Yr 12 Ext 1 Assignment 1
Sequences and Series
Due Friday Week 2B

Please complete your assignment neatly on your own paper (Do not tear out of book!)

1) Show that $\sum_{k=1}^{8}(4k+1)$ represents an arithmetic series and hence evaluate this sum.

2) An arithmetic series has a seventh term of 8 and the difference between the twelfth and the eleventh terms is -4.
   i. Find the common difference d.
   ii. What is the first term a.
   iii. Calculate the sum of the first 17 terms.

3) Find the limiting sum of the series $1+\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+\ldots$.

4) The decimal (i.e. $0{\cdot}34343434\ldots$) can be considered as the sum of a geometric sequence.
   i. What is the value of the first term, a, and the common ratio, r?

   ii. Hence or otherwise express as a fraction.

5) The second term of a geometric series is 120 and the fifth term is $50{\cdot}625$.
   i. Find the common ratio and the first term of the series.
   ii. Find the limiting sum of the series.
   iii. Hence, find the difference between the limiting sum and the sum of the first 40 terms giving your answer in scientific notation correct to 2 significant figures.

6) The third term of a geometric series is 3 and the seventh term is 12. Find the 14^{th} term of this series.

7) At the beginning of each year a man invests $\$1500$ in a superannuation fund, on which he is paid $10{\cdot}5\%$ p.a. interest. Find:
   a. The total amount of interest for the first year.
   b. The total his investment amounts to after 20 years.

8) Mrs Strife borrows *\$80 000* at *18%* p.a. monthly reducible interest for *25* years and agrees to repay the loan in equal monthly instalments.

i. Calculate the value of each monthly instalment.

ii. What will be the total amount of interest paid on this loan?

9) Over the years the statistics showed the population of a particular town was decreasing in such a way that at the end of each year the population could be determined in the following way:

10% of the population at the beginning of the year moved out and
500 new citizens moved in during the year.

At the beginning of *1990*, before the *10%* moved out, the population of the town was *10 000*.

i. Show that at the end of *1992* the population of the town was:
$10\,000(0{\cdot}9)^3 + 500(1 + 0{\cdot}9 + 0{\cdot}9^2)$.

ii. This trend continued indefinitely. Find the population of the town at the end of the year *2009*. (i.e. the end of the *20*th year)

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1) $T_3 - T_2 = d \quad T_2 - T_1 = d$

$(4(3)+1) - (4(2)+1) = 4$

$(4(2)+1) - (4(1)+1) = 4$ ✓

$d = 4$ arithmetic series

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_8 = \frac{8}{2}[2(5) + (8-1)4]$

$S_8 = 152$ ✓

2) i) $d = -4$ ✓

ii) $T_7 = a + 6d = 8$

$a = 8 - 6(-4)$

$a = 32$ ✓

iii) $S_{17} = \frac{17}{2}[2 \times 32 + (17-1) \times -4]$

$(16) -4$

$64 - 64$

$S_{17} = \cancel{-68}\ 0$

3) $a = 1 \quad r = \frac{3}{4}$ as $|r| < 1$

a limiting sum exists

$S_\infty = \frac{1}{1 - \frac{3}{4}} \quad S_\infty = 4$ ✓

4) i) $0.34 + 0.0034 + 0.000034 \ldots$

$a = 0.34 \quad r = \frac{1}{100}$

iii) $S_\infty = \frac{0.34}{1 - \frac{1}{100}}$

$S_\infty = \frac{34}{100} \times \frac{100}{\cancel{94}\ 99}$

$S_\infty = \frac{34}{99}$ or $\frac{\cancel{17}}{\cancel{47}}$

5) i) $T_2 = ar = 120$ —— ①

$T_5 = ar^4 = 50.625$ —— ②

② ÷ ①: $r^3 = 0.421875$

$r = 0.75$ —— ③

③ into ①: $0.75a = 120$

$a = 160$ ✓

ii) $S_\infty = \frac{160}{1 - \frac{3}{4}} \quad S_\infty = 640$ ✓

iii) $S_{40} = \frac{a(1 - r^n)}{1 - r}$

$S_{40} = \frac{160\left(1 - \left(\frac{3}{4}\right)^{40}\right)}{0.25}$

$S_{40} = 639.9935638$ ✓

$S_\infty - S_{40} = 0.0064$ ✓

6) ?

7) $C.I = 1500(1.105)^1$ ✓

a) $= \$\ 57.50$

b) 1st deposit: $1500(1.105)^{20}$

2nd deposit: $1500(1.105)^{19}$

Total: $1500(1.105 + 1.105^2 \ldots 1.105^{20})$

S_{20} of G.P $= 1500\left[\frac{1.105(1.105^{20} - 1)}{1.105 - 1}\right]$

$S_{20} = \$100,495.56$

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8) i) $Q = \dfrac{(R-1)(PR^n - A_n)}{(R^n - 1)}$

where Q = monthly installment
$R = 1 + \frac{r}{100}$ where r = rate
A_n = Amount owing
n = number of periods

$Q = \dfrac{(1.015-1)(80{,}000 \times 1.015^{300})}{1.015^{300} - 1}$

$Q = \$1213.94$

ii) $Q \times 300 - 80{,}000 = \$284{,}183.19$

9) i) End of first year:
$P_1 = 10000 \times 0.9 + 500$
End of second year:
$P_2 = P_1 \times 0.9 + 500$
End of third year:
$P_3 = P_2 \times 0.9 + 500$
$P_3 = (P_1 \times 0.9 + 500) \times 0.9 + 500$
$P_3 = [(10000 \times 0.9 + 500) \times 0.9 + 500] \times 0.9 + 500$
$P_3 = (10^4 \times (0.9)^2 + (500 \times 0.9) + 500) \times 0.9 + 500$
$P_3 = 10^4 \times (0.9)^3 + 500 \times (0.9)^2 + 500 \times (0.9) + 500$
$P_3 = 10{,}000 \times (0.9)^3 + 500(1 + 0.9 + (0.9)^2)$

ii) G.P: $10{,}000 \times (0.9)^{20} + 500(1 + 0.9 + 0.9^2 + 0.9^3 \ldots)$
$\therefore 10{,}000 \times (0.9)^{20} + 500\left[\dfrac{1(1-0.9^{20})}{1-0.9}\right]$
$\therefore 10{,}000 \times (0.9)^{?} + 500(8.7842\ldots)$
$= 5608$