Name: _____

Teacher: _____

Randwick Girls High School

Year 12 Extension 1 Mathematics

HSC Assessment Task 2 2015

Weighting: 20%

Examiner: L. Delley

General Instructions

- Reading Time: 5 minutes
- Working time: 45 minutes
- Answer on the paper provided
- Write using black or blue pen.
- Show all necessary working
- Marks may be deducted for careless or badly arranged work
- Begin each question on a new page.

Section	Marks
Question 1–5 Multiple Choice	/5
Question 6 Permutations and Combinations	/9
Question 7 Geometrical Applications of Calculus	/13
Question 8 Integration and Exponentials	/11
Total	/38

Section 1. Multiple Choice
5 Marks

Circle the best response on your multiple choice answer sheet.

Question 1
A coat hanger has 4 knobs and each knob can be painted in any colour. If 6 colours of paint are available, how many different ways can the knobs be painted?

(A) 24

(B) 360

(C) 720

(D) 1296

Question 2

Barbara, Sean and six other people go through a doorway one at a time. In how many ways can the eight people go through the doorway if Sean goes through the doorway after Barbara with no one in between?

(A) 2

(B) 720

(C) 5040

(D) 40320

Question 3

The indefinite integral of $\frac{dy}{dx} = \frac{3}{x^4}$ would be:

(A) $y = \frac{15}{x^5} + c$

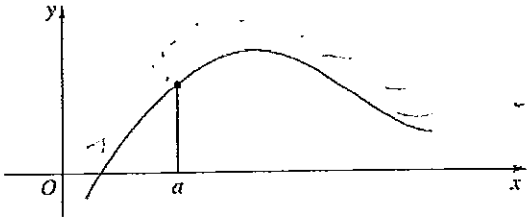
(B) $y = -\frac{1}{x^3} + c$

(C) $y = \frac{12}{x^3} + c$

(D) $y = -\frac{4}{4x^3} + c$

Question 4

The diagram shows the graph of $y = f(x)$.



Which of the following statements is true?

(A) $f'(a) > 0$ and $f''(a) < 0$

(B) $f'(a) > 0$ and $f''(a) > 0$

(C) $f'(a) < 0$ and $f''(a) < 0$

(D) $f'(a) < 0$ and $f''(a) > 0$

Question 5

$\int e^{4x+1}dx$ evaluates to:

(A) $4e^{4x+1}$

(B) $4e^{4x+1} + c$

(C) $\frac{1}{4}e^{4x+1} + c$

(D) $\frac{1}{4}e^{4x+1}$

End of Multiple Choice Section

Short Answer Questions

- Begin each question on a new page
- Show all working

Question 6: Permutations and Combinations (9 marks) **Marks**

a) David's briefcase lock contains 4 dials and each dial has the option of showing a number from $0 - 9$

i) How many different codes are possible? 1

ii) If David forgot the code to his briefcase, what is the probability that he will guess it correctly within 5 tries? 1

b) Simplify $\frac{n!}{(n-3)!}$ 3

c) i) In how many ways can 8 people sit around a circular table with no restrictions? 2

ii) Eleni and Loukia are members of a committee which has 8 members. This committee sits around a circular table when they meet. In how many ways can Eleni and Loukia sit together at the next meeting? 2

Question 7: Geometrical Applications of Calculus (13 marks) **Marks**

a) The profit after starting a particular business over a time t years is given by:

$$p(t) = \sqrt{t} - \frac{1}{2}t$$

At what time would the profit be a maximum? **3**

b) Consider the curve $y = \dfrac{x}{x^2+1}$

i) Find any stationary points and determine their nature **3**

ii) Find any points of inflexion **2**

iii) Find any asymptotes **2**

iv) Sketch the curve, showing all essential features **3**

Question 8: Integration and Exponentials (11 marks) **Marks**

a) Evaluate: $\int_{-2}^{2}(t - t^3 - t^5)dt$ **2**

b) If $f(x) = \dfrac{e^x}{x+1}$ find $f'(x)$ **2**

c) The region beneath the curve $y = e^{-x}$, which is above the x-axis and between the lines $x = 0$ and $x = 1$, is rotated about the x-axis.

i) Sketch the region **1**

ii) Find the exact volume of the resulting solid of revolution. **3**

d) Use the substitution $u = 1 - x$ to evaluate $\int_0^1 x\sqrt{1-x}\,dx$ **3**

End of Exam

Year 12 Extension 1 Assessment 2 Solutions

/38

1. D
2. C
3. B
4. A
5. C

6. a) i) number of options $= 10 \times 10 \times 10 \times 10$
$= 10000$ (1)

ii) Probability (guess correctly) $= \frac{5}{10000}$ } either
$= \frac{1}{2000}$ } (1)

b) $\frac{n!}{(n-3)!} = \frac{n(n-1)(n-2)(n-3)\times\cdots\times 3\times 2\times 1}{(n-3)(n-4)(n-5)\times\cdots\times 3\times 2\times 1}$ (1) (1)

$= n(n-1)(n-2)$ (1)

(1) expand $n!$ correctly
(1) expand $(n-3)!$ correctly
(1) correct answer

c) i) number of options $= (8-1)!$ (1)
$= 5040$ (1) CE

ii) Eleni and Loukia can sit in 2 ways ($2!$)
Remaining people can sit in $6!$ ways (1)
∴ number of seating options $= 2! \times 6!$
$= 1440$ (1)

- subtract a mark for any incorrect working.
or
(1) $= 2! \times 6!$ (1)

7. $P(t) = \sqrt{t} - \frac{1}{2}t$

a) $= t^{\frac{1}{2}} - \frac{1}{2}t$

$P'(t) = \frac{1}{2}t^{-\frac{1}{2}} - \frac{1}{2}$ (1)

$P''(t) = -\frac{1}{4}t^{-\frac{3}{2}}$

Stationary points occur when $P'(t) = 0$

ie $\frac{1}{2}t^{-\frac{1}{2}} - \frac{1}{2} = 0$

$\frac{1}{2\sqrt{t}} - \frac{1}{2} = 0$

$2 - 2\sqrt{t} = 0$

$2\sqrt{t} = 2$

$\sqrt{t} = 1$

$t = 1$ (1)

at $t = 1$, $P''(t) = -\frac{1}{4} < 0$ ∩ ∴ maximum at $t = 1$ (1)

(1) Finds $P'(t)$
(1) Finds $t = 1$
(1) Proves maximum (CE applies)

b) $y = \frac{x}{x^2+1}$

i)

$u = x \qquad v = x^2+1$

$u' = 1 \qquad v' = 2x$

$y' = \frac{u'v - v'u}{v^2}$

$= \frac{1(x^2+1) - 2x(x)}{(x^2+1)^2}$

$y' = \boxed{\frac{-x^2+1}{(x^2+1)^2}}$ ← (1) either

$y'' = \frac{-2x(x^2+1)^2 - 4x(x^2+1)(1-x^2)}{(x^2+1)^4}$

$= \frac{-2x(x^2+1)\left[x^2+1+2(1-x^2)\right]}{(x^2+1)^{4-3}}$

$= \frac{2x(-x^2+3)}{(x^2+1)^3}$

$= \boxed{\frac{2x(x^2-3)}{(x^2+1)^3}}$

Stationary points: $y' = 0$

$x^2 = 1$

$x = 1, \ y = \frac{1}{2} \rightarrow (1, \frac{1}{2})$

$x = -1, \ y = -\frac{1}{2} \rightarrow (-1, -\frac{1}{2})$ (1)

(1)

at $x=1$, $y'' < 0 \cap$ ∴ max at $(1, \frac{1}{2})$

at $x=-1$, $y'' > 0 \cup$ ∴ min at $(-1, -\frac{1}{2})$

or test either side of $x = \pm 1$

ii) Possible POI when $y'' = 0$

$2x(x^2-3) = 0$

(1) $\begin{cases} x=0, \ y=0 \rightarrow (0,0) \\ x=\sqrt{3}, \ y=0.433.. \rightarrow (\sqrt{3}, 0.433) \\ x=-\sqrt{3}, \ y=-0.433.. \rightarrow (-\sqrt{3}, -0.433) \end{cases}$

Check for concavity change:

x	-2	$-\sqrt{3}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\sqrt{3}$	2
y''	$-$	0	$+$	$+$	0	$-$	$-$	0	$+$
	$\cap$	↕	$\cup$	$\cup$	↑	$\cap$	$\cap$	↑	$\cup$

Change in concavity (1)

∴ Points of inflexion at $(0,0)$ $(\sqrt{3}, 0.433)$ and $(-\sqrt{3}, -0.433)$

iii) Vertical asymptote:

$x^2+1 \neq 0$

$x^2 \neq -1$

no solution. (1)

horizontal asymptote:

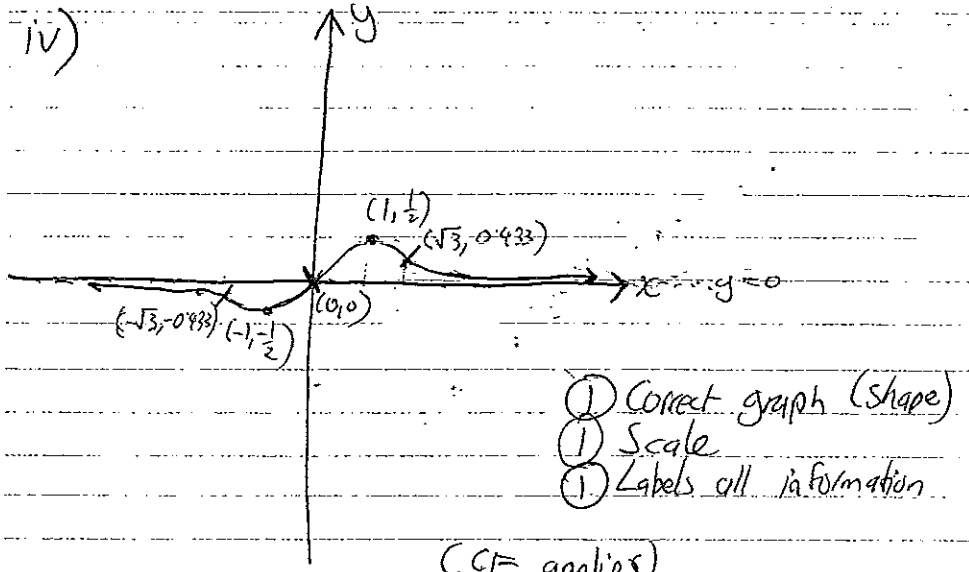
$\lim_{x\to\infty} \frac{x}{x^2+1} = \lim_{x\to\infty} \frac{\frac{x}{x^2}}{\frac{x^2}{x^2}+\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{\frac{1}{x}}{1+\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{0}{1}$

$= 0$ (1)

iv)



(1) Correct graph (shape)
(1) Scale
(1) Labels all information

(CE applies)

8.

a) $\int_{-2}^{2} (t - t^3 - t^5)\,dt = \left[\frac{t^2}{2} - \frac{t^4}{4} - \frac{t^6}{6}\right]_{-2}^{2}$ (1)

$= \left(\frac{2^2}{2} - \frac{2^4}{4} - \frac{2^6}{6}\right) - \left(\frac{(-2)^2}{2} - \frac{(-2)^4}{4} - \frac{(-2)^6}{6}\right)$

$= \left(\frac{-38}{3}\right) - \left(\frac{-38}{3}\right)$

$= 0$ (1)

b) $f(x) = \frac{e^x}{x+1}$

$u = e^x \qquad v = x+1$

$u' = e^x \qquad v' = 1$

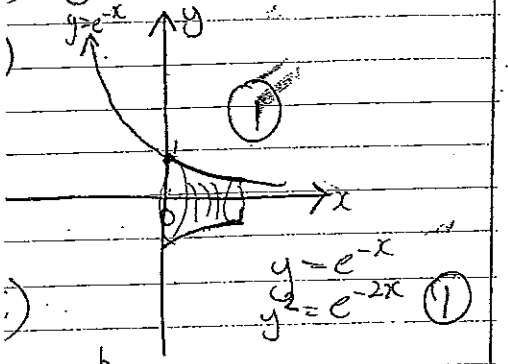
$f'(x) = \frac{u'v - v'u}{v^2}$

$= \frac{e^x(x+1) - 1(e^x)}{(x+1)^2}$ (1)

$= \frac{xe^x + e^x - e^x}{(x+1)^2}$

$= \frac{xe^x}{(x+1)^2}$ (1)

c) $y = e^{-x}$



$y = e^{-x}$

$y^2 = e^{-2x}$ (1)

$= \pi \int_a^b y^2\,dx$

$= \pi \int_0^1 e^{-2x}\,dx$

$= \pi \left[\frac{e^{-2x}}{-2}\right]_0^1$ (1)

$= \pi \left[\left(\frac{e^{-2}}{-2}\right) - \left(\frac{e^0}{-2}\right)\right]$

$= \pi \left(\frac{-e^{-2}}{2} + \frac{1}{2}\right) u^3$ (1)

1) Finds y^2 value of x

1) Correct integration

1) Correct answer

1) Correct sketch

d) $\int_0^1 x\sqrt{1-x}\,dx$

let $u = 1-x \longrightarrow x = 1-u$

$\frac{du}{dx} = -1$ (1)

$du = -1\,dx$

x	u	
1	0	When $x=1$, $u=0$
0	1	When $x=0$, $u=1$

$\int_{x=0}^{x=1} x\sqrt{1-x}\,dx = -\int_{u=1}^{u=0} (1-u)u^{\frac{1}{2}}\,du$ (1)

$= -\int_1^0 \left(u^{\frac{1}{2}} - u^{\frac{3}{2}}\right)du$

$= -\left[\frac{2u^{\frac{3}{2}}}{3} - \frac{2u^{\frac{5}{2}}}{5}\right]_1^0$

$= -\left[(0-0) - \left(\frac{2}{3} - \frac{2}{5}\right)\right]$

$= -\left(-\frac{4}{15}\right)$

$= \frac{4}{15}$ (1)

(1) Find du

(1) Correct u integral

(1) Correct answer