Homebush Boys High School

HSC
ASSESSMENT TASK 2

2018
MATHEMATICS

Time allowed –90 minutes

Name: .....

Class: ..... Teacher: .....

General Instructions

- Write using blue or black pen
- All necessary working should be shown for every question
- Approved calculators may be used
- Begin each question on a new page clearly marked "Question 1", "Question 2", etc.
- Write your name on every sheet of paper handed in.
- Attempt all questions
- Questions 1 to 10 are multiple choice

Total marks (60)

Question	Mark	Out of
1-10		10
11		13
12		13
13		13
14		11
Total		60

Multiple choice Questions 1 to 10

- **Each question is worth 1 mark**
- **Circle your chosen answer on the question paper**

Question 1

What is the value of $\frac{dy}{dx}$ if $y=2\sqrt{x}$?

(A) $\frac{dy}{dx}=\frac{1}{\sqrt{x}}$ (B) $\frac{dy}{dx}=\frac{2}{\sqrt{x}}$

(C) $\frac{dy}{dx}=\frac{\sqrt{x}}{2}$ (D) $\frac{dy}{dx}=2$

Question 2

What is the gradient of the curve $y=x^2-x-6$ at $(6, 24)$?

(A) 11 (B) 12

(C) 23 (D) 24

Question 3

What is the equation of the tangent to the curve $y=x^2-2x$ at the point $(1,-1)$?

(A) $y=-1$ (B) $y=1$

(C) $y=x+2$ (D) $y=x-2$

Question 4

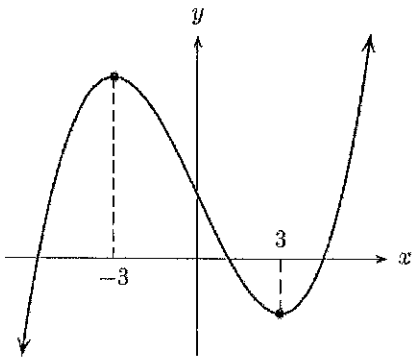
The curve $y = ax^2 - 6x + 3$ has a stationary point at $x = 1$. What is the value of a?

(A) 2 (B) -1

(C) 3 (D) -3

Question 5

From the graph of $y = f(x)$, when is $f'(x)$ negative?



(A) $x < -3$ or $x > 3$

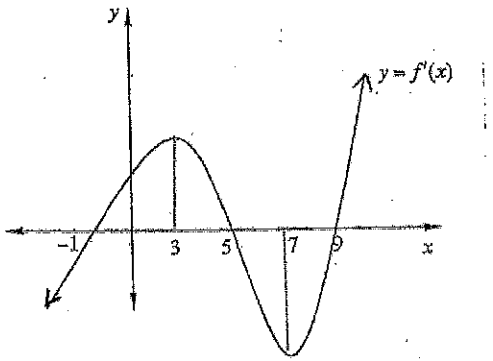
(B) $-3 < x < 3$

(C) $x \leq -3$ or $x \geq 3$

(D) $-3 \leq x \leq 3$

Question 6

The diagram represents a sketch of the **gradient function** of the curve $y = f(x)$.



A maximum turning point on $y = f(x)$ occurs at:

(A) $x = -1$

(B) $x = 3$

(C) $x = 5$

(D) $x = 7$

Question 7

Which of the following is an expression for $\int\left(x^2 + \frac{1}{x^2}\right)dx$?

(A) $\frac{x^3}{3} - \frac{1}{x} + c$ (B) $\frac{x^3}{3} + \frac{1}{x} + c$

(C) $\frac{x^3}{3} - \frac{1}{2x} + c$ (D) $\frac{x^3}{3} + \frac{1}{2x} + c$

Question 8

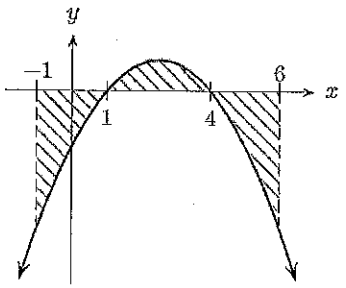
Evlualte: $\int_{-1}^{1} x^3 dx$

(A) -1 (B) $\frac{1}{4}$

(C) 0 (D) $\frac{1}{2}$

Question 9

Which of the following expressions gives the total area of the shaded region in the diagram?



(A) $\int_{-1}^{6} f(x)dx$

(B) $-\int_{-1}^{0} f(x)dx + \int_{0}^{6} f(x)dx$

(C) $-\int_{-1}^{1} f(x)dx + \int_{1}^{4} f(x)dx - \int_{4}^{6} f(x)dx$

(D) $\int_{1}^{4} f(x)dx + 2\int_{4}^{6} f(x)dx$

Question 10

What is the equation of the normal to the curve $y = x^2 - 4x$ at (1, -3)?

(A) $x + 2y - 7 = 0$

(B) $x - 2y - 7 = 0$

(C) $2x - y + 1 = 0$

(D) $2x + y + 1 = 0$

End of Multiple choice section

Question 11 (13 marks) Begin a new booklet

(a) If $y=\frac{\sqrt{x+1}}{x}$ show that $y'=\frac{-x-2}{2x^2\sqrt{x+1}}$. 3

(b) A function is defined by $f(x)=x^3-3x^2-9x+22$

(i) Find the coordinates of the turning points of the graph $y=f(x)$, and determine their nature. 3

(ii) Find the coordinates of the point of inflexion 2

(iii) Hence, sketch the graph of $y=f(x)$, showing the turning points, the point of inflexion and where the curve meets the y-axis. 2

(iv) For what values of x is the graph of $y=f(x)$ concave up? 1

(c) Given that $f(x)=ax^4+5x-6$. Find the value of a if $f''(1)=-2$ 2

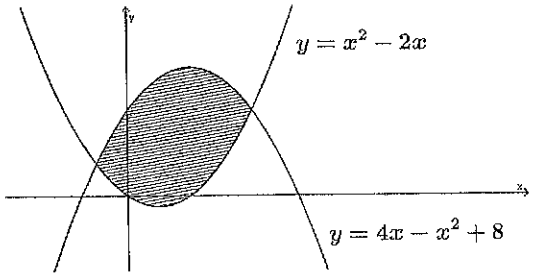
Question 12(13 marks) Begin a new booklet

(a) Find the primitive of $\frac{x}{3}+\frac{1}{x^2}$ 2

(b) Find the values of k if $\int_1^k (x+1)dx=6$ 3

(c) Evaluate $\int_0^3 x\sqrt{x}\,dx$. Leave your answer in simplified exact form. 3

(d) The shaded region is bounded by the parabolas $y=x^2-2x$ and $y=4x-x^2+8$

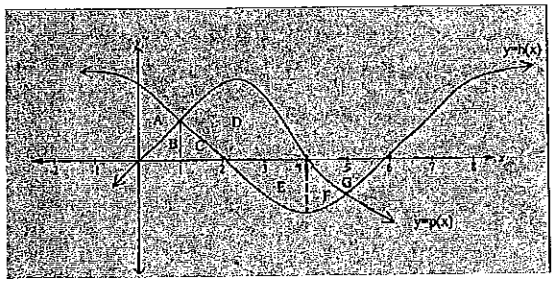


(i) Find the coordinates where the curves $y=4x-x^2+8$ and $y=x^2-2x$ intersect. 2

(ii) Calculate the area bounded by the two curves 3

Question 13 (13 marks) Begin a new page

(a) The two curves $y = p(x)$ and $y = h(x)$ are sketched below. Different areas enclosed by the curves and the axes are labelled A to G.



The integrals below represent the sum of which areas.

(i) $\int_0^4 p(x)dx$ 1

(ii) $\int_1^5 (p(x)-h(x))dx$ 1

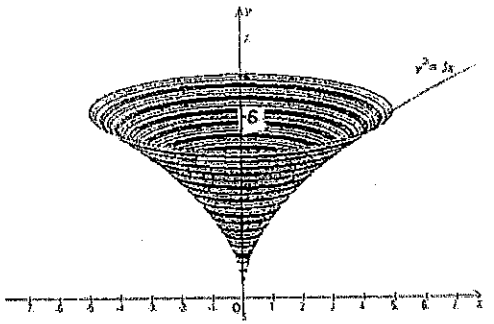
(iii) Give an integral which would define the area denoted by letter G. 1

(b) Find:

(i) $\int(\sqrt{2-x})dx$ 2

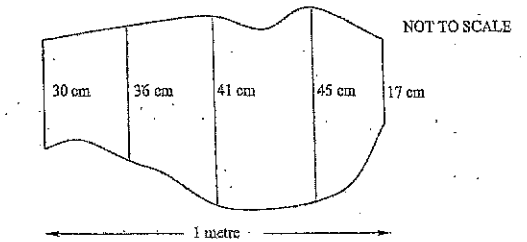
(ii) $\int\frac{4x-x^4}{x^3}dx$ 2

(c) The diagram illustrates the shape of a vessel obtained by rotating, about the y-axis, the part of the parabola $y^2 = 5x$ between y = 0 and y = 5



Show that the volume of the vessel is 25π cubic units. 3

(d) Sammy needs to estimate the area of the following hole in the wall, which is divided into four equal intervals.



(i) Copy and complete the table below:

Distance from the left edge(cm)	0				100
Height of hole(cm)	30	36			

1

(ii) Use Simpson's Rule and all the values from the table to find an approximation for the area of the hole. 2

Question 14 (11 marks) Begin a new booklet

(a) (i) Sketch $y=f(x)$ given that $f'(x)<0$ for $x<0$, $f'(x)>0$ for $x>0$ and $f(0)=f'(0)=0$ — 2

(ii) State a possible equation for $y=f(x)$ — 1

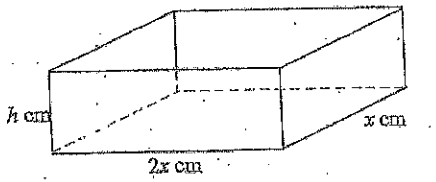
(b) Let $f(x)=3x^2+1$

(i) Copy the following table and find the missing values. — 1

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	1					4

(ii) Use these six values of the function and the trapezoidal rule to find the approximate value of: $\int_0^1 (3x^2+1)dx$ — 2

(c) Joe is building a small toy box with no lid. The box is twice as long as it is wide. The box has a total external surface area of 3750cm^2.



(i) Show that the height h of the toy box is given by $h=\frac{625}{x}-\frac{x}{3}$ — 1

(ii) Find the dimensions of the box which gives a maximum volume. — 2

(iii) Joe decides that height of the box will be $10\frac{5}{6}$ cm. Find the new dimensions of the box and hence find its volume if the surface area is to remain at 3750cm^2. — 2

End of Examination

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Multiple Choice

1 A
2 A
3 A
4 C
5 B
6 C
7 A
8 C
9 C
10 B

Question 11

a) $y=\frac{\sqrt{x+1}}{x}$

$y'=\frac{x\cdot\frac{1}{2\sqrt{x+1}}-\sqrt{x+1}}{x^2}$

$=\frac{x-2(x+1)}{2x^2\sqrt{x+1}}$

$=\frac{-x-2}{2x^2\sqrt{x+1}}$

Hence shown.

b) i) $f(x)=x^3-3x^2-9x+22$

$f'(x)=3x^2-6x-9$

For Stationary points $f'(x)=0$

$\therefore 3x^2-6x-9=0$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x=-1, 3$

$f(-1)=-1-3+9+22$

$=27$

$f(3)=27-27-27+22$

$=-5$

$\therefore$ Turning pts. are $(-1,27)$ and $(3,-5)$

Test nature of points.

$f''(x)=6x-6$

For $(-1,27)$; $f''(-1)=-12<0$

$\therefore (-1,27)$ is maximum.

For $(3,-5)$; $f''(3)=12>0$

$\therefore (3,-5)$ is minimum.

ii) For POI:

$f''(x)=0$

$6x-6=0$

$x=1$

$f(1)=1-3-9+22$

$=11$

$\therefore$ point is $(1,11)$

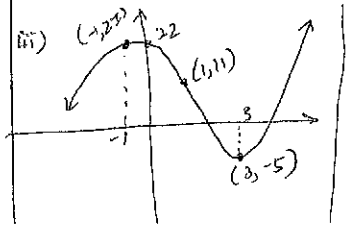
Test:

x	1⁻	1	1⁺
$f''(x)$	-ve	0	+ve

Change in concavity

$\therefore (1,11)$ is POI.

iii)



iv) Concave up for $x>1$

Q13

b ii) $\int \frac{4x}{x^3} - \frac{x^4}{x^3}\,dx$

$= \int \left(\frac{4}{x^2} - x\right) dx$

$= -\frac{8}{x^3} - \frac{x^2}{2} + C$

c) $V = \pi \int_0^5 x^2\,dy$

$= \pi \int_0^5 \frac{y^4}{25}\,dy \quad (\because y^2 = 5x)$

$= \frac{\pi}{25}\left[\frac{y^5}{5}\right]_0^5$

$= \frac{\pi}{25}\left(\frac{5^5}{5} - 0\right)$

$= 25\pi$ cu. units.

Hence shown.

d) i)

Distance	0	25	50	75	100
Height	30	36	41	45	17

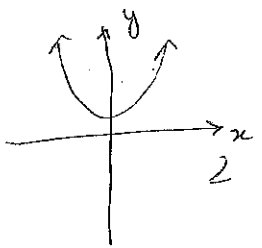
ii) $A \approx \frac{h}{3}\left[(30+17) + 4(36+45) + 2(41)\right]$

$\approx \frac{25}{3}(47 + 324 + 82)$

$\approx 3775\ cm^2$

Question 14

a) i)



ii) $y = ax^2$, $a > 0$ /

b) i) $f(x) = 3x^2 + 1$

x	0	0.2	0.4	0.6	0.8	1
$f(x)$	1	1.12	1.48	2.08	2.92	4

ii) $\int_0^1 (3x^2+1)\,dx \approx \frac{0.2}{2}\left[(1+4) + 2(1.12+1.48+2.08+2.92)\right]$

≈ 2.02 sq. units.

c) i) $S.A = 2x^2 + 2(2x)h + 2xh$

$\therefore \quad 2x^2 + 6xh = 3750$

$x^2 + 3xh = 3750$

$3xh = 3750 - x^2$

$h = \frac{3750}{3x} - \frac{x^2}{3x}$

$= \frac{625}{x} - \frac{x}{3}$

Hence shown.

ii) $V = 2x(x)h$

$= 2x^2\left(\frac{625}{x} - \frac{x}{3}\right)$

$= 1250x - \frac{2}{3}x^3$

$V' = 1250 - 2x^2$

$V'' = -4x$.

14(c) Continue.

For $V' = 0$

$1250 - 2x^2 = 0$

$2x^2 = 1250$

$x^2 = 625$

$x = 25$

$V'' < 0$ at $x = 25$ /

$\therefore$ Maximum volume occur at $x = 25$

$\therefore$ Dimensions of the box :

$50, 25, \frac{50}{3}$

iii) Since Surface area remain same. and height is changing

$\therefore \quad h = 10\frac{5}{6} = \frac{65}{6}$

$\therefore \quad \frac{625}{x} - \frac{x}{3} = \frac{65}{6}$

$3750 - 2x^2 = 65x$

$2x^2 + 65x - 3750 = 0$

$x = \frac{-65 \pm \sqrt{8125 + 30000}}{4}$

$= \frac{-65 \pm 185}{4}$

$= \frac{120}{4}, \left(\frac{-250}{4}\right)$ Reject

$= 30$

$\therefore$ New Dimension with the same S.A are : $30, 60, 10\frac{5}{6}$

& Volume $= 30 \times 60 \times 10\frac{5}{6}$

$= 19500\ cm^3$