Year 12 2 unit

Series and Sequences (G)

1 (i) In an arithmetic progression, the first term is 40 and the sixth term is 25. Find

(a) the common difference

(b) the least value of n such that the sum of the first n terms of the series is negative.

(Barker College 1982)

(ii) Find the limiting sum of the series $2 + 1 + \frac{1}{2} + \ldots.$

2 Which term of the sequence -8, -2, 4, 10, ... is the number 208? (SCEGGS 1991)

3 For an arithmetic progression the fifth term is 16 and the eleventh term is 40. Find the first term and the common difference. Hence find how many terms must be added to obtain a sum of 312. (Shore 1991)

4 (i) *Write the formulae for the*

(α) *nth term*

(β) *sum to n terms of an A.P.*

(Ascham 1990)

(ii) *Hence write expressions for the fourth term of an A.P. and the first five terms of an A.P. in terms of a and d.*

(iii) *In an A.P. the ratio of the second term to the fourth term is 11 : 13 and the sum of the first five terms is 30. Find the first term and common difference.*

5 Find the value of (Sydney Girls' High School) 1994

$$\sum_{n=1}^{20} (5n - 1)$$

6 The third term of a G.P. is $\frac{27}{4}$ and the fifth term is $\frac{234}{16}$. Find the sequence(s). (Danebank 1993)

7 The first three terms of a geometric series are (Catholic Trial) 1989

2, 6, 18.

Find the value of the fifteenth term leaving your answer in index form.

8 The numbers 4, a, b form a G.P. whilst the numbers a, b, 12 form an A.P. Find the values of a, b. (PLC 1984)

9 Given the arithmetic series 180 + 174 + 168 + ... (Kambala 1987)

a) Find the value of the twenty-third term.

b) Find the least number of terms such that the sum should be negative.

10 In an arithmetic sequence, the fourth term is 12 and the fourteenth term is 62. (Catholic Trial 1994)

(i) Find the first term and the common difference.

(ii) Calculate the sum of the first 50 terms of this sequence.

Series & Sequences (G)

① i) $T_1 = 40$
$T_6 = 25$

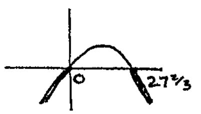
a) AP $a = 40$ —①
$a + 5d = 25$ —②

Substitute ① into ②
$40 + 5d = 25$
$5d = -15$
$d = -3$

b) $S_n = \frac{n}{2}[2a + (n-1)d]$
$= \frac{n}{2}[2\times 40 + (n-1)\times -3]$
$= \frac{n}{2}[80 - 3n + 3]$
$= \frac{n}{2}[83 - 3n]$

Solve $S_n < 0$
$\frac{n}{2}[83 - 3n] < 0$

Quadratic inequality.
cuts axis at $n = 0, \frac{83}{3}$



$n < 0$ Not valid, $n > 27\frac{2}{3}$

$\therefore n = 28$

$\therefore$ For 28 terms the sum is negative.

ii) $2 + 1 + \frac{1}{2} + \dots$

GP $a = 2$
$r = \frac{1}{2}$

Limiting Sum
$S = \frac{a}{1-r}$
$= \frac{2}{1 - \frac{1}{2}}$
$= \frac{2}{\frac{1}{2}}$
$= 4$

② $-8, -2, 4, 10, \dots$

AP $a = -8$, $d = 6$ $T_n = 208$ Find n.

$T_n = a + (n-1)d$
$208 = -8 + (n-1).6$
$208 = -8 + 6n - 6$
$208 = 6n - 14$
$6n = 222$
$n = \frac{222}{6}$
$n = 37$

$\therefore T_{37} = 208$

③ $T_5 = 16$
$T_{11} = 40$

AP $a + 4d = 16$ —①
$a + 10d = 40$ —②

Subtract
$6d = 24$
$d = 4$

Subst. into ①
$a + 16 = 16$
$a = 0$

$\therefore$ First term $a = 0$
Common diff. $d = 4$

$S_n = 312$, Find n

$S_n = \frac{n}{2}[2a + (n-1)d]$
$312 = \frac{n}{2}[0 + (n-1).4]$
$312 = \frac{n}{\cancel{2}} \cdot \cancel{4}^{2}(n-1)$
$312 = 2n(n-1)$
$312 = 2n^2 - 2n$
$2n^2 - 2n - 312 = 0$
$n^2 - n - 156 = 0$
$(n-13)(n+12) = 0$
$n = 13$ $n = -12$ Not valid

$\therefore S_{13} = 312$

④ i) AP
α) $= a + (n-1)d$

β) $S_n = \frac{n}{2}[2a + (n-1)d]$
or
$S_n = \frac{n}{2}[a + l]$

ii) $T_4 = a + 3d$

$S_5 = \frac{5}{2}[2a + (5-1)d]$
$= \frac{5}{2}[2a + 4d]$
$= \frac{5}{2}\times 2a + \frac{5}{2}\times 4d$
$= 5a + 10d$

iii) $T_2 : T_4 = 11 : 13$
$S_5 = 30$

$\frac{T_2}{T_4} = \frac{11}{13}$

$\frac{a+d}{a+3d} = \frac{11}{13}$
cross multiply
$13(a+d) = 11(a+3d)$
$13a + 13d = 11a + 33d$
$2a - 20d = 0$
$a - 10d = 0$ —①

$S_5 = 30$
$5a + 10d = 30$
$a + 2d = 6$ —②

Solve simultaneously
$a - 10d = 0$ —①
$a + 2d = 6$ —②
Subtract
$-12d = -6$
$d = \frac{-6}{-12}$
$d = \frac{1}{2}$

Substitute into ①
$a - 10\times\frac{1}{2} = 0$
$a - 5 = 0$
$a = 5$

$\therefore a = 5$, $d = \frac{1}{2}$

⑤ $\sum_{n=1}^{20} (5n-1)$

$= (5-1) + (10-1) + (15-1) + \dots (100-1)$
$= 4 + 9 + 14 + \dots + 99$

AP $a = 4$
$d = 5$
$n = 20$

$S_n = \frac{n}{2}[a + l]$
$S_{20} = \frac{20}{2}[4 + 99]$
$= 10 \times 103$
$= 1030$

⑥ GP $T_3 = ar^2 = \frac{27}{4}$ —①

$T_5 = ar^4 = \frac{234}{16}$ —②

Divide T_5 by T_3

$\frac{ar^4}{ar^2} = \frac{\frac{234}{16}}{\frac{27}{4}}$

$r^2 = 2\frac{1}{6}$

$r^2 = \frac{13}{6}$

$r = \pm\sqrt{\frac{13}{6}}$

Find a (sub. into ①)
when $r = \sqrt{\frac{13}{6}}$ or $-\sqrt{\frac{13}{6}}$

$a.\left(\sqrt{\frac{13}{6}}\right)^2 = \frac{27}{4}$

$a.\frac{13}{6} = \frac{27}{4}$

$a = \frac{\frac{27}{4}}{\frac{13}{6}}$

$= 3\frac{3}{26}$

the sequences are
$a = 3\frac{3}{26}$ $r = \sqrt{\frac{13}{6}}$

$3\frac{3}{26}, \frac{81}{26}\sqrt{\frac{13}{6}}, 6\frac{3}{4}, \dots$

$a = 3\frac{3}{26}$, $r = -\sqrt{\frac{13}{6}}$

$3\frac{3}{26}, -\frac{81}{26}\sqrt{\frac{13}{6}}, 6\frac{3}{4}, \dots$

⑦ $2, 6, 18, \dots$

GP $a = 2$
$r = 3$

$T_n = ar^{n-1}$
$T_{15} = 2.3^{15-1}$
$= 2.3^{14}$

⑧ GP $4, a, b$

$\frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\frac{a}{4} = \frac{b}{a}$

$a^2 = 4b$ —①

AP $a, b, 12$
$T_2 - T_1 = T_3 - T_2$
$b - a = 12 - b$
$2b = a + 12$
$b = \frac{1}{2}(a+12)$ —②

Substitute ② into ①
$a^2 = 4 . \frac{1}{2}(a+12)$
$a^2 = 2(a+12)$
$a^2 = 2a + 24$
$a^2 - 2a - 24 = 0$
$(a-6)(a+4) = 0$
$a = 6$ $a = -4$
$\therefore b = \frac{1}{2}\times 18$ $b = \frac{1}{2}(-4+12)$
$= 9$ $= \frac{1}{2}\times 8$
$= 4$

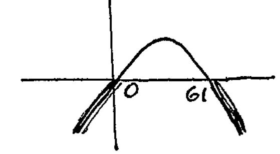
$\therefore a = 6, b = 9$
or
$a = -4, b = 4$

⑨ $180 + 174 + 168 + \dots$

AP $a = 180$
$d = -6$

a) $T_n = a + (n-1)d$
$T_{23} = 180 + (23-1)\times -6$
$= 180 + 22\times -6$
$= 180 - 132$
$= 48$

b) $S_n = \frac{n}{2}(2a + (n-1)d)$
Solve $S_n < 0$
$\frac{n}{2}(360 + (n-1).-6) < 0$
$\frac{n}{2}(360 - 6n + 6) < 0$
$\frac{n}{2}(366 - 6n) < 0$
$n(183 - 3n) < 0$
Quadratic inequality



$n < 0$ Not valid, $n > 61$

$\therefore n = 62$

$\therefore S_{62} < 0$

⑩ AP $T_4 = a + 3d = 12$ —①
$T_{14} = a + 13d = 62$ —②

i) subtract
$10d = 50$
$d = 5$
Subst. into ①
$a + 15 = 12$
$a = -3$
$\therefore$ first term $a = -3$
common diff. $d = 5$

ii) $S_n = \frac{n}{2}[2a + (n-1)d]$
$S_{50} = \frac{50}{2}[-6 + 49\times 5]$
$= 25 \times 239$
$= 5975$