+x$
$x = -1$

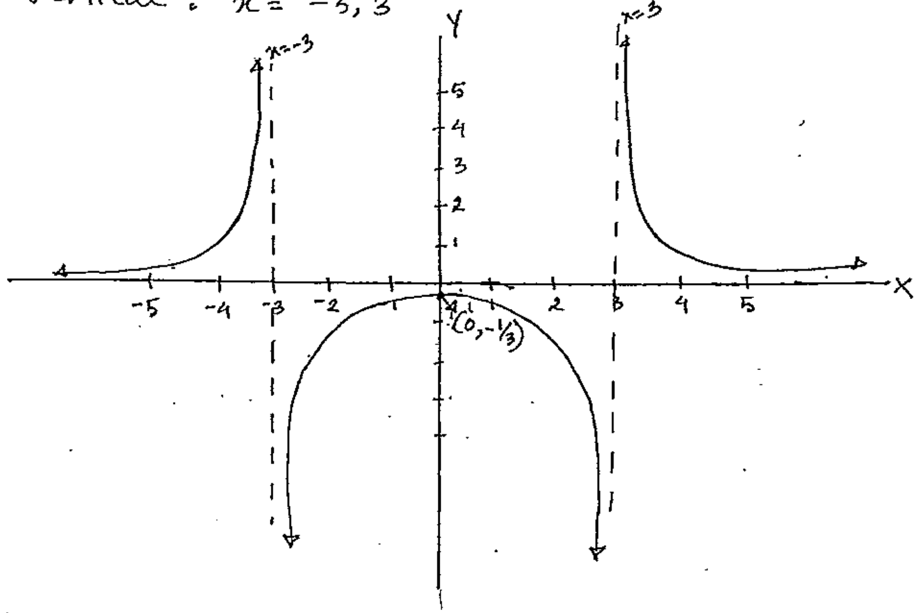
$-2x-5 = 4+x$
$x = -3$

$x = -1$ or $x = -3$

c) i) $f(-x) = \frac{3}{(-x)^2-9} = \frac{3}{x^2-9} = f(x)$ ∴ even

ii) horizontal: $y = 0$
vertical: $x = -3, 3$

iii)



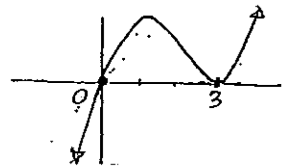
11. a) $\frac{AR}{RS} = \frac{AY}{YC}$ (intercepts on parallel lines)

$\frac{AX}{XB} = \frac{AY}{YC}$ (intercepts on parallel lines)

$\therefore \frac{AR}{RS} = \frac{AX}{XB}$

b) i) $x^3 - 6x^2 + 9x$
$= x(x^2 - 6x + 9)$
$= x(x-3)^2$

ii) $x(x-3)^2 \leq 0$



$\therefore x \leq 0,\ x = 3$

c) $\frac{1}{x-1} + \frac{1}{x+1} - \frac{2}{x^2-1}$

$= \frac{x+1+x-1}{x^2-1} - \frac{2}{x^2-1}$

$= \frac{2x}{x^2-1} - \frac{2}{x^2-1}$

$= \frac{2x-2}{x^2-1}$

$= \frac{2(3\sqrt{3}+2)-2}{(3\sqrt{3}+2)^2-1}$

$= \frac{6\sqrt{3}+4-2}{27+12\sqrt{3}+4-1}$

$= \frac{6\sqrt{3}+2}{12\sqrt{3}+30}$

$= \frac{3\sqrt{3}+1}{6\sqrt{3}+15}$

$= \frac{(3\sqrt{3}+1)(6\sqrt{3}-15)}{(6\sqrt{3}+15)(6\sqrt{3}-15)}$

$= \frac{54 - 45\sqrt{3} + 6\sqrt{3} - 15}{108 - 225}$

$= \frac{39 - 39\sqrt{3}}{-117}$

$= \frac{39(1-\sqrt{3})}{-117}$

$= \frac{1-\sqrt{3}}{-3}$

$= \frac{\sqrt{3}-1}{3}$