Santa Sabina College

Assessment Task 1
23 November 2017

HSC Mathematics

General Instructions

- Working Time – 75 minutes
- Write using blue or black pen. Pencil may be used to draw diagram.
- Board approved scientific calculators may be used.
- Write your student number on each new answer booklet.
- The marks indicated are a guide only.

Total marks – 45

SECTION I – (5 marks)

- Attempt all questions in this section.
- Answers Objective Response question (1) to (5) on the separate answer sheet.
- Allow about 8 minutes to complete this section.

SECTION II – (40 marks)

- Attempt ALL questions in this section
- Start a new answer booklet for each question.
- Clearly label each question part
- Work down the page.
- All necessary working must be shown.
- Full marks may not be awarded for careless or badly arranged work
- Allow about 67 minutes for this section.

SECTION I
Objective Response Questions
(5 marks)

Use the Objective Response answer sheet for Question 1 – 5.

1. Which of the following is the exact value of $\sin 120°$?

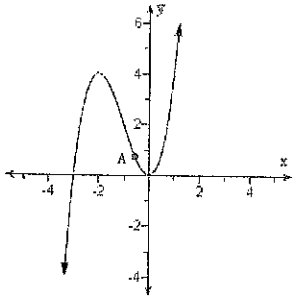
(A) $\frac{\sqrt{3}}{2}$

(B) $\sqrt{3}$

(C) $\frac{-\sqrt{3}}{2}$

(D) $\frac{2}{\sqrt{3}}$

2. The function $y = f(x)$ is shown below. Which of the following is true at the point A?



(A) $f'(x) > 0, f''(x) > 0$

(B) $f'(x) > 0, f''(x) < 0$

(C) $f'(x) < 0, f''(x) > 0$

(D) $f'(x) < 0, f''(x) < 0$

3. Given $\cos x = \frac{4}{7}$ and $\tan x < 0$, what is the exact value of $\operatorname{cosec} x$?

(A) $\frac{\sqrt{33}}{7}$

(B) $\frac{7}{\sqrt{33}}$

(C) $-\frac{\sqrt{33}}{7}$

(D) $-\frac{7}{\sqrt{33}}$

4. Which of the following is the solution to $2\cos\theta = 1$ for $0° \leq \theta \leq 360°$?

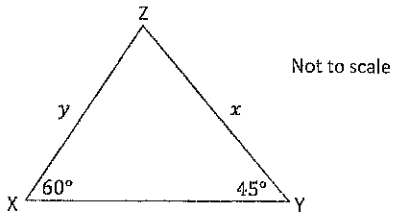
(A) $\theta = 60°$

(B) $\theta = 60°, 300°$

(C) $\theta = 120°, 240°$

(D) $\theta = 60°, 120°, 240°, 300°$

5. In the diagram, XYZ is a triangle where $\angle ZXY = 60°$ and $\angle ZYX = 45°$.
What is the exact value for $\frac{x}{y}$?



Not to scale

(A) $\frac{\sqrt{6}}{2}$

(B) $\sqrt{3}$

(C) $\frac{\sqrt{3}}{2\sqrt{2}}$

(D) 1.2

End of Section I

SECTION II
Total marks (40)
Attempt Questions 6-8

Start each question in a new answer booklet.
Correctly label each booklet with your student number and the appropriate question number.

Question 6 **Start a new answer booklet** **(12 marks)**

(a) HIJK is a quadrilateral with vertices $H(-4,-3)$, $I(0,-1)$, $J(6,-2)$ and $K(2,-4)$.
Draw this information onto a number plane in your writing booklet.

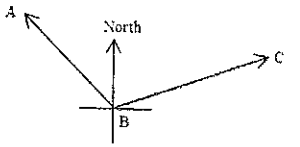
(i) Prove that HIJK is a parallelogram but *not* a rectangle. 3

(ii) Show that the equation of HI is $x - 2y - 2 = 0$ 1

(iii) Find the exact perpendicular distance from K to HI. 2

(iv) Hence or otherwise find the area of the parallelogram $HIJK$. 2

(b) Aaron and Carl leave point B at the same time. Aaron walks on a bearing of 310° at a speed of 1.8km/h and Carl walks on a bearing of 070° at a speed of 2.4km/h.



Not to scale

Copy the diagram above into your booklet and label the given information on your diagram.

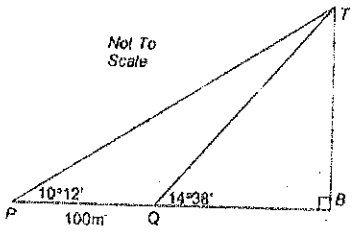
(i) How far apart are Aaron and Carl after 2 hours? Answer to 1 decimal place. 2

(ii) What is the true bearing of Carl from Aaron after 2 hours? 2

End of Question 6

Question 7 Start a new answer booklet (15 marks)

(a) 3



The angles of elevation of the top of a tree T from two points on the ground P and Q, are 10°12' and 14°38' respectively.

Find the height of the tree BT to the nearest metre.

(b) For the curve $f(x) = x^3(4-x)$

(i) Find the coordinates of the stationary points and determine their nature. 3

(ii) Find the coordinates of any points of inflexion. 2

(iii) Sketch the graph of $y = f(x)$ for $-1 \leq x \leq 4$ 3

(iv) For the given domain $-1 \leq x \leq 4$ when is the curve concave up? 1

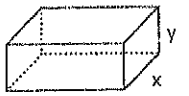
(c) Prove that $\frac{1}{secA-1} + \frac{1}{secA+1} = 2cotAcosecA$ 3

End of Question 7

Question 8 Start a new answer booklet (13 marks)

(a) A rectangular box, open at the top, is to be constructed out of a thin sheet of metal. It has a base which is twice as long as it is wide.

Not to Scale



(i) The box is to have a volume of 2304 cubic units. If its width is x units and its height is y units, find an equation for y in terms of x. 1

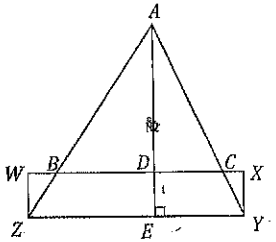
(ii) Show that the area A, in square units, of sheet metal required is given by 2

$$A = 2x^2 + \frac{6912}{x}.$$

(iii) Hence find the least area of sheet metal required to make such a box. 3

(b) A triangle ZAY is constructed using the base of a rectangle $WXYZ$, with intersection points B and C as shown. AE is the altitude of the triangle ZAY.

Not to Scale



(i) Copy the diagram into your writing booklet and prove that ΔBAC is similar to ΔZAY. 3

(ii) The area of ΔZAY is three times the area of the rectangle $WXYZ$. Show that 2

$$\frac{AD}{AE} = \frac{5}{6}$$

(iii) Hence or otherwise, show that $6CX + 6BW = ZY$ 2

End of Task

HSC Maths Task 1 Nov 2017

1 $\sin 120° = \sin 60°$

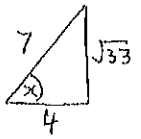
$= \frac{\sqrt{3}}{2}$

(A)

2. $f'(x) < 0$, $f''(x) > 0$

(C)

3. $\cos x = \frac{4}{7}$ $\tan x < 0$



S	A
T	C

cos +
tan −
sin −

$\sin x = -\frac{\sqrt{33}}{7}$

$\text{cosec } x = -\frac{7}{\sqrt{33}}$

(D)

4. $2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$ + in 1st & 4th Quadrant

$\theta = 60°, 300°$

(B)

5. $\frac{x}{\sin 60} = \frac{y}{\sin 45}$

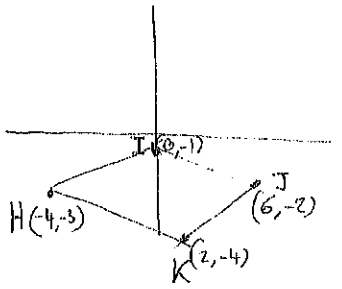
$\frac{x}{y} = \frac{\sin 60}{\sin 45} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{1}$

$= \frac{\sqrt{6}}{2}$

(A)

Q 6 a)

i.



$m_{HI} = \frac{-1+3}{0+4} = \frac{1}{2}$

$m_{JK} = \frac{-2+4}{6-2} = \frac{1}{2}$

$m_{IJ} = \frac{-2+1}{6-0} = -\frac{1}{6}$

$m_{KH} = \frac{-4+3}{2+4} = -\frac{1}{6}$

$m_{HI} = m_{JK}$
$\therefore HI \parallel JK$

$m_{IJ} = m_{KH}$
$\therefore IJ \parallel HK$

Parallelogram → 2 pairs of parallel sides.

$m_{HI} \times m_{IJ} \neq -1$ ∴ Not a rectangle.

ii. Equation of HI

$y - (-3) = \frac{1}{2}(x - (-4))$

$2y + 6 = x + 4$

$x - 2y - 2 = 0$

iii $x - 2y - 2 = 0$ K (2,-4)

$a=1$ $b=-2$ $c=-2$ $x=2$ $y=-4$

$\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$

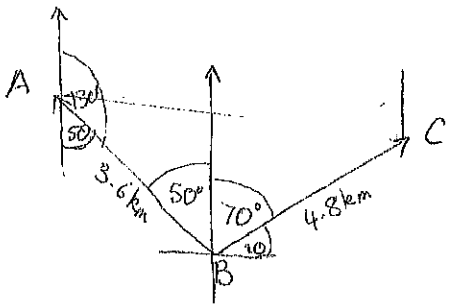
$= \frac{2 + (-2)(-4) + (-2)}{\sqrt{1+4}}$

$= \frac{8}{\sqrt{5}}$

iv. $A = b \times h$

base $= \sqrt{(-4-0)^2 + (-3+1)^2} \times \frac{8}{\sqrt{5}} = \sqrt{20} \times \frac{8}{\sqrt{5}} = 16 \text{ units}^2$

6 b)



i. $c^2 = a^2 + b^2 - 2bc\cos C$

$AC^2 = 3.6^2 + 4.8^2 - 2(3.6)(4.8)\cos 120°$

$AC^2 = 53.28$

$AC = \boxed{7.3\text{km}}$

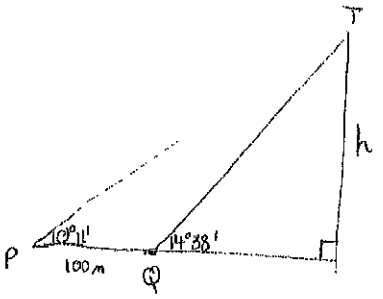
ii. $\frac{\sin \angle BAC}{4.8} = \frac{\sin 120}{7.3}$

$\sin \angle BAC = 34.7°$

$130 - 34.7° = \boxed{95.29°}$

True bearing $\boxed{095°}$

7 a)



$\tan 14°38' = \frac{h}{x}$

$x = \frac{h}{\tan 14°38'}$

$\tan 10°12' = \frac{h}{100+x}$

$\tan(10°12') \times (100 + x) = h$

$100\tan 10°12 + x\tan 10°12' = h$

$x = \frac{h}{\tan 14°38'}$

$17.99283993 + h(0.689111971 5) = h$

$h = \frac{17.99283993}{0.310888028 5}$

$h = 58\text{m}$ (nearest metre)

7 b) $f(x) = x^3(4-x)$

$= 4x^3 - x^4$

i. $f'(x) = 12x^2 - 4x^3 = 0$

$4x^2(3-x) = 0$

$x = 0 \quad x = 3$

$f''(x) = 24x - 12x^2$

$f''(0) = 0$ possible pt. of inflection

$f''(3) = -36 < 0$ max

x	-1	0	1
$f''(x)$	-36	0	+12

concavity change

(3,27) is a maximum pt

(0,0) is a horizontal pt. of inflection

ii. $f''(x) = 24x - 12x^2 = 0$

$12x(2-x) = 0$

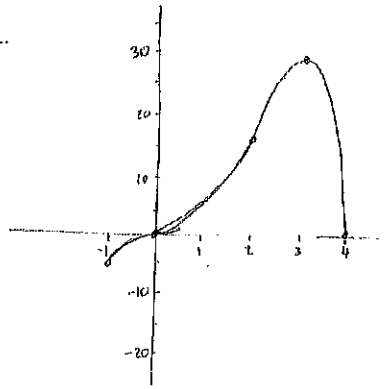
$x = 0 \quad x = 2$

x	1	2	3
$f''(x)$	+12	0	-36
	+		-

(2,16) is a pt. of inflection

iii.

x	-1	0	4
$f(x)$	-5	0	0



iv. Concave up when $0 < x < 2$

7 c) Prove $\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2\cot A \operatorname{cosec} A$

$LHS = \frac{\sec A + 1 + \sec A - 1}{\sec^2 A - 1}$

$= \frac{2\sec A}{\tan^2 A}$

$= \frac{2}{\cos A} \div \frac{\sin^2 A}{\cos^2 A}$

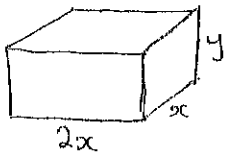
$= \frac{2}{\cos A} \times \frac{\cos^2 A}{\sin^2 A}$

$= 2 \times \frac{\cos A}{\sin A} \times \frac{1}{\sin A}$

$= 2\cot A \operatorname{cosec} A$

$= RHS$

8a)



i. Volume $= 2x^2y = 2304$

$$y = \frac{2304}{2x^2} = \boxed{\frac{1152}{x^2}}$$

ii. $A = 2xy + 4xy + 2x^2$

$= 2x^2 + 6xy$

$= 2x^2 + 6x\left(\frac{1152}{x^2}\right)$

$= 2x^2 + \frac{6912}{x}$ as required.

iii. $\frac{dA}{dx} = 4x - \frac{6912}{x^2} = 0$

$4x^3 - 6912 = 0$

$4x^3 = 6912$

$x^3 = 1728$

$x = 12$

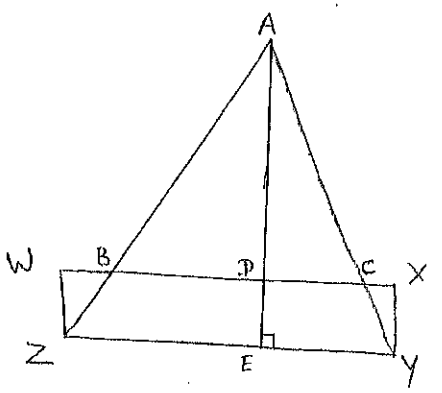
$\frac{d^2A}{dx^2} = 4 + \frac{13824}{x^3} > 0$ when $x = 12$

$\therefore$ concave up $\rightarrow$ minimum area when $x = 12$

Minimum Area is $2 \times 12^2 + \frac{6912}{12} = 864$ units^2

8b

i.



$\angle ZAY = \angle BAX$ (common)

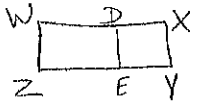
$\angle AZY = \angle ABX$ (WX || ZY properties of a rectangle. Corresponding angles in parallel lines)

$\angle ACB = \angle AYZ$ (Angle sum of $\triangle$)

$\therefore \triangle AZY \,|||\, \triangle ABX$ equiangular

8b

ii. Area 

$= 3$ Area 

$\frac{1}{2} ZY \times AE = 3\ ZY \times DE$

$ZY \times AE = 6\ ZY \times DE$

$AE = 6(AE - AD)$

$AE = 6AE - 6AD$

$6AD = 5AE$

$$\boxed{\frac{AD}{AE} = \frac{5}{6}}$$

iii Since $\triangle ZAY \;|||\; \triangle BAC$

$\frac{BC}{ZY} = \frac{5}{6}$ (from $\frac{AD}{AE} = \frac{5}{6}$)

$\frac{ZY - BW - CX}{ZY} = \frac{5}{6}$

$6ZY - 6BW - 6CX = 5ZY$

$$\boxed{ZY = 6BW + 6CX}$$ as required