Student Name

St. Catherine's School Waverley

September 2016

Mathematics Extension 1

Total Marks – 55

Section I

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided.

Section II

45 marks

- Attempt Questions 11-13
- Allow about 1 hour and 15 minutes for this section
- Answer each question in the booklet provided.

General Instructions

- Reading Time – 5 minutes
- Working Time – 1.5 hours
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- Show all necessary working in Questions 11-13
- Task weighting – 50%

Section I

Total marks - 10
Attempt Questions 1-10
All questions are of equal value.
Answer either *A,B,C* or *D* on the multiple choice answer sheet provided.

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1 What is the solution to the inequality $\frac{3}{x-2} \leq 4$?

A $x \geq \frac{11}{4}$

B $2 \leq x \leq \frac{11}{4}$

C $2 < x \leq \frac{11}{4}$

D $x < 2 \ or \ x \geq \frac{11}{4}$

2 A: (-4,-3) and B: (1, 5). A point P divides AB internally in the ratio 3:2. What are the coordinates of P?

A $(-\frac{1}{4}, 3)$

B $(-\frac{1}{2}, 6)$

C $(-1, \frac{9}{5})$

D $(-\frac{3}{4}, \frac{1}{2})$

3 The roots of the polynomial $x^3 - 7x + 6 = 0$ are

A -1, -2, 3

B 0, 6 and -6

C 2, 1 and $\frac{3}{2}$

D 1, 2 and -3

4 The general solution to $\tan 3x = -\sqrt{3}$ is

A $x = 180n + 40^0$

B $x = 60n + 40^0$

C $x = 60n + 120^0$

D $x = 180n + 120^0$

5 Find the value of k if $(x-2)$ is a factor of $P(x) = x^3 - 3x^2 + kx + 12$

A $k = 0$

B $k = -4$

C $k = 2$

D $k = -2$

6 The acute angle between $y = \sqrt{3}\,x + 1$ and $y = x$ is closest to

A 15^0

B 60^0

C 45^0

D 0^0

7 $\cos(A+B)\cos A - \sin(A+B)\sin A$ is

A $\cos A$

B $\sin A$

C $\cos(2A+B)$

D $\sin(2A+B)$

8 $\cos x - \sqrt{3}\sin x = 2\cos(x+\alpha)$. Then α to the nearest degree is

A 60^0

B 30^0

C 120^0

D -30^0

9 When a polynomial is divided by $(x-2)$, the remainder is 5. When the polynomial is divided by x, the remainder is 3. What is the remainder when the polynomial is divided by $x(x-2)$

A $5x+3$

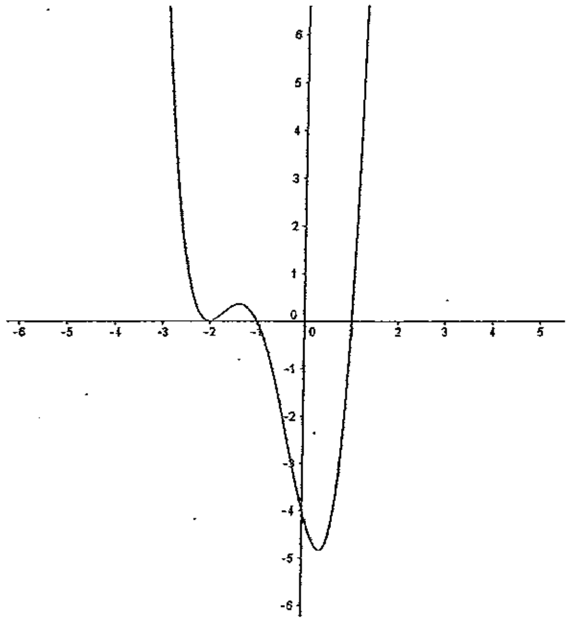
B $3x+5$

C 8

D $x+3$

Please turn over for Question 10.

10



The possible equation of the graph is

A $y=(x+2)^2(x+1)(x-1)$

B $y=(x^2-2)(x-1)(x+1)$

C $y=(x+2)^2(1-x)(x+1)$

D $y=(x-2)^2(x-1)(x+1)$

Section II

Total marks - 45
Attempt Questions 11-13
The questions are of equal value

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

Question 11 (Start a new booklet) 15 marks

a) Solve for x: **3**

$$\frac{2x+3}{x-2} > 1$$

b) The acute angle between the lines $y = 2x - 3$ $\quad and\ mx + y - 5 = 0$ is 45^0. **3**

Find the possible value(s) of m.

c) Point P with coordinates *(11, -11)* divides the interval joining A *(-1, 7)* and B *(5,-2)* externally in the ratio *k:1*. Find the value of *k*. **2**

d) The polynomial $P(x) = ax^3 - 4bx^2 + x - 4$ leaves a remainder of 17 when divided by $(x-3)$ and a remainder of -11 when divided by $(x+1)$ **3**

Find the value of a and b.

Question 11 is continued on page 9

Question 11 continued...

e) A polynomial $P(x)$ is odd i.e. $P(-x) = -P(x)$.

(i) Given that k is a zero of an odd polynomial $P(x)$, show that $-k$ is also a zero for $P(x)$. **1**

(ii) Show that x is a factor of $P(x)$. **1**

iii (ii) An odd polynomial P(x) has zeros at $x = 2\ and\ x = -1$ and *P(x)* is of degree 5. **1**

Explain why P(x) can be written in the form $P(x) = A\,x\,(x+2)(x-2)(x+1)(x-1)$, where A is a constant.

(iii) iv Write down an expression for P(x) if P (3) = 240. **1**

Please turn over for Question 12.

Question 12 (Start a new booklet) 15 marks

a) (i) Expand $\cos(A + B)$ 1

(ii) Hence show that $\cos 2\theta = 2\cos^2\theta - 1$ 2

(iii) Hence or otherwise show that the exact value of 2

$cos22\frac{1}{2}^0 = \frac{1}{2}\sqrt{2+\sqrt{2}}$

b) Show that $\frac{\sin 2A - \cos 2A + 1}{\sin 2A + \cos 2A + 1} = \tan A$ 3

c) Find the general solution to 3

$3\cos 2\theta = 2\cos^2\theta$

d) Use the substitution $t = tan\frac{x}{2}$ and solve the following equation 4

$7sinx - 4cosx = 4 \quad 0 \leq x \leq 360^0$

Please turn over for Question 13

Question 13 (Start a new booklet)

a) Find the values of k, for which $y = x^2 - kx + 4$ is positive definite 2

b) (i) Show that $(x - 1)$ is a factor of the polynomial 1
$P(x): 2x^3 + 11x^2 + 2x - 15 = 0$

(ii) Hence completely factories $P(x)$ 2

(iii) Draw a clear sketch of the above polynomial clearly indicating the x and y intercepts. 2

c) If α and β are the roots of the equation 4

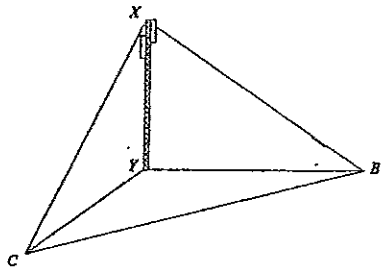
$3x^2 + 5x - 6 = 0$, find without evaluating α and β the quadratic equation

whose roots are α^2 and β^2

Question 13 continues on page 12.

Question 13 continued...

d) From a point C due south of a tower XY, the angle of elevation of the tower is 22^0. From a point B due east of the tower, the angle of elevation is 35^0.
The distance between B and C is 50 metres.



(i) Show that $CY = h\tan 68^0$ and $BY = h\tan 55^0$ 1

(ii) Hence or otherwise show that the height of the tower is 2

$$h = \frac{50}{\sqrt{\tan^2 68^0 + \tan^2 55^0}}$$

(iii) Find the value of h to the nearest metre 1

END of Paper

Yr 11 Extension 1 – End of Preliminary examination
Questions 1 to 12

1. $\frac{3}{x-2} \le 4$

$3(x-2) \le 4(x-2)^2$

$(x-2)(3-4(x-2)) \le 0$

$(x-2)(11-4x) \le 0$

$x \ne 2$. $x < 2$; $x \ge \frac{11}{4}$ $\boxed{D}$

2. $\left(\frac{3(1)+2(-4)}{5}, \frac{3(5)+2(-3)}{5}\right)$ $\boxed{C}$

$\left(-1, \frac{9}{5}\right)$

3. $1+2-3 = 0$ $\boxed{D}$

$1\times2+2\times-3+1\times-3 = -7$

$1\times2\times-3 = -6$.

4. $3x = 180n + 120^\circ$ $\boxed{B}$

$x = 60n + 40$

5. $P(2) = 0$ $\therefore k = -4$ $\boxed{B}$

6. $m_1 = \sqrt{3}$ $m_2 = 1$ $\tan\theta = \left|\frac{\sqrt{3}-1}{1+\sqrt{3}}\right|$

OR: $y = \sqrt{3}x + 1$ makes an angle of 60^0 with the positive direction of the x axis ($\tan^{-1}\sqrt{3} = 60^\circ$)

a $y = x$ make 45^0 ($\tan^{-1} 1 = 45^\circ$)

$\therefore$ the angle between the line is 15^0 $\boxed{A}$

7. $\cos(A+B+A) = \cos(2A+B)$ $\boxed{C}$

8. $\cos x - \sqrt{3}\sin x = 2(\cos x \cos\alpha - \sin x \sin\alpha)$

$1 = 2\cos\alpha$

$\sqrt{3} = 2\sin\alpha$

$\tan\alpha = \sqrt{3}$ $\boxed{A}$

$\alpha = 60^\circ$

9. $P(x) = x(x-2)Q(x) + ax + b$

($x(x-2)$ is quadratic $\therefore ax+b$, a linear expression has degree 1 less)

$P(2) = 5$; $5 = 0 + 2a + b$

$P(0) = 3$; $3 = b$.

$b = 3$, $a = 1$

$\therefore$ The remainder is $x + 3$ $\boxed{D}$

10A. double root at $x = -2$

Single roots at $x = -1$ and $x = 1$

$\boxed{A}$

Qn	Solutions	Marks	Comments: Criteria
	Question 11		all method.
	$\frac{2x+3}{x-2} > 1$ $\quad x \neq 2$ don't penalise.		
	$(x-2)^2 \cdot \frac{2x+3}{x-2} > (x-2)^2$		
	$(x-2)(2x+3) > (x-2)^2$	1	
	$(x-2)(2x+3-x+2) > 0$		
	$(x-2)(x+5) > 0$	1.	
	$x < -5$ or $x > 2$	1.	
b)	grad. of $y = 2x+3$; $m_1 = 2$		
	grad. of $mx + y - 5 = 0$ $\quad m_2 = -m$		
	$\tan 45^\circ = \left\lvert\frac{2+m}{1-2m}\right\rvert =$		
	$1 = \left\lvert\frac{2+m}{1-2m}\right\rvert$		
	$2+m = 1-2m$ (1) $\quad 2+m = -1+2m$ (1)		
	$3m = -1$ $\quad 3 = m$		
	$m = -\frac{1}{3}$		
c)	A: $(-1, 7)$ $\quad$ B: $(5, -2)$		minus 1 if an internal calculation was done.
	k $\quad -1$		
	P: $(11, -11)$		minus 1 for incorrect application of formula.
	$11 = \frac{5k+1}{k-1}$ (1M) $\quad 11k - 11 = 5k + 1$		
	$6k = 12$ (1M)		
	$k = 2$		

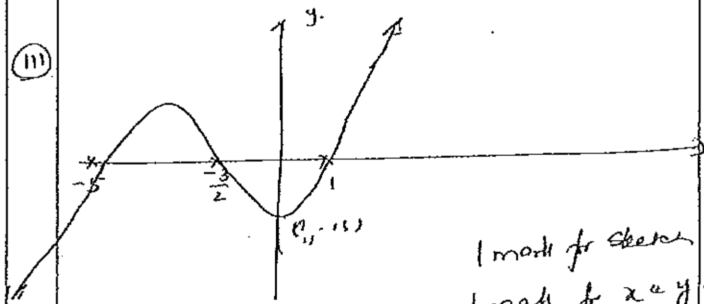
Qn	Solutions	Marks	Comments: Criteria
	You could do work with the y. coordinate It is not necessary to do both.		
d)	$P(x) = ax^3 - 4bx^2 + x - 4$		
	$P(3) = 17$ $P(-1) = -18$.	(1M)	
	$17 = 27a - 36b + 3 - 4$ $18 = 27a - 36b$ $2 = 3a - 4b$ —— ①		for two eqns
	$-18 = -a - 4b - 1 - 4$ $a + 4b = 6$. —— ②		1M
	Solve ① & ② simultaneously ①+② $4a = 8$ $a = 2$ sub. $2 + 4b = 6$ $4b = 4$ $b = 1$		
e)	K is a zero; $\therefore P(K) = 0$.		
i)	$P(-K) = -P(K)$ $P(x)$ is odd $= 0$ $\therefore -K$ is a zero.		
ii)	$P(0) = -P(0)$ $\therefore P(0) = 0$ $\therefore x\ (= (x-0))$ is a factor.		

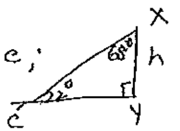
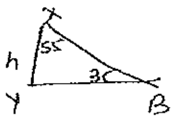
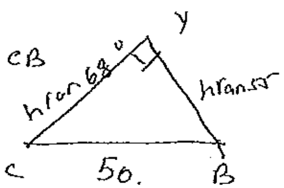
Qn	Solutions	Marks	Comments: Criteria
	$x = 2$ is a zero $\therefore x = -2$ is also a zero $(x-2)$ is a factor & $(x+2)$ is a factor Similarly $(x+1)$ and $(x-1)$ are factors. $P(x)$ being odd x is a factor. $P(x)$ is of degree 5 $\therefore P(x) = A\,x(x+1)(x-1)(x-2)(x+2)$ where A is a constant.	(1M)	must justify
	$P(3) = 240$ $240 = A(3)(5)(1)(4)(2)$ $= 120A$ $A = 2$		
	$\therefore P(x) = 2x(x+1)(x-1)(x-2)(x+2)$		

Qn	Solutions	Marks	Comments: Criteria
	Question 12		
	$\cos(A+B) = \cos A\cos B - \sin A\sin B$		1M or 0
	put $A = B = \theta$	1M	
	$\cos 2\theta = \cos^2\theta - \sin^2\theta$		
	$= \cos^2\theta - (1-\cos^2\theta)$	1M	
	$= 2\cos^2\theta - 1$		
	$\cos 45 = 2\cos^2 22\frac{1}{2} - 1$	½	
	$2\cos^2 22\frac{1}{2} = \cos 45 + 1$		
	$= \frac{1}{\sqrt{2}} + 1$		
	$= \frac{1+\sqrt{2}}{\sqrt{2}}$	1M	
	$\cos^2 22\frac{1}{2} = \frac{1+\sqrt{2}}{2\sqrt{2}}$		$(-\frac{1}{2})$ if $\cos 22\frac{1}{2} > 0$ is not noted.
	$\cos 22\frac{1}{2}° > 0 \therefore \cos 22\frac{1}{2}° = \sqrt{\frac{1+\sqrt{2}}{2\sqrt{2}}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$		
	$= \frac{\sqrt{2+\sqrt{2}}}{2}$.	1M	
b)	$\frac{\sin 2A - \cos 2A + 1}{\sin 2A + \cos 2A + 1}$		
	$= \frac{\sin 2A - (1 - 2\sin^2 A) + 1}{\sin 2A + 2\cos^2 A - 1 + 1}$		
	$= \frac{\sin 2A + 2\sin^2 A}{\sin 2A + 2\cos^2 A}$		

Qn	Solutions	Marks	Comments: Criteria
	$= \frac{2\sin A\cos A + 2\sin^2 A}{2\sin A\cos A + 2\cos^2 A}$		
	$= \frac{2\sin A\,\cancel{(\cos A + \sin A)}}{2\cos A\,\cancel{(\sin A + \cos A)}}$	1M	
	$= \tan A$		
c)	$3\cos 2\theta = 2\cos^2\theta$		
	$3(2\cos^2\theta - 1) = 2\cos^2\theta$		
	$6\cos^2\theta - 3 = 2\cos^2\theta$		
	$4\cos^2\theta = 3$	1M.	
	$\cos\theta = \pm\frac{\sqrt{3}}{2}$		
	$\cos\theta = \frac{\sqrt{3}}{2}$		
	$\theta = 360n \pm 30°$	M	
	$\cos\theta = -\frac{\sqrt{3}}{2}$		
	$\theta = 360n \pm 150°$.	1M.	-1 if one solution is wrong.
d)	$7\sin x - 4\cos x = 4$		
	$t = \tan\frac{x}{2}$; $\sin x = \frac{2t}{1+t^2}$; $\cos x = \frac{1-t^2}{1+t^2}$		
	note $\frac{x}{2} \neq 90°$		
	$x \neq 180°$		

Qn	Solutions	Marks	Comments: Criteria
	$7\left(\frac{2t}{1+t^2}\right) - 4\left(\frac{1-t^2}{1+t^2}\right) = 4$		
	$14t - 4 + 4t^2 = 4 + 4t^2$	1½	
	$14t = 8$		
	$t = \frac{4}{7}$.	1	
	$\tan\frac{\theta}{2} = \frac{4}{7}$		
	$\frac{\theta}{2} = 29.7°, 209.7°$		
	$\theta = 59.5°$		
	Test $x = 180°$	M	
	$7\sin 180° - 4\cos 180°$		
	$= -4(-1)$		
	$= 4$		
	$\therefore x = 180°$ is also a solution.		

Qn	Solutions	Marks	Comments: Criteria
	Question 13		
	$y = x^2 - kx + 4$ is positive definite when its $\Delta = (-k)^2 - 4(1)(4) < 0$		
	$k^2 - 16 < 0$ (1M)		
	$(k-4)(k+4) < 0$ also coeff of $x^2 = 1 > 0$ ½ m		
	$-4 < k < 4$ (½)		
b)	$P(x) = 2x^3 + 11x^2 + 2x - 15$		
(i)	$P(1) = 2 + 11 + 2 - 15$		
	$= 0$		
	$\therefore x - 1$ is a factor.		
(ii)	$2x^3 + 11x^2 + 2x - 15$		
	$= (x-1)(2x^2 + 13x + 15)$	1M	
	by observation or long division)		
	$= (x-1)(2x+3)(x+5)$	½	
(iii)			
	1 mark for sketch		
	1 mark for x & y intercepts		

Qn	Solutions	Marks	Comments: Criteria
c)	$3x^2+5x-6=0$ $\alpha+\beta=\frac{-5}{3}$ $\qquad\qquad \alpha^2\beta^2=4$ $\alpha\beta=-\frac{6}{3}=-2$ $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ $=\frac{25}{9}+4$ $=\frac{61}{9}.$ The required equation is $x^2-(\alpha^2+\beta^2)x+\alpha^2\beta^2=0$ $x^2-\frac{61}{9}x+4=0$ $9x^2-61x+36=0.$	$(\alpha^2+\beta^2)$ 1M 1M 1M	1M
d)	In Δ XYC;  $\frac{CY}{h}=\tan 68^\circ$ $CY=h\tan 68^\circ$ In Δ XYB  $\frac{BY}{h}=\tan 55^\circ$ $BY=h\tan 55^\circ$		1M.
(i)	In Δ YCB  (Angle between two pts in East & South is 90°) $50^2=h^2\tan^2 68+h^2\tan^2 55$; (with reasons $=h^2(\tan^2 68+\tan^2 55)$ $h=\frac{50}{\sqrt{\tan^2 68+\tan^2 55}}$ 1M $=17$ .. 1M (to n. metre)	1M	