

Mathematics General 2

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black pen
- Board-approved calculators may be used
- A formulae and data sheet is provided at the back of this paper
- In Questions 26–30, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I Pages 2–10

25 marks

- Attempt Questions 1–25
- Allow about 35 minutes for this section

Section II Pages 13–35

75 marks

- Attempt Questions 26–30
- Allow about 1 hour and 55 minutes for this section

Section I

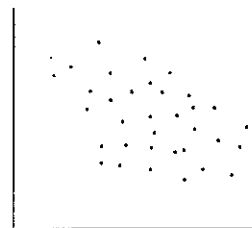
25 marks

Attempt Questions 1–25

Allow about 35 minutes for this section

Use the multiple-choice answer sheet for Questions 1–25.

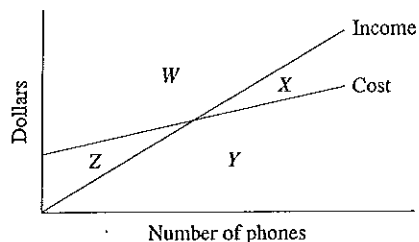
- 1 What is 208.345 correct to two significant figures?
(A) 208
(B) 210
(C) 208.34
(D) 208.35
- 2 Which of the following equations has $x = 5$ as the solution?
(A) $x - 5 = 10$
(B) $5 - x = 10$
(C) $\frac{x}{2} = 10$
(D) $2x = 10$
- 3 The graph shows a scatterplot for a set of data.



Which of the following is the best approximation for the correlation coefficient of this set of data?

- (A) -1
- (B) -0.3
- (C) 0.3
- (D) 1

- 4 A company manufactures phones. The company's income equation and cost equation are drawn on the same graph.



Which region of the graph is the profit zone?

- (A) W
 (B) X
 (C) Y
 (D) Z
- 5 Which expression is equivalent to $2(3x - 4) + 27$?
- (A) $6x - 2$
 (B) $6x - 4$
 (C) $6x - 6$
 (D) $6x - 10$
- 6 Ben has 6 shirts, 4 ties and 2 jackets.

How many different outfits of a shirt, a tie and a jacket can he choose?

- (A) 12
 (B) 16
 (C) 24
 (D) 48

- 7 Which set of data is classified as categorical and nominal?

- (A) blue, green, yellow
 (B) small, medium, large
 (C) 5.2 cm, 6 cm, 7.21 cm
 (D) 4 people, 5 people, 9 people

- 8 The table shows the future value of an investment of \$1000, compounding yearly, at varying interest rates for different periods of time.

Future values of an investment of \$1000

Number of years	Interest rate per annum				
	1%	2%	3%	4%	5%
1	1010.00	1020.00	1030.00	1040.00	1050.00
2	1020.10	1040.40	1060.90	1081.60	1102.50
3	1030.30	1061.21	1092.73	1124.86	1157.63
4	1040.60	1082.43	1125.51	1169.86	1215.51
5	1051.01	1104.08	1159.27	1216.65	1276.28

Based on the information provided, what is the future value of an investment of \$2500 over 3 years at 4% pa?

- (A) \$1124.86
 (B) \$2812.15
 (C) \$3624.86
 (D) \$5312.15
- 9 An old washing machine uses 130 L of water per load. A new washing machine uses 50 L per load.
- How much water is saved each year if two loads of washing are done each week using the new machine?
- (A) 2600 L
 (B) 4160 L
 (C) 5200 L
 (D) 8320 L

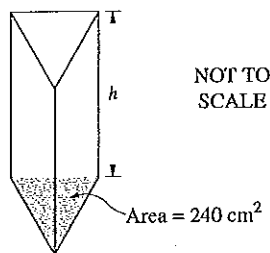
- 10 Caroline drinks two small bottles of wine over a three-hour period. Each of these bottles contains 2.3 standard drinks. Caroline weighs 53 kg.

What is her approximate blood alcohol content (BAC) at the end of this period?

- (A) 0.081
 (B) 0.065
 (C) 0.0017
 (D) 0.0014
- 11 The concentration of a drug in a certain medication is 100 mg/5 mL. A patient is prescribed 2000 mg of the drug.

How much medication should be given to the patient?

- (A) 4 mL
 (B) 25 mL
 (C) 100 mL
 (D) 400 mL
- 12 A container is in the shape of a triangular prism which has a capacity of 12 litres. The area of the base is 240 cm^2 .



What is the distance, h , between the two triangular ends of the container?

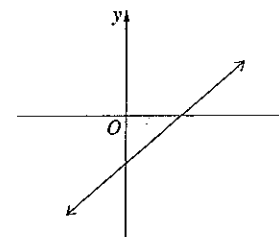
- (A) 5 cm
 (B) 20 cm
 (C) 25 cm
 (D) 50 cm

- 13 The speed limit outside a school is 40 km/h. Year 11 students measured the speed of passing vehicles over a period of time. They found the set of data to be normally distributed with a mean speed of 36 km/h and a standard deviation of 2 km/h.

What percentage of the vehicles passed the school at a speed greater than 40 km/h?

- (A) 2.5%
 (B) 5%
 (C) 47.5%
 (D) 95%

- 14 The graph shows a line which has an equation in the form $y = mx + b$.



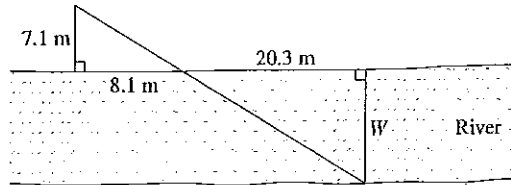
Which of the following statements is true?

- (A) m is positive and b is negative
 (B) m is negative and b is positive
 (C) m and b are both positive
 (D) m and b are both negative
- 15 Calls on a mobile phone plan are charged at the rate of 54 cents per 30 seconds, or part thereof.

What is the cost of a call lasting 2 minutes and 15 seconds?

- (A) \$2.16
 (B) \$2.32
 (C) \$2.43
 (D) \$2.70

- 16 The width (W) of a river can be calculated using two similar triangles, as shown in the diagram.



NOT TO SCALE

What is the approximate width of the river?

- (A) 17.8 m
 (B) 19.3 m
 (C) 23.2 m
 (D) 24.9 m
- 17 Ariana is charged compound interest at the rate of 0.036% per day on outstanding credit card balances. She has \$780 outstanding for 24 days.

How much compound interest is she charged?

- (A) \$6.74
 (B) \$6.77
 (C) \$786.74
 (D) \$786.77
- 18 The value of E varies directly with the square of S .

It is known that $E = 20$ when $S = 10$.

What is the value of E when $S = 40$?

- (A) 40
 (B) 80
 (C) 320
 (D) 400

- 19 A soccer referee wrote down the number of goals scored in 9 different games during the season.

2, 3, 3, 3, 5, 5, 8, 9,

The last number has been omitted. The range of the data is 10.

What is the five-number summary for this data set?

- (A) 2, 3, 5, 8.5, 12
 (B) 2, 3, 5, 8.5, 10
 (C) 2, 3, 5, 8, 12
 (D) 2, 3, 5, 8, 10
- 20 Isabella works a 35-hour week and is paid at an hourly rate of \$18. Any overtime hours worked are paid at time-and-a-half. In a particular week, she earned \$1008.

How many hours in total did Isabella work in this week to earn this amount?

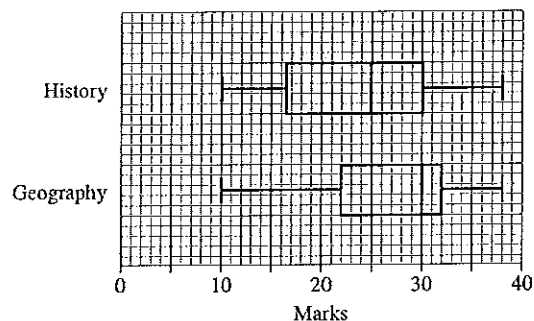
- (A) 37.3
 (B) 42
 (C) 49
 (D) 56
- 21 A grouped data frequency table is shown.

<i>Class interval</i>	<i>Frequency</i>
1–5	3
6–10	6
11–15	8
16–20	9

What is the mean for this set of data?

- (A) 6.5
 (B) 10.5
 (C) 11.9
 (D) 12.4

- 22 The box-and-whisker plots show the results of a History test and a Geography test.



In History, 112 students completed the test. The number of students who scored above 30 marks was the same for the History test and the Geography test.

How many students completed the Geography test?

- (A) 8
 (B) 50
 (C) 56
 (D) 112
- 23 A group of 485 people was surveyed. The people were asked whether or not they smoke. The results are recorded in the table.

	Smokers	Non-smokers	Total
Male	88	176	264
Female	68	153	221
	156	329	485

A person is selected at random from the group.

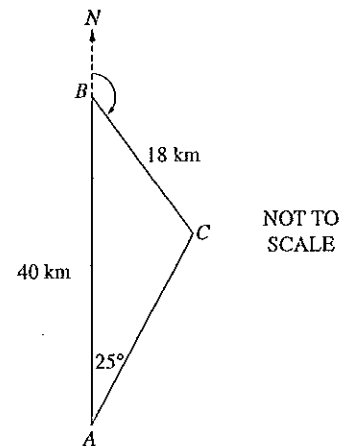
What is the approximate probability that the person selected is a smoker OR is male?

- (A) 33%
 (B) 18%
 (C) 68%
 (D) 87%

- 24 Which of the following correctly expresses Q as the subject of $e = iR + \frac{Q}{C}$?

- (A) $Q = Ce + CiR$
 (B) $Q = Ce - CiR$
 (C) $Q = \frac{e + iR}{C}$
 (D) $Q = \frac{e - iR}{C}$

- 25 The diagram shows towns A , B and C . Town B is 40 km due north of town A . The distance from B to C is 18 km and the bearing of C from A is 025° . It is known that $\angle BCA$ is obtuse.



What is the bearing of C from B ?

- (A) 070°
 (B) 095°
 (C) 110°
 (D) 135°

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Centre Number

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Student Number

Section II

75 marks
 Attempt Questions 26–30
 Allow about 1 hour and 55 minutes
 for this section

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided on pages 34 and 35. If you use this space, clearly indicate which question you are answering.

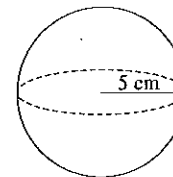
Write your Centre Number and Student Number at the top of this page.

Please turn over

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Question 26 (15 marks)

- (a) Calculate the surface area of a sphere with a radius of 5 cm, correct to the nearest whole number. 1



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- (b) Simplify $\frac{8x^4y}{24x^3y^5}$. 2

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- (c) Peta's car uses fuel at the rate of 5.9 L/100 km for country driving and 7.3 L/100 km for city driving. On a trip, she drives 170 km in the country and 25 km in the city. 2

Calculate the amount of fuel she used on this trip.

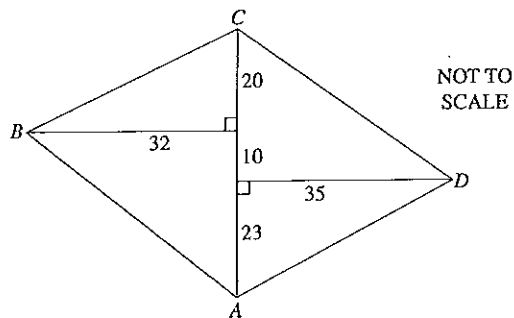
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Question 26 continues on page 15

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Question 26 (continued)

- (d) The field diagram shows a block of land $ABCD$ that has been surveyed. All measurements are in metres. 2



Calculate the length of AB , correct to the nearest metre.

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- (e) Jenny earns a yearly salary of \$63 752. Her annual leave loading is $17\frac{1}{2}\%$ of four weeks pay. 3

Calculate her total pay for her four weeks of annual leave.

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Question 26 continues on page 16

Question 26 (continued)

- (f) Theo is completing his tax return. He has a gross salary of \$82 521 and income from a rental property totalling \$10 920. He is claiming \$13 420 in allowable deductions.

- (i) Determine Theo's taxable income. 1

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- (ii) Using the tax table below, calculate Theo's tax payable. 2

Taxable income	Tax on this income
\$0 - \$18 200	Nil
\$18 201 - \$37 000	19c for each \$1 over \$18 200
\$37 001 - \$80 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$80 001 - \$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 547 plus 45c for each \$1 over \$180 000

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- (iii) In addition to the above tax, Theo must also pay a Medicare levy of \$1600.42. 2

Theo has already paid \$20 525 as Pay As You Go (PAYG) tax.

Should Theo receive a tax refund or will he owe more tax? Justify your answer with calculations.

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End of Question 26

Do NOT write in this area.

Question 27 (15 marks)

- (a) Alice intends to buy a car and insure it.

2

Briefly describe what each of these types of insurance covers:

- Compulsory third-party insurance (CTP)
- Non-compulsory third-party property insurance.

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- (b) A small population consists of three students of heights 153 cm, 168 cm and 174 cm. Samples of varying sizes can be taken from this population.

2

What is the mean of the mean heights of all the possible samples? Justify your answer.

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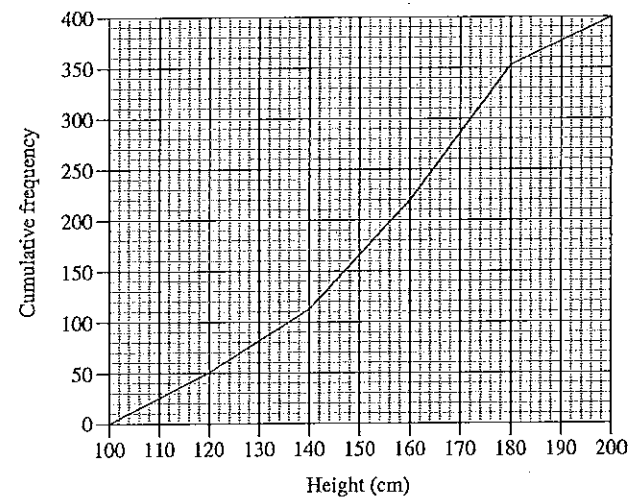
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Question 27 continues on page 18

Question 27 (continued)

- (c) The heights of 400 students were measured. The results are displayed in this cumulative frequency polygon.

2



Use the polygon to estimate the interquartile range.

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Question 27 continues on page 19

Do NOT write in this area.

Question 27 (continued)

- (d) Marge borrowed \$19 000 to buy a used car. Interest on the loan was charged at 4.8% pa at the end of each month. She made a repayment of \$436 at the end of every month. The table below sets out her monthly repayment schedule for the first four months of the loan.

Month	Amount owing at start of month	Interest charged	Repayment	Amount owing at end of month
1	A	\$76.00	\$436.00	\$18 640.00
2	\$18 640.00	X	\$436.00	\$18 278.56
3	\$18 278.56	\$73.11	\$436.00	\$17 915.67
4	\$17 915.67	\$71.66	\$436.00	B

- (i) Some values in the table are missing. Write down the values for A and B. 2

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- (ii) Calculate the value of X. 2

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- (iii) Marge repaid this loan over four years. 1

What is the total amount that Marge repaid?

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Question 27 continues on page 20

Question 27 (continued)

- (e) Melbourne is located at (38°S, 145°E) and Dubai is located at (24°N, 55°E).

- (i) Calculate the difference in longitude between Melbourne and Dubai. 1

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- (ii) Show that the time difference between Melbourne and Dubai is 6 hours. 1

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- (iii) A plane leaves Melbourne on Friday at 11.30 pm. The flight time to Dubai is 15 hours. 2

What will be the time and the day in Dubai when the plane is due to land?

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End of Question 27

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Question 28 (15 marks)

(a) Jacob has a large jar of silver coins. He adds 20 gold coins into the jar. He then seals the jar and shakes it to ensure that the gold coins are mixed in thoroughly with the silver coins. Jacob then opens the jar and takes a handful of coins. In his hand he has 33 silver coins and 4 gold coins.

(i) Based on Jacob's handful, if a coin is selected at random from the jar, what is the probability that it is a gold coin? 1

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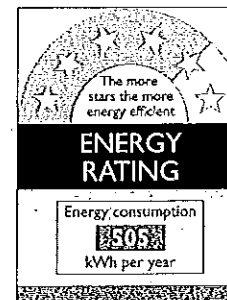
(ii) Jacob returns the handful of coins to the jar. Estimate the total number of coins in the jar. 2

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Question 28 continues on page 22

Question 28 (continued)

(b) The cost of buying a new heater is \$990. It uses energy according to the following energy label. 2



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Energy is charged at the rate of \$0.35/kWh.

How much will it cost in total to purchase and then run this heater for five years?

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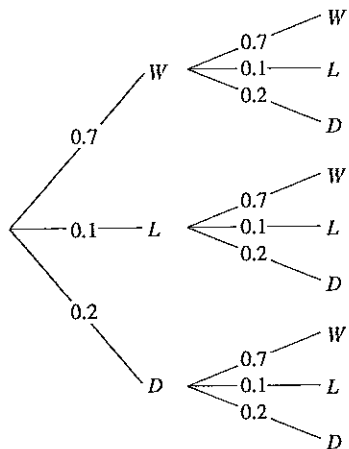
Question 28 continues on page 23

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Question 28 (continued)

- (c) A cricket team is about to play two matches. The probability of the team having a win, a loss or a draw is 0.7, 0.1 and 0.2 respectively in each match. The possible results in the two matches are displayed in the probability tree diagram.



- (i) What is the probability of the team having a win and a draw, in any order? 2

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- (ii) Paul claims that 1.4 is the probability of the team winning both matches. 1

Give one reason why this is NOT correct.

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Question 28 continues on page 24

Question 28 (continued)

- (d) The table gives the contribution per period for an annuity with a future value of \$1 at different interest rates and different periods of time. 2

Contribution per period for an annuity with a future value of \$1

Number of periods	Interest rate (% per period)					
	0.25%	0.5%	0.75%	1%	1.25%	1.5%
6	0.1656	0.1646	0.1636	0.1625	0.1615	0.1605
12	0.0822	0.0811	0.0800	0.0788	0.0778	0.0767
18	0.0544	0.0532	0.0521	0.0510	0.0499	0.0488
24	0.0405	0.0393	0.0382	0.0371	0.0360	0.0349
30	0.0321	0.0310	0.0298	0.0287	0.0277	0.0266
36	0.0266	0.0254	0.0243	0.0232	0.0222	0.0212

Margaret needs to save \$75 000 over 6 years for a deposit on a new apartment. She makes regular quarterly contributions into an investment account which pays interest at 3% pa.

How much will Margaret need to contribute each quarter to reach her savings goal?

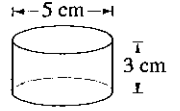
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Question 28 continues on page 25

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Question 28 (continued)

- (e) A company makes large marshmallows. They are in the shape of a cylinder with diameter 5 cm and height 3 cm, as shown in the diagram.



- (i) Find the volume of one of these large marshmallows, correct to one decimal place. 2

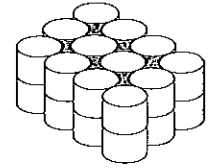
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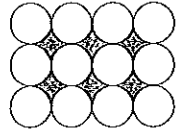
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- (ii) A cake is to be made by stacking 24 of these large marshmallows and filling the gaps between them with chocolate. The diagrams show the cake and its top view. The shading shows the gaps to be filled with chocolate. 3



Cake



Top view

What volume of chocolate will be required? Give your answer correct to the nearest whole number.

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End of Question 28

Question 29 (15 marks)

- (a) Two unbiased coins are tossed.

- (i) What is the probability that one coin shows heads and the other shows tails? 1

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- (ii) A game is played in which one player tosses the two coins. The rules are as follows: 2

- If both coins show heads, the player wins \$40
- If both coins show tails, the player wins \$20
- If one coin shows heads and the other shows tails, the player loses \$30.

What is the financial expectation of this game?

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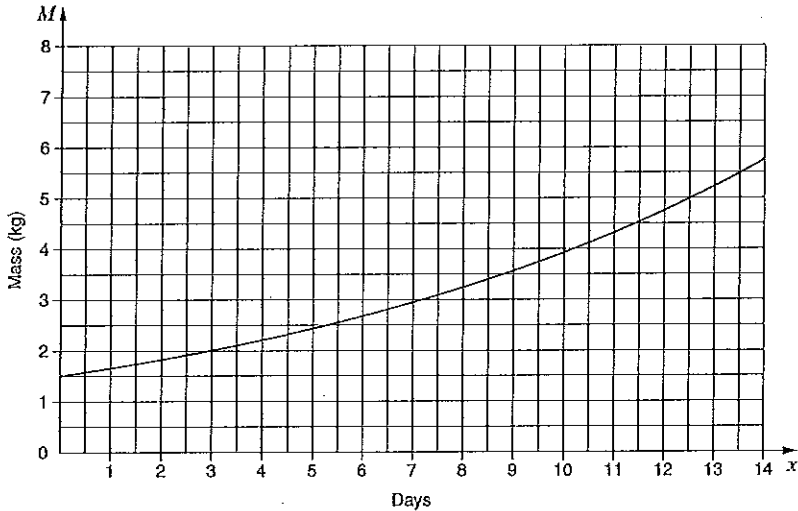
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Question 29 continues on page 27

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Question 29 (continued)

- (b) The mass M kg of a baby pig at age x days is given by $M = A(1.1)^x$ where A is a constant. The graph of this equation is shown.



- (i) What is the value of A ? 1

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- (ii) What is the daily growth rate of the pig's mass? Write your answer as a percentage. 1

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Question 29 continues on page 28

Question 29 (continued)

- (c) The ages of members of a dance class are shown in the back-to-back stem-and-leaf plot.

Women		Men
2	3	4 6
4	4	2 2 5 6 8
8 8 5 4 0 0	5	3
9 4 3 3	6	3

Pat claims that the women who attend the dance class are generally older than the men.

Is Pat correct? Justify your answer by referring to the median and skewness of the two sets of data.

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Question 29 continues on page 29

Question 29 (continued)

- (d) Five students sat both a Physics and a Chemistry exam. Their results are shown in the table. The mean and standard deviation of each exam are also shown.

						Mean	Standard deviation
Physics (x)	75	55	81	64	72	69.4	9.05
Chemistry (y)	84	48	77	53	68	66	13.73

The correlation coefficient for this data set is approximately 0.9.

- (i) Verify the value of the correlation coefficient, using your calculator, and give your value correct to three decimal places. 1

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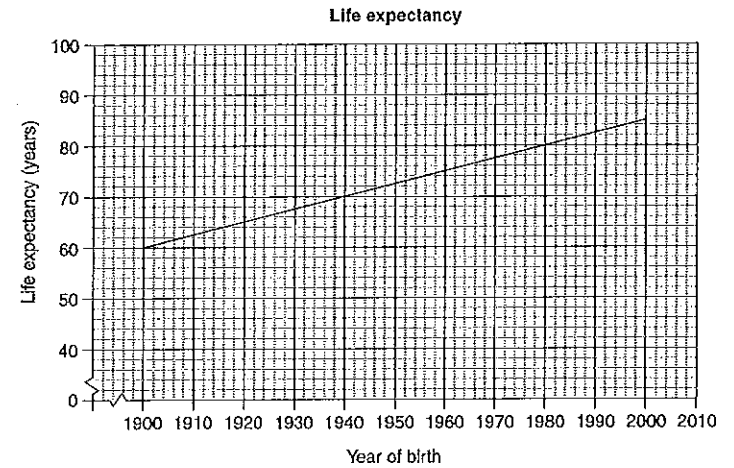
- (ii) By using the appropriate formulae from the Formulae and Data Sheet, and the given information, determine the equation of the least-squares line of best fit. 3

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Question 29 continues on page 30

Question 29 (continued)

- (e) The graph shows the life expectancy of people born between 1900 and 2000.



- (i) According to the graph, what is the life expectancy of a person born in 1932? 1

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- (ii) With reference to the value of the gradient, explain the meaning of the gradient in this context. 2

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End of Question 29

Question 30 (continued)

(d) The formula to calculate z-scores can be rearranged to give

$$\bar{x} = x - sz$$

where \bar{x} is the mean

x is the score

s is the standard deviation

z is the z-score.

(i) In an examination, Aaron achieved a score of 88, which corresponds to a z-score of 2.4. 1

Substitute these values into the rearranged formula above to form an equation.

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(ii) In the same examination, Brock achieved a score of 52, which corresponds to a z-score of -1.2. 3

Using this information, form another equation and solve it simultaneously with the equation from part (i) to find the values of \bar{x} and s .

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Section II extra writing space

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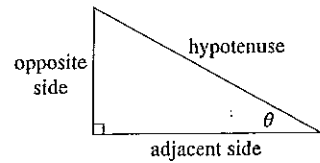
Lined writing area for Section II.

FORMULAE AND DATA SHEET

Financial Mathematics	Data Analysis
<p>Simple Interest</p> $I = Prn$ <p>P is initial amount r is interest rate per period, expressed as a decimal n is number of periods</p> <p>Compound Interest</p> $A = P(1 + r)^n$ <p>A is final amount P is initial amount r is interest rate per period, expressed as a decimal n is number of compounding periods</p> <p>Present value and future value</p> $PV = \frac{FV}{(1 + r)^n}, \quad FV = PV(1 + r)^n$ <p>r is interest rate per period, expressed as a decimal n is number of compounding periods</p> <p>Straight-line method of depreciation</p> $S = V_0 - Dn$ <p>S is salvage value of asset after n periods V_0 is initial value of asset D is amount of depreciation per period n is number of periods</p> <p>Declining-balance method of depreciation</p> $S = V_0(1 - r)^n$ <p>S is salvage value of asset after n periods V_0 is initial value of asset r is depreciation rate per period, expressed as a decimal n is number of periods</p>	<p>Mean of a sample</p> $\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$ <p>z-score</p> <p>For any score x,</p> $z = \frac{x - \bar{x}}{s}$ <p>\bar{x} is mean s is standard deviation</p> <p>Outlier(s)</p> <p>score(s) less than $Q_L - 1.5 \times IQR$ or score(s) more than $Q_U + 1.5 \times IQR$</p> <p>Q_L is lower quartile Q_U is upper quartile IQR is Interquartile range</p> <p>Least-squares line of best fit</p> $y = \text{gradient} \times x + \text{y-intercept}$ $\text{gradient} = r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}}$ $\text{y-intercept} = \bar{y} - (\text{gradient} \times \bar{x})$ <p>r is correlation coefficient \bar{x} is mean of x scores \bar{y} is mean of y scores</p> <p>Normal distribution</p> <ul style="list-style-type: none"> approximately 68% of scores have z-scores between -1 and 1 approximately 95% of scores have z-scores between -2 and 2 approximately 99.7% of scores have z-scores between -3 and 3

Spherical Geometry	Surface Area
<p>Circumference of a circle</p> $C = 2\pi r \text{ or } C = \pi D$ <p>r is radius D is diameter</p> <p>Arc length of a circle</p> $l = \frac{\theta}{360} 2\pi r$ <p>r is radius θ is number of degrees in central angle</p> <p>Radius of Earth (taken as) 6400 km</p> <p>Time differences For calculation of time differences using longitude: $15^\circ = 1$ hour time difference</p>	<p>Sphere</p> $A = 4\pi r^2$ <p>r is radius</p> <p>Closed cylinder</p> $A = 2\pi r^2 + 2\pi rh$ <p>r is radius h is perpendicular height</p>
<p>Area</p> <p>Circle</p> $A = \pi r^2$ <p>r is radius</p> <p>Sector</p> $A = \frac{\theta}{360} \pi r^2$ <p>r is radius θ is number of degrees in central angle</p> <p>Annulus</p> $A = \pi(R^2 - r^2)$ <p>R is radius of outer circle r is radius of inner circle</p> <p>Trapezium</p> $A = \frac{h}{2}(a + b)$ <p>h is perpendicular height a and b are the lengths of the parallel sides</p> <p>Area of land and catchment areas unit conversion: $1 \text{ ha} = 10\,000 \text{ m}^2$</p>	<p>Volume</p> <p>Prism or cylinder</p> $V = Ah$ <p>A is area of base h is perpendicular height</p> <p>Pyramid or cone</p> $V = \frac{1}{3}Ah$ <p>A is area of base h is perpendicular height</p> <p>Volume and capacity unit conversion: $1 \text{ m}^3 = 1000 \text{ L}$</p> <p>Approximation Using Simpson's Rule</p> <p>Area</p> $A \approx \frac{h}{3}(d_f + 4d_m + d_l)$ <p>h is distance between successive measurements d_f is first measurement d_m is middle measurement d_l is last measurement</p> <p>Volume</p> $V \approx \frac{h}{3}(A_L + 4A_M + A_R)$ <p>h is distance between successive measurements A_L is area of left end A_M is area of middle A_R is area of right end</p>

Trigonometric Ratios



$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Sine rule

In $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle

In $\triangle ABC$,

$$A = \frac{1}{2}ab \sin C$$

Cosine rule

In $\triangle ABC$,

$$c^2 = a^2 + b^2 - 2ab \cos C$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Units of Memory and File Size

1 byte = 8 bits

1 kilobyte = 2^{10} bytes = 1024 bytes

1 megabyte = 2^{20} bytes = 1024 kilobytes

1 gigabyte = 2^{30} bytes = 1024 megabytes

1 terabyte = 2^{40} bytes = 1024 gigabytes

Blood Alcohol Content Estimates

$$BAC_{\text{male}} = \frac{10N - 7.5H}{6.8M}$$

or

$$BAC_{\text{female}} = \frac{10N - 7.5H}{5.5M}$$

N is number of standard drinks consumed

H is number of hours of drinking

M is person's mass in kilograms

Distance, Speed and Time

$$D = ST, \quad S = \frac{D}{T}, \quad T = \frac{D}{S}$$

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$\text{stopping distance} = \left\{ \begin{array}{l} \text{reaction-time} \\ \text{distance} \end{array} \right\} + \left\{ \begin{array}{l} \text{braking} \\ \text{distance} \end{array} \right\}$$

Probability of an Event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Straight Lines

Gradient

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient-Intercept form

$$y = mx + b$$

m is gradient

b is y-intercept

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2016 Higher School Certificate Solutions Mathematics General 2

Section I

Multiple Choice Summary

1 B	2 D	3 B	4 B	5 C	6 D	7 A	8 B	9 D	10 A
11 C	12 D	13 A	14 A	15 D	16 A	17 B	18 C	19 A	20 C
21 D	22 C	23 C	24 B	25 D					

Multiple Choice Solutions

1. B
The digits 2 and 0 are significant. Hence the required number is either 200 or 210. The number given (208.345) is closer to 210.
2. D
Substituting the provided solution into $2x = 10$ gives $2 \times 5 = 10$ which is a true statement.
3. B
The line of best fit would have negative gradient. Hence the correlation coefficient is negative, so it could be (A) or (B). The dots do not form a single straight line, so the correlation coefficient is not -1. (B) is the only possibility. i.e. a weak negative correlation.
4. B
In region X income is greater than cost, so the company would be making a profit.
5. C
$$2(3x - 4) + 2 = 6x - 8 + 2$$
$$= 6x - 6$$
6. D
Multiplying the number of choices at each stage gives $6 \times 4 \times 2 = 48$ outfits.
7. A
Options (A) and (B) appear to be categories. Options (C) and (D) are numerical. The words 'small, medium and large' in Option (B) indicate an order, so they are ordinal, whereas the words 'blue, green, yellow' in Option (A) are just labels or names with no specific order, so they are nominal.
8. B
Using the table provided in the question, \$1000 invested at 4% per annum compounded annually for 3 years gives a future value of \$1124.86.

Method 1

	\$1000	becomes	\$1124.86
	$\times 2.5 \downarrow$		$\downarrow \times 2.5$
Therefore	\$2500	becomes	\$2812.15

Method 2

$$\begin{aligned} \text{Future Value} &= \$1124.86 \times \frac{2500}{1000} \\ &= \$2812.15 \end{aligned}$$

9. D

Method 1

The number of litres of water saved in every load is $130 - 50 = 80$.
The number of loads in one year is $52 \times 2 = 104$.

Saving	80 litres	in	1 load
	$\times 104 \downarrow$		$\downarrow \times 104$
Saves	8320 litres	in	104 loads

Method 2

$$\begin{aligned} \text{Volume of water saved} &= (130 - 50)L \times 2 \text{ loads/week} \times 52 \text{ weeks} \\ &= 8320 \text{ L} \end{aligned}$$

10. A

From the Formulae and Data Sheet, the formula for females is:

$$\begin{aligned} BAC_{\text{female}} &= \frac{10N - 7.5H}{5.5M} \\ &= \frac{10 \times 4.6 - 7.5 \times 3}{5.5 \times 53} \\ &= 0.0806... \end{aligned}$$

11. C

Concentration of	100 mg	in	5 mL
	$\times 20 \downarrow$		$\downarrow \times 20$
Is equivalent to	2000 mg	in	100 mL

12. D

The triangular prism has a capacity of 12 litres, which is 12 000 mL.
(1 L = 1000 mL)

\therefore the volume is 12 000 cm³
(1 mL = 1 cm³)

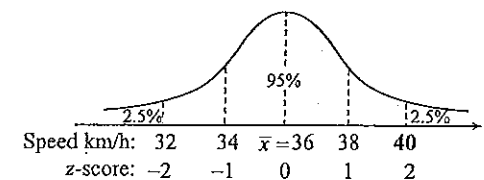
From the Formulae and Data Sheet, the volume of a prism is given by:

$$\begin{aligned} V &= Ah \\ \therefore 12000 &= 240h \\ 240h &= 12000 \\ h &= 12000 \div 240 \\ &= 50 \end{aligned}$$

\therefore the required distance is 50 cm.

13. A

From the Formulae and Data Sheet approximately 95% of scores have z-scores between -2 and 2, which puts them in the grey region in the diagram.
 \therefore 5% of the scores are outside the grey region and is shared equally to each section.
 $5\% \div 2 = 2.5\%$, making 2.5% of the scores lie above 40.



14. A
The line in the diagram is sloping uphill from left to right, so the gradient (m) is positive.
The y -intercept (b) is below 0, so it is negative.

15. D
Method 1
The call lasted 2 minutes and 15 seconds but the user will be charged from 2 minutes and 30 seconds, which is five 30-second periods.
Cost = $\$0.54 \times 5$
= $\$2.70$

Method 2

Paying	\$0.54	for	30 sec
	$\times 5 \downarrow$		$\downarrow \times 5$
Will cost	\$2.70	for	2.5 min

16. A
Method 1: Using the scale factor
In the similar triangles, the sides of 8.1 m and 20.3 m are 'matching sides'.
Scale Factor = $20.3 \div 8.1$
= 2.506...

Horizontal sides	8.1	7.1
	$\times 2.506... \downarrow$	$\downarrow \times 2.506...$
Vertical sides	20.3	17.79...

\therefore the river is approximately 17.8 m wide.

Method 2: Using an equation

In the similar triangles, the sides of 8.1 m and 20.3 m are matching sides.
The sides marked 7.1 m and W are also matching sides.

$$\frac{W}{7.1} = \frac{20.3}{8.1}$$

$$W = \frac{20.3}{8.1} \times 7.1$$

$$W = 17.79...$$

\therefore the river is approximately 17.8 m wide.

17. B
From the Formulae and Data Sheet, the future value of this debt is given by the compound interest formula:

$$A = P(1+r)^n$$

$$= 780 \left(1 + \frac{0.036}{100} \right)^{24}$$

$$= 786.767...$$

\therefore the interest charged is given by:

$$I = A - P$$

$$= \$786.77 - \$780$$

$$= \$6.77$$

18. C
Method 1
Using a scale factor:

If	$E = 20$	then	$S^2 = 10^2 = 100$
	$\times 16 \downarrow$		$\downarrow \times 16$
Hence	$E = 320$	if	$S^2 = 40^2 = 1600$

Method 2

Using an equation:
 E varies directly with the square of S

$$\therefore E = k \times S^2$$

But $E = 20$ when $S = 10$

$$\therefore 20 = k \times 10^2$$

$$100k = 20$$

$$k = 0.2$$

$$E = 0.2 \times S^2$$

Now $S = 40$

$$E = 0.2 \times 40^2$$

$$E = 320$$

19. A
The range is 10, so the last number must be $2 + 10 = 12$.
The scores, in ascending order, are:

2 3 | 3 3 5 5 8 | 9 12

The lower extreme is 2 and the upper extreme is 12. The median (or middle score) is 5.

Lower quartile = $(3+3) \div 2 = 3$ Upper quartile = $(8+9) \div 2 = 8.5$

\therefore the five-number summary is: 2, 3, 5, 8.5, 12

20. C
Isabella's normal rate is \$18 per hour. For 35 hours Isabella is paid \$630.
 \therefore Overtime pay = $\$1008 - \630
= $\$378$

Isabella's time-and-a-half rate = $\$18 \times 1.5 = \27

\therefore Overtime hour worked = $378 \div 27 = 14$

Total hour worked = $35 + 14 = 49$

21. D
As we do not have access to the raw data we will use the class centres to calculate an estimate for the mean:

Class Interval	Class centre x	Frequency f	$f \times x$
1 - 5	3	3	9
6 - 10	8	6	48
11 - 15	13	8	104
16 - 20	18	9	162
Total:		26	323

\therefore Estimate for mean = $323 \div 26 = 12.423...$

22. C (Note that all four answers are possible but C is the most obvious.)
 From the History box-and-whisker plot, 25% of the 112 students scores above 30.
 $\therefore 25\% \times 112 = 28$ students
 From the Geography box-and-whisker plot, 50% of the students scores above 30.
 $\therefore 50\% \times \text{Geography students} = 28$
 \therefore there will be $2 \times 28 = 56$ Geography students

23. C
 There are 264 males. There are also 68 females who smoke. This makes 332 who are smokers or males. The probability of choosing one of the people out of group of 485 is given by:

$$\frac{332}{485} \times 100\% = 68.4536\dots\%$$

$$\div 68\%$$

24. B

$$e = iR + \frac{Q}{C}$$

$$iR + \frac{Q}{C} = e$$

$$\frac{Q}{C} = e - iR$$

$$Q = Ce - CiR$$

25. D
 To find the required bearing, $\angle NBC$, it is necessary to find the size of $\angle BCA$.
 From the Formulae and Data Sheet, the sine rule is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Substituting the values we are know:

$$\frac{40}{\sin C} = \frac{18}{\sin 25^\circ}$$

$$\frac{\sin C}{40} = \frac{\sin 25^\circ}{18}$$

$$\sin C = \frac{\sin 25^\circ}{18} \times 40$$

$$\sin C = 0.939\dots$$

$$C = 69.909\dots^\circ$$

But it is given in the question that $\angle BCA$ is obtuse, so it must be the supplement of C .

$$\angle BCA = 180^\circ - 69.909\dots^\circ$$

$$= 110.09\dots^\circ$$

$$= 110^\circ \text{ (to the nearest degree)}$$

\therefore using the angle sum of a triangle,

$$\angle ABC = 180^\circ - 25^\circ - 110^\circ$$

$$= 45^\circ$$

ABN is a straight line, so the bearing of C from B is given by:

$$\angle NBC = 180^\circ - 45^\circ$$

$$= 135^\circ$$

- (f) (i) Gross income = salary + rental income
 $= \$82521 + \10920
 $= \$93441$
 Taxable income = Gross income - allowable deductions
 $= \$93441 - \13420
 $= \$80021$
- (ii) For a person in the tax bracket from \$80001 to \$180000, the tax payable is:
 $\$17547$ plus 37c for each \$1 over \$80000
 \therefore Tax payable = $\$ [17547 + 0.37 \times (80021 - 80000)]$
 $= \$17554.77$
- (iii) Theo has already paid \$20525 in PAYG tax.
 Total tax Theo should have paid = Tax payable + Medicare Levy
 $= \$17554.77 + \1600.42
 $= \$19155.19$
 He has already paid more tax than this, so he will receive a refund.
 \therefore Refund = $\$20525 - \$19155.19 = \$1369.81$

Question 27

- (a) **Compulsory third-party (CTP) insurance** is required by law to be paid for every car. If the driver of the car causes injury or death of other drivers, passengers, pedestrians or cyclists the insurance company pays the costs incurred.

In addition to CTP, some people pay for non-compulsory **third-party property insurance**, which covers the driver of the car for damage to other people's property, such as cars and houses. It does not cover damage to your own car, so it is cheaper than comprehensive insurance.

- (b) Population: 153, 168, 174.
 Mean of population = $(153 + 168 + 174) \div 3 = 165$

From a population of three students, samples of size 1, 2 and 3 can be taken.

Samples:

	Sample	Mean of sample
All samples of size 1	153	153
	168	168
	174	174
All samples of size 2	153, 168	160.5
	153, 174	163.5
	168, 174	171
Sample of size 3	153, 168, 174	165
	Total	1155

Mean of sample means = $1155 \div 7 = 165$ which is the same as the population mean.

\therefore the mean of the means of the samples is equal to the mean of the population.

Section II

Question 26

- (a) From the Formulae and Data Sheet, the surface area of a sphere is given by:

$$\begin{aligned} A &= 4\pi r^2 \\ &= 4 \times \pi \times 5^2 \\ &= 314.159\dots \end{aligned}$$

\therefore the surface area is 314 cm² (correct to the nearest whole number).

$$\begin{aligned} \text{(b)} \quad \frac{8x^4y}{24x^3y^5} &= \frac{1}{3} \frac{x^4y^1}{x^3y^5} \\ &= \frac{x}{3y^4} \end{aligned}$$

- (c)
- Method 1**

Country driving:

Using	5.9 L	for	100 km
	$\times 1.7 \downarrow$		$\downarrow \times 1.7$
Will require	10.03 L	for	170 km

City driving:

Using	7.3 L	for	100 km
	$+ 4 \downarrow$		$\downarrow + 4$
Will require	1.825 L	for	25 km

\therefore the total amount of fuel used is $10.03 + 1.825 = 11.855$ L

Method 2

$$\begin{aligned} \text{Fuel for Country driving} &= 5.9 \times \frac{170}{100} & \text{Fuel for City driving} &= 7.3 \times \frac{25}{100} \\ &= 10.03 & &= 1.825 \end{aligned}$$

\therefore the total amount of fuel used is $10.03 + 1.825 = 11.855$ L

- (d)
- AB
- is the hypotenuse of a right-angled triangle with sides 32 and
- $10 + 23 = 33$
- .

Using Pythagoras' Theorem:

$$\begin{aligned} (AB)^2 &= 32^2 + 33^2 \\ &= 2113 \\ \therefore AB &= \sqrt{2113} \\ &= 45.967\dots \end{aligned}$$

\therefore the length of AB is 46 metres, correct to the nearest metre.

- (e) Using 52 weeks in 1 year:

$$\text{\$}63752 \text{ for 1 year} = \text{\$}1226 \text{ for 1 week} = \text{\$}4904 \text{ for 4 weeks}$$

Leave loading of 17.5% is added to this amount, so she is paid 117.5% of her normal pay.

$$\therefore \text{Total holiday pay} = \frac{117.5}{100} \times \text{\$}4904 = \text{\$}5762.20$$

- (ii)
- Method 1**

Using a scale factor:

$$\begin{array}{ccc} \text{In the recapture} & 4 \text{ golds} & \text{out of} & 37 \text{ coins} \\ & \times 5 \downarrow & & \downarrow \times 5 \end{array}$$

This is equivalent to 20 golds for 185 coins

\therefore there are approximately 185 coins in the jar

Method 2Using an equation: Let T be the estimate of the total number of coins

$$\begin{aligned} \therefore \frac{20}{T} &= \frac{4}{37} \\ \frac{T}{20} &= \frac{37}{4} \\ T &= \frac{37}{4} \times 20 \\ T &= 185 \end{aligned}$$

- (b)
- $\text{Cost} = \$990 + 505 \times \0.35×5
-
- $= \$1873.75$

- (c) (i) There are 2 possibilities: WD and DW
-
- $P(1 \text{ win and 1 draw}) = P(WD) + P(DW)$
-
- $= 0.7 \times 0.2 + 0.2 \times 0.7$
-
- $= 0.28$

- (ii)
- Method 1**

A probability of 1 is for an event that is certain to happen.

Probabilities cannot be greater than 1. i.e. $0 \leq P(E) \leq 1$

The number 1.4 is greater than 1, so Paul's claim is not correct.

Method 2

$$\begin{aligned} P(\text{win both matches}) &= 0.7 \times 0.7 \\ &= 0.49 \end{aligned}$$

This is not 1.4

\therefore Paul's claim is not correct.

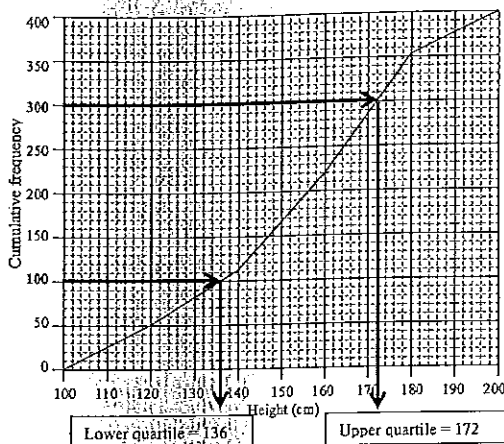
- (d) The interest rate of 3% per annum is equivalent to 0.75% per quarter.
-
- There are 24 quarters in 6 years.
-
- \therefore
- the required number from the table is 0.0382.
-
- Amount =
- $75000 \times 0.0382 = 2865$
-
- \therefore
- Margaret will need to contribute \\$2865 each quarter.

- (e) (i) From the Formulae and Data Sheet, the volume of a cylinder is
-
- $V = Ah$
- , where
- $A = \pi r^2$
-
- $\therefore V = \pi r^2 h$
-
- $= \pi \times 2.5^2 \times 3$
-
- $= 58.90\dots$
-
- \therefore
- the volume is 58.9 cm
- ³
- , correct to one decimal place.

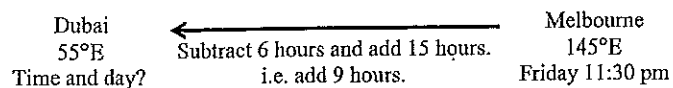
- (c) As 400 students were measured, the quartiles divide the group into sub-groups of 100.

From the diagram, the inter-quartile range is estimated to be

$$\begin{aligned} IQR &= Q_U - Q_L \\ &= 172 - 136 \\ &= 36 \end{aligned}$$



- (d) (i) $A = \$19000$ (the amount of money borrowed)
 $B = \$17915.67 + \$71.66 - \$436$
 $= \$17551.33$
- (ii) $X = 4.8 + 12 + 100 \times \18640
 $= \$74.56$
- (iii) Total repaid $= \$436 \times 4 \times 12$
 $= \$20928$
- (e) (i) Difference in longitude $= 145^\circ - 55^\circ = 90^\circ$
- (ii) From the Formula and Data Sheet: 15° longitude $= 1$ hour time difference.
 \therefore Time difference $= 90 \div 15$ hours
 $= 6$ hours (as required)
- (iii) Dubai is 6 hours behind Melbourne but the flight time is 15 hours.



When 9 hours is added to 11:30 pm on Friday, the time in Dubai will be 8:30 am on Saturday.

Question 28

- (a) (i) $P(\text{gold coin}) = \frac{4}{33+4}$
 $= \frac{4}{37}$

- (c) Only 3 women are under the age of 50, compared with 7 men.
 Only 2 men are over the age of 50, compared with 10 women.
 The women's ages are negatively skewed and men's ages are positively skewed.
 The median age for women (55) is 10 years greater than the median age for men (45).
 \therefore Pat's statement is correct.

- (d) (i) By calculator, the correlation coefficient is 0.907, correct to three decimal places.
- (ii) **Note:**
 A spreadsheet gives the equation to be $y = 1.3759x - 29.4844$, to 4 decimal places.
 The question is unclear regarding the value of r to be used (0.9, 0.907 or 0.9068...) and so there is a variety of possible answers.
 In the following solution, unrounded answers are used at all times.

From the Formulae and Data Sheet:

$$\begin{aligned} \text{gradient} &= r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}} \\ &= 0.9068 \dots \times \frac{13.73}{9.05} \\ &= 1.375 \dots \end{aligned}$$

From the Formulae and Data Sheet: y -intercept $= \bar{y} - (\text{gradient} \times \bar{x})$
 $= 66 - (1.375 \dots \times 69.4)$
 $= -29.4765 \dots$

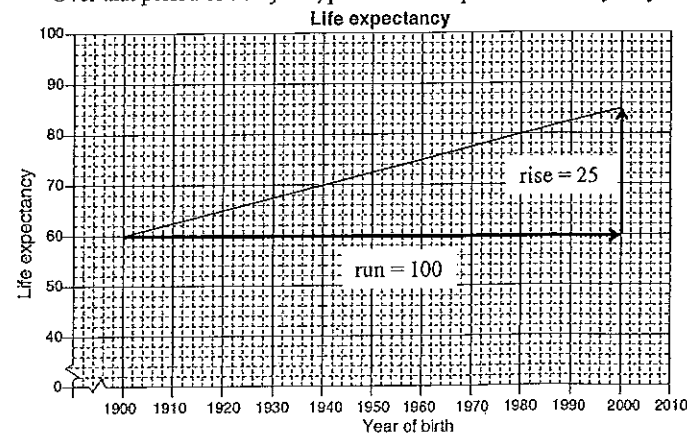
From the Formulae Sheet and Data Sheet, the equation of the line of best fit is:

$$y = \text{gradient} \times x + y\text{-intercept}$$

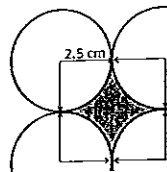
$$\therefore y = 1.375 \dots \times x - 29.4765 \dots$$

Rounding to 1 decimal place gives: $y = 1.4x - 29.5$

- (e) (i) 68 years
- (ii) A person born in 1900 had a predicted life span of 60 years.
 A person born in 2000 has a predicted life span of 85 years.
 Over that period of 100 years, predicted life span increased by 25 years.



- (ii) Consider the shaded region in the diagram. The radius of each circle is 2.5 cm, so it can be inscribed in a square with sides 5 cm.



Shaded area = area of square – area of circle

$$A = 5 \times 5 - \pi \times 2.5^2$$

$$= 5.365\dots$$

There are 6 of these regions and the chocolate in each region is 6 cm deep.

The volume is given by:

$$V = A \times 6 \times 6$$

$$= 193.14165\dots$$

∴ the volume of chocolate required is 193 cm³, correct to the nearest whole number.

Question 29

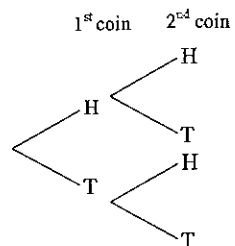
- (a) (i) **Method 1**

There are 4 possibilities: HH, HT, TH and TT.

In 2 out of 4 of these one coin shows heads and the other shows tails.

∴ the required probability is $\frac{1}{2}$ or 0.5 or 50%.

Method 2



$$P(\text{one head and one tail}) = P(HT) + P(TH)$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{2}$$

∴ the required probability is $\frac{1}{2}$ or 0.5 or 50%.

- (ii)

Outcome	Profit	Probability
HH	\$40	0.25
TT	\$20	0.25
HT or TH	-\$30	0.5

$$\text{Financial expectation} = \$40 \times 0.25 + \$20 \times 0.25 + (-\$30) \times 0.5$$

$$= \$0$$

- (b) (i) **Method 1**

When $x = 0$ reading the value of M from the graph gives

$$M = 1.5$$

∴ the value of $A = 1.5$

Method 2

Substitute $x = 0$, $M = 1.5$ into $M = A(1.1)^x$

$$1.5 = A(1.1)^0$$

$$A = 1.5$$

- (ii) In the given equation, 1.1 represents 110%. This indicates that the mass increases by 10% every day.

- (c) The required area is comprised of the sum of:
 Region A: a sector of a circle
 Region B: an isosceles triangle
 Region C: a rectangle

Region A: From the Formula and Data Sheet, the area of a sector is given by:

$$A = \frac{\theta}{360} \pi r^2$$

$$= \frac{(360 - 100)}{360} \times \pi \times 45^2$$

$$= 4594.57\dots$$

Region B: From the Formula and Data Sheet, the area of a triangle is given by:

$$A = \frac{1}{2} ab \sin C$$

$$= \frac{1}{2} \times 45 \times 45 \times \sin 100^\circ$$

$$= 997.117\dots$$

Region C: We need the length of AB .

From the Formula and Data Sheet, the cosine rule is given by:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$AB^2 = 45^2 + 45^2 - 2 \times 45 \times 45 \times \cos 100^\circ$$

$$AB^2 = 4753.27\dots$$

$$AB = \sqrt{4753.27\dots}$$

$$AB = 68.943\dots$$

$$\therefore \text{Area of rectangle} = AB \times 20$$

$$= 1378.879\dots$$

$$\therefore \text{Total area} = 4594.57\dots + 997.117\dots + 1378.879\dots$$

$$= 6970.577\dots$$

∴ the required area is 6971 m², correct to the nearest square metre.

- (d) (i) Given that $x = 88$ and $z = 2.4$ and $\bar{x} = x - sz$: ∴ $\bar{x} = 88 - 2.4s$

- (ii) Given that $x = 52$ and $z = -1.2$ and $\bar{x} = x - sz$: $\bar{x} = 52 - (-1.2)s$
 ∴ $\bar{x} = 52 + 1.2s$

Equation ①: $\bar{x} = 52 + 1.2s$

Equation ②: $\bar{x} = 88 - 2.4s$

Substituting ① into ② gives: $52 + 1.2s = 88 - 2.4s$
 $1.2s + 2.4s = 88 - 52$

$$3.6s = 36$$

$$s = 10$$

Substituting $s = 10$ into ① gives: $\bar{x} = 52 + 1.2s$
 $\bar{x} = 52 + 1.2 \times 10$
 $\bar{x} = 64$

∴ the required values are $\bar{x} = 64$ and $s = 10$.

End of solutions

From the Formula and Data Sheet, gradient is given by:

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

$$= \frac{25}{100}$$

$$= 0.25$$

In this context the gradient represents the average annual rate of increase in predicted life expectancy.

The rate is 0.25 years per year. Note: This is equivalent to 3 months per year.

Question 30

- (a) The water can be considered to be a rectangular prism with base area 30 m^2 .

The height of the prism is $20 \text{ mm} = 0.02 \text{ m}$.

From the Formulae and Data Sheet, the volume of a prism is given by:

$$V = Ah$$

$$= 30 \times 0.02$$

$$= 0.6$$

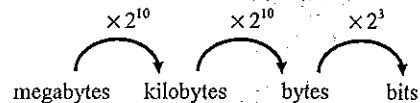
\therefore the volume occupied is 0.6 m^3 .

It is stated on the Formulae and Data Sheet that every cubic metre holds 1000 litres.

\therefore the capacity is $0.6 \times 1000 = 600$ litres.

- (b) (i) Number of gigabytes remaining $= 6.44 - 3.44 = 3$.
The Formulae and Data Sheet indicates that 1 gigabyte = 1024 megabytes.
 \therefore the number of megabytes remaining is $3 \times 1024 = 3072$, as required.

- (ii) Using the Formula and Data Sheet:



$$\therefore 3072 \text{ megabytes} = 3072 \times 2^{23} \text{ bits.}$$

It is given that 1 000 000 bits = 1 megabit.

$$\therefore 3072 \times 2^{23} \text{ bits} = (3072 \times 2^{23} + 1\,000\,000) \text{ megabits}$$

$$= 25\,769.803... \text{ megabits}$$

Also, 7 minutes = 420 seconds

$$\begin{array}{r} \text{Therefore } 25\,769.803... \text{ megabits in } 420 \text{ seconds} \\ + 420 \downarrow \\ \text{Is equivalent to } 61.356... \text{ megabits in } 1 \text{ second} \end{array}$$

\therefore the speed required is 61 Mbps, correct to the nearest whole number.