NAME _____

CLASS _____

Randwick Girls' High School

MATHEMATICS

YEAR 12 ASSESSMENT TASK II
11th March 2014

Examiner: Mr Stewart

General Instructions

- Reading time – 5 minutes
- Working Time – 45 minutes
- Write using a black or blue pen
- Approved calculators may be used
- All necessary working should be shown for every question.
- Work down the page, not across!
- Begin each question on a new page.
- Questions 1-5 Multiple choice (circle correct answer on sheet)

Question	Marks
1-5	/5
6	/6
7	/6
8	/9
9	/5
10	/7
Total	/38

Marks

Question 6. **(6 marks)**

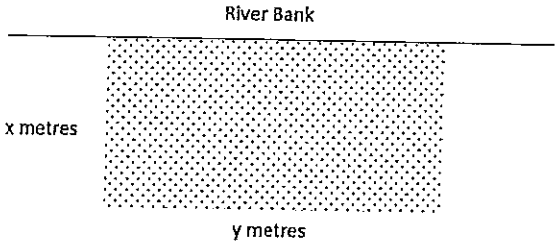
a) Find the locus of a point P(x,y) which moves in a plane such that 3

$PA^2 + PB^2 = 44$ where A = (-3,2) and B = (3,-2)

b) Show that the locus is a circle and find its centre and radius. 3

Question 7. **(6 marks)**

A farmer has a straight river running through his property. He wishes to create a rectangular field using the river as one side. He has 200 m of fencing and wishes to make a field with the largest possible area.



a) If the length of the field is y m and the width is x m, show that the area is given by: 3

$$A = 200x - 2x^2$$

b) Find the value of x that will give the field its greatest area. 3

c) Hence find the maximum area of the field. 1

Question 8. **(9 marks)**

For the curve $y = x^3 + \frac{3}{2}x^2 - 6x + 7$

a) Find the stationary points and determine their nature 4

b) Find any points of inflexion 2

c) Sketch the curve showing all important features. 3

Marks

Question 9 (5 marks)

For the parabola defined by $x^2 + 4x - 8y + 12 = 0$, find the:

a) Coordinates of the vertex. 2

b) Coordinates of the focus. 2

c) Equation of the directrix. 1

Question 10. (7marks)

a) Find the equation of the normal to the curve $f(x) = x^4 + 3x^2$ at the point where x =1 4

b) Find where the curve $y = 2x^3 + 9x^2$ is monotonic increasing. 3

Multiple Choice – Circle the most correct answer.

1. From the equation $(y+5)^2 = -2(x-3)$ it is possible to find:

A the directrix B the vertex

C the focus D all of the above

2. The curve $y = x^4$ has:

A no points of inflexion B 1 point of inflexion

C 2 points of inflexion D 3 points of inflexion

3. The second derivative of the function $f(x) = (3-x)^{\frac{1}{2}}$ is:

A $f''(x) = -\frac{1}{2\sqrt{3-x}}$ B $f''(x) = \frac{1}{4(3-x)^{\frac{3}{2}}}$

C $f''(x) = -\frac{1}{4(3-x)^{\frac{3}{2}}}$ D $f''(x) = -4(3-x)^{-\frac{3}{2}}$

4. A primitive function of $(x-3)^2$ might be:

A $\frac{1}{3}x^3 - 3x^2 + 9x + 5$ B $\frac{1}{3}x^3 - 3x^2 + 9x - 5$

C both A and B D neither A nor B

5. The locus of a point that is equidistant from a fixed point and a fixed line is a:

A circle B parabola

C straight line D focal chord

26 a) $PA^2 + PB^2 = 44$

$(x+3)^2 + (y-2)^2 + (x-3)^2 + (y+2)^2 = 44$ 1

$x^2 + 6x + 9 + y^2 - 4y + 4 + x^2 - 6x + 9 + y^2 + 4y + 4 = 44$

$2x^2 + 2y^2 + 26 = 44$ 1

$2x^2 + 2y^2 = 18$

$x^2 + y^2 = 9$ 1 /3

b) Since it is in the form of $x^2 + y^2 = r^2$ it is a circle, 1

centre $(0,0)$ 1

radius $= 3$ 1 /3

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7. a) $2x + y = 200$

$y = 200 - 2x$ 1

$A = xy$

$= x(200 - 2x)$

$= 200x - 2x^2$ 1/2

) $A(x) = 200x - 2x^2$

$A'(x) = 200 - 4x$ 1

$A''(x) = -4x$

Max or Min when $A'(x) = 0$

$200 - 4x = 0$

$4x = 200$

$x = 50$ 1

$A''(50) = -200$

< 0 ie. Max 1/3

) $A = 200x - 2x^2$

$A(50) = 200(50) - 2(50)^2$

$= 5000\ m^2$ /1

Q8. a) $y = x^3 + \frac{3}{2}x^2 - 6x + 7$

$y' = 3x^2 + 3x - 6$ 1

$y'' = 6x + 3$

S.P. when $y' = 0$

$3x^2 + 3x - 6 = 0$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2, 1$ 1

$x = -2$, $y = 17$ $(-2, 17)$

$x = 1$, $y = 3\frac{1}{2}$ $(1, 3\frac{1}{2})$ 1

$y''(-2) = 6(-2) + 3$

< 0 Max. TP

$y''(1) = 6(1) + 3$ 1

> 0 Min T.P. /4

b) P.o.I when $y'' = 0$

$6x + 3 = 0$

$6x = -3$

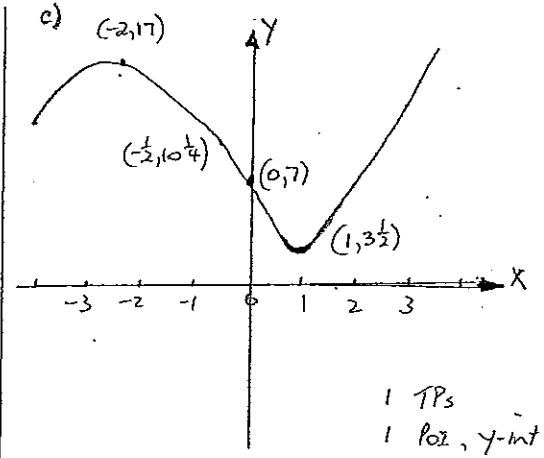
$x = -\frac{1}{2}$

$x = -\frac{1}{2}$, $y = 10\frac{1}{4}$ $(-\frac{1}{2}, 10\frac{1}{4})$ 1

x	-1	$-\frac{1}{2}$	0
y''	-3	0	3

Concavity changes 1 /2

c)



1 TPs
1 Pos, y-int
1 shape /3

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9 a) $(x^2 + 4x + 4) = 8y - 12 + 4$

$(x+2)^2 = 8y - 8$ 1

$= 8(y-1)$

Vertex: $(-2, 1)$ 1/2

b) $4 \times 2 = 8$

$\therefore a = 2$ 1

focus: $(-2, 3)$ 1/2

c) directrix: $y = -1$ 1/4

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10 a) $f(x) = x^4 + 3x^2$

$f'(x) = 4x^3 + 6x$

$f'(1) = 4 + 6$

$= 10$ 1

$m_1 = 10$

$m_2 = -\frac{1}{10}$ 1

$f(1) = 1 + 3$

$= 4$ $(1, 4)$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{10}(x - 1)$ 1

$10y - 40 = -x + 1$

$x + 10y - 41 = 0$ 1 /4

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10 b) $y = 2x^3 + 9x^2$

$y' = 6x^2 + 18x$ 1

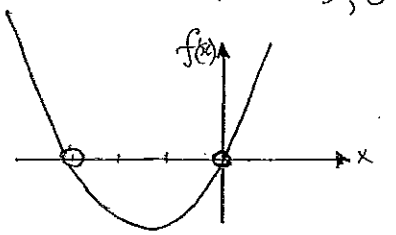
Monotonic Increasing $f'(x) > 0$

$6x^2 + 18x > 0$

Put $6x^2 + 18x = 0$

$6x(x+3) = 0$ 1

$x = -3, 0$



Monotonic increasing when $x < -3$ and $x > 0$ 1 /3

Multiple Choice

1. D
2. A
3. C
4. C
5. B