YEAR 12 ~~4~~ 3/4U UNIT TEST BINOMIAL THEOREM, PERMUTATIONS & COMBINATIONS 26-2-96

Name..........................

Question 1.(4 marks)

By expanding both sides of the identity

$(1+x)^{k+3} = (1+x)^k(1+x)^3$ find an expression for $\binom{k+3}{r}$.

Question 2.(6 marks)

Find the numerically greatest term of the expansion $(3x-y)^{11}$ when $x=4$ and $y=5$. [leave answer as a product of its prime factors]

Question 3.(4 marks)

By considering the expansion of $x(1+x)^n$ and differentiating w.r.t.x. show that $(n+2).2^{n-1} = \sum_{r=0}^{n}(r+1).{}^nC_r$.

Question4.(2 marks)

In how many ways can the letters of the state "NEW SOUTH WALES" be arranged in a row?

Question 5.(4 marks)

Twelve cards are numbered 1,2,3,......11,12. Three boxes are labelled A,B,C. In how many ways can I put 6 cards in one box,4 cards in a second box and 2 cards in a third?

Question 6.(4 marks)

Four men and their wives belong to a social club.A committee of three is to be formed and it is decided that no man should be on the committee if his wife is also on it.In how many ways can the committee be formed?

Question 7.

Seven men and two women are to be seated in

(i) **a straight line (1 mark,2 marks,3 marks)**

(ii) **a circle.(1 mark,2 marks,2 marks)**

In how many ways can this be done if :

(a) there are no restrictions.

(b) the 2 women are to be seated together.

(c) a particular man is not to sit between the women.

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Q1: $(1+x)^{k+3} = (1+x)^k(1+x)^3$

$\therefore$ LHS $= (1+x)^{k+3}$

$= 1 + \binom{k+3}{1}x + \binom{k+3}{2}x^2 + \dots + \binom{k+3}{r}x^r + \dots + x^{k+3}$

RHS $= (1+x)^k(1+x)^3$

$= \left[1 + \binom{k}{1}x + \binom{k}{2}x^2 + \dots + \binom{k}{r-3}x^{r-3} + \binom{k}{r-2}x^{r-2} + \binom{k}{r-1}x^{r-1} + \binom{k}{r}x^r + \dots + x^k\right] \times \left[1 + \binom{3}{1}x + \binom{3}{2}x^2 + x^3\right]$

$\therefore$ Equating coefficients of x^r terms in LHS and RHS:

$\binom{k+3}{r} = 1\times\binom{k}{r} + \binom{3}{1}\times\binom{k}{r-1} + \binom{3}{2}\times\binom{k}{r-2} + 1\times\binom{k}{r-3}$

$\Rightarrow \binom{k+3}{r} = \binom{k}{r} + \binom{3}{1}\binom{k}{r-1} + \binom{3}{2}\binom{k}{r-2} + \binom{k}{r-3}$

Q2: For numerically greatest term of $(3x-y)^{11}$ when $x=4$ and $y=5$ consider $(3x+y)^{11}$.

Now for greatest terms $\frac{T_r}{T_{r+1}} > 1$

$\therefore \frac{{}^{11}C_{r-1}(3x)^{12-r}(y)^{r-1}}{{}^{11}C_r(3x)^{11-r}y^r} > 1$

$\therefore \frac{11!\,(11-r)!\,r!\,(3x)}{(12-r)!\,(r-1)!\,11!\,y} > 1$

$\therefore \frac{r\cdot 3x}{(12-r)y} > 1$

when $x=4$, $y=5$

$\therefore \frac{12r}{(12-r).5} > 1$

$\therefore 12r > 60-5r$

$\therefore 17r > 60$

$\therefore r > 3\frac{9}{17}$ $\therefore r=4$

$\therefore$ numerically greatest term is T_4

where $T_4 = {}^{11}C_3(12)^8(5)^3$

$= 165\times(2^2\times3)^8\times5^3$

$= 5\times3\times11\times2^{16}\times3^8\times5^3$

$= 2^{16}\times3^9\times5^4\times11$

Q3

$x(1+x)^n = x\left[1 + {}^nC_1x + {}^nC_2x^2 + \dots + {}^nC_nx^n\right]$

$= x + {}^nC_1x^2 + {}^nC_2x^3 + \dots + {}^nC_nx^{n+1}$

diff. b.s. w.r.t. x:

$x.n(1+x)^{n-1} + (1+x)^n.1 = 1 + 2{}^nC_1x + 3.{}^nC_2x^2 + \dots + (n+1){}^nC_nx^n$

let $x=1$

$\therefore n(2)^{n-1} + 2^n = 1 + 2.{}^nC_1 + 3.{}^nC_2 + \dots + (n+1).{}^nC_n$

$\therefore 2^{n-1}.(n+2) = \sum_{r=0}^{n}(r+1).{}^nC_r$

Q4: 'NEW SOUTH WALES' consists of 13 letters of which there are 2 E's, 2 W's and 2 S's and 7 other letters.

$\therefore$ No of arrangements in a row $= \frac{13!}{2!2!2!}$

$= 778377600$

Q5: $\boxed{A}$ $\boxed{B}$ $\boxed{C}$

6 cards, 4 cards, 2 cards

The 3 boxes can be arranged in 3! ways

$\therefore$ No. of ways $= \left[3!\times\underbrace{{}^{12}C_6}_{\substack{\text{6 from 12} \\ \text{This leaves} \\ \text{6 cards.}}}\times\underbrace{{}^6C_4}_{\substack{\text{4 from 6} \\ \text{This leaves} \\ \text{2 cards.}}}\times{}^2C_2\right]\div 3!$

as an order has been implied.

$= 13860$

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Q6: Let men be A, B, C and D and their wives be a, b, c and d.

$\therefore$ Number of committees

= 3 men, 0 women or 2 men, 1 woman (not married to these 2 men)

or 1 man, 2 women (not married to this man) or 0 men, 3 women

$$= {}^4C_3 + {}^4C_2 \times {}^2C_1 + {}^4C_1 \times {}^3C_2 + {}^4C_3$$

$$= 32$$

or $\dfrac{8 \times 6 \times 4}{3!} = 32$

(8 ← any man or wife; 6 ← any of remaining 3 couples; 4 ← any of remaining 2 couples)

(order implied when there is no order)

Q7: (i) straight line.

(a) No restrictions $= 9!$
$= 362880$

(b) Consider the 2 women as a unit. This is done in $2!$ ways. This leaves $(9-2)+1 = 8$ people.

$\therefore$ No. of ways $= 2! \times 8!$
$= 80640$

(c) Take the case where the particular man does sit between the women

$\therefore$

W M W _ _ _ _ _ _
_ W M W _ _ _ _ _
_ _ W M W _ _ _ _
_ _ _ W M W _ _ _
_ _ _ _ W M W _ _
_ _ _ _ _ W M W _
_ _ _ _ _ _ W M W

7 ways

The women in each line can be seated in $2!$ ways ~~& other men~~ in $6!$ ways.

$\therefore$ No. of ways man seated between women is $2! \times 7 \times 6! = 10080$

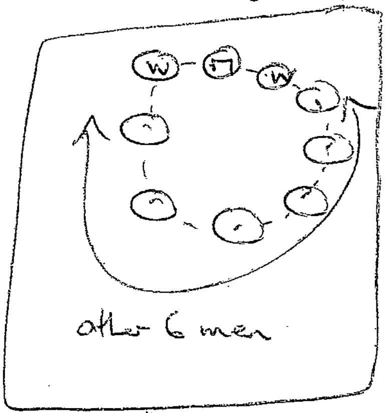
$\therefore$ No. of ways particular man does not sit between the women $= 9! - 2! \times 7 \times 6!$
$= 352800$

(ii) circle

(a) No restrictions $= 1 \times 8!$
$= 40320$

(b) No. of ways women together $= 1 \times 2! \times 7!$
$= 10080$

(c) No. of ways $= 8! - 1 \times 2! \times 6!$



$= 38880$

Case for particular man sitting between the 2 women

$1 \times 2! \times 6!$

Tait is gay!