Year 12
2 unit

Series and Sequences (H)

1

(a) Evaluate : (i) $\sum_{n=1}^{6}(5+4n)$ (Trinity Grammar School 1993)

(ii) $\sum_{r=2}^{\infty}(0.5)^r$

(b) In an arithmetic sequence, the sum of the first three terms is 54, and the tenth term is 50.

Find : (i) The first term and the common difference,

(ii) The sum of the first 20 terms.

2. Alex accepted a job that pays an initial salary of \$28 000 per annum. After each year of service she will receive an increment of \$950 until she reaches the maximum salary of \$40 350.

(i) What will her salary be after 5 years of service? (Catholic Trial 1996)

(ii) How long will she have to work before she reaches her maximum salary?

(iii) Calculate her total earnings for the first 10 years of service.

3. For the sequence, 3, 5, 7, 9, ..., (Catholic Trial 1983)

find (a) the nth term;

(b) the sum of the first 30 terms. (Leave your answer in index form.)

4. Find the sum of the multiples of 3 between 100 and 250. (The Scot's College 1995)

5. At a disaster refugee centre, there was originally 150 tonnes of supplies for distribution. Thereafter 14 tonnes were flown in on the first day of the disaster , 16 tonnes the next day . 18 tonnes the next day and so on to continue. (South Sydney High School 1994)

(i) Find a formula for the total mass (T.a) of supplies available after n days of the disaster.

(ii) However, on the first day of disaster 8 tonnes of supplies was distributed, 11 tonnes on the next day, 14 tonnes on the day after and so on to continue. Find a formula for the total mass (T.d) of supplies distributed after n days.

(iii) Using the results in parts (i) and (ii), or otherwise, find how many days after the disaster, will the supplies run out (ie. no supplies left) .

6. a) The third and seventh terms of an arithmetic series are 17 and 37 respectively. Find the first term and the common difference for this series. (The Scot's College 1992)

b) The first three terms of an arithmetic series are 15, 13, 11. Show that the sum of the first n terms is given by $S_n = 16n - n^2$.

c) The first three terms of a geometric sequence are 8, 4, 2.
If the last term of the sequence is $\frac{1}{128}$, how many terms are there in the sequence?

d) A plant is observed over a period of time. Its initial height is 30cm. It grows 5cm during the first week of observation. Each suceeding week the growth is 80% of the previous week's growth. Assuming this pattern continues, calculate the plant's ultimate height.

Series & Sequences (H)

1) a) $\sum_{n=1}^{6}(5+4n)$

$= (5+4)+(5+8)+(5+12) + \dots + (5+24)$

$= 9+13+17+\dots+29$

AP $a=9$, $d=4$

$S_n = \frac{n}{2}[a+l]$

$S_6 = \frac{6}{2}[9+29]$

$= 3\times 38$

$= 114$

ii) $\sum_{r=2}^{\infty}(0.5)^r$

$=(0.5)^2+(0.5)^3+(0.5)^4+\dots$

Find limiting sum

GP $a=(0.5)^2$, $r=0.5$

$S = \frac{a}{1-r}$

$= \frac{(0.5)^2}{1-0.5}$

$= \frac{(0.5)^2}{0.5}$

$= 0.5$

b) AP $S_3 = 54$, $T_{10} = 50$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_3 = \frac{3}{2}[2a+2d]$

$54 = 3a+3d$

$a+d = 18$ —①

$T_n = a+(n-1)d$

$T_{10} = a+9d$

$a+9d = 50$ —②

Solve simultaneously

$a+d=18$ —①

$a+9d=50$ —②

Subtract

$8d = 32$

$d=4$

Subst. into ①

$a+4=18$

$a=14$

i) First term $a=14$

Common diff. $d=4$

ii) $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{20} = \frac{20}{2}[28+19\times 4]$

$= 10\times 104$

$= 1040$

② AP $\begin{cases} a=28000 \\ d=950 \end{cases}$

$28000, 28950, 29900, \dots, 40350$

i) $T_n = a+(n-1)d$

$T_5 = 28000 + 4\times 950$

$= \$31800$

ii) $T_n = 40350$

$a+(n-1)d = 40350$

$28000+(n-1).950 = 40350$

$950(n-1) = 12350$

$n-1 = 13$

$n = 14$

Maximum salary reached after 14 years.

iii) $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{10} = \frac{10}{2}[56000+9\times 950]$

$= 5\times 64550$

$= \$322750$

③ 3, 5, 7, 9 ...

a) AP $a=3$, $d=2$

$T_n = a+(n-1)d$

$= 3+(n-1).2$

$= 3+2n-2$

$= 2n+1$

b) $S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{30} = \frac{30}{2}[6+29\times 2]$

$= 15\times 64$

$= 960$

④ Multiples of 3 between 100 and 250

$\left.\begin{array}{l} 34\times 3 = 102 \\ 83\times 3 = 249 \end{array}\right\} \begin{array}{l} n = 83-34+1 \\ = 50 \end{array}$

$102, 105, 108, \dots, 249$

AP $a=102$, $d=3$, $n=50$

$S_n = \frac{n}{2}[a+l]$

$S_{50} = \frac{50}{2}[102+249]$

$= 25\times 351$

$= 8775$

⑤ Supplies available originally 150 t

i) $150 + 14+16+18+\dots$

AP $a=14$, $d=2$

$T_n = a+(n-1)d$

$= 14+(n-1).2$

$= 14+2n-2$

$= 2n+12$

Total mass after n days

$\therefore$ T.a. $= 150+2n+12$

$= 162+2n$

ii) Distributed

$8+11+14+\dots$

AP $a=8$, $d=3$

$T_n = a+(n-1)d$

$= 8+(n-1).3$

$= 8+3n-3$

$= 3n+5$

T.d. $= 3n+5$

iii) No supplies left when T.a. = T.d.

$162+2n = 3n+5$

$158 = n$

$\therefore n = 158$

$\therefore$ Supplies will run out after 158 days.

⑥ a) AP $T_3 = 17$, $T_7 = 37$

Solve simultaneously

$a+2d = 17$ —①

$a+6d = 37$ —②

Subtract

$4d = 20$

$d = 5$

Subst. into ①

$a+10 = 17$

$a = 7$

$\therefore$ First term $a=7$

Common diff. $d=5$

b) $15, 13, 11, \dots$

AP $a=15$, $d=-2$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$= \frac{n}{2}[30+(n-1).-2]$

$= \frac{n}{2}[30-2n+2]$

$= \frac{n}{2}[32-2n]$

$= n[16-n]$

$= 16n-n^2$

c) GP $8, 4, 2, \dots, \frac{1}{128}$

$a=8$, $r=\frac{1}{2}$

$T_n = ar^{n-1}$

$\frac{1}{128} = 8.\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{1024} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{2^{n-1}} = \frac{1}{1024}$

Reciprocals.

$2^{n-1} = 1024$

$2^{n-1} = 2^{10}$

$\therefore n-1 = 10$

$n = 11$

$\therefore T_{11} = \frac{1}{128}$

There are eleven terms.

d) $30\text{ cm}, \underbrace{5, 4, 3.2,}_{\text{growth}}$

GP $a=5$, $r = 80\% = \frac{4}{5}$

Ultimate growth is given by the limiting sum of the GP.

$S = \frac{a}{1-r}$

$= \frac{5}{1-4/5}$

$= \frac{5}{1/5}$

$= 25$

$\therefore$ Ultimate height

$= 30+25$

$= 55$ cm.