J.M.J.Ch

MARCELLIN COLLEGE RANDWICK

YEAR 11
ACCELERATED MATHEMATICS
PRELIMINARY TASK 1
2018

STUDENT NAME:

TEACHER: MR CAPIZZI

General Instructions

- Working time – 45 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 1-3
- **Total marks - 41**

Attempt Questions 1 – 3

Start a new page for each question.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 1 (10 marks) | Marks

(a) Evaluate $\frac{15\cdot 7}{\sqrt{1\cdot 6+2\cdot 9}}$ correct to 1 decimal place. | 2

(b) Express $1\cdot\dot{2}\dot{4}$ as a fraction in simplest form. | 2

(c) By rationalising the denominator, express $\frac{8}{3-\sqrt{5}}$ in the form $a+b\sqrt{5}$. | 2

(d) Factorise i) $2x^2+3x-2$. | 1

ii) $2-16x^3$. | 1

(e) Simplify $\frac{x}{x^2-4}+\frac{2}{x-2}$. | 2

Question 2 (17 marks) *START A NEW PAGE* | Marks

(a) Find the values of x for which $|2x+1|\leq 5$. | 2

(b) Solve $4^{2x+1}=64$. | 2

(c) Show that $1-\sqrt{3}$ is a solution of the equation $x^2-2x-2=0$. | 2

(d) i. Sketch the graph of $y=x^2-6$. | 1

ii. On the same set of axes, sketch the graph of $y=|x|$. | 1

iii. Find the coordinates of the two points where the functions intersect. | 2

iv. Hence solve the inequality $x^2-6\leq|x|$. | 2

(e) State the domain and range of the function $y=2\sqrt{25-x^2}$. | 2

(f) Sketch the region defined by: | 3

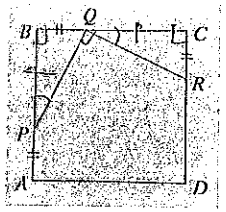
$$y\leq 1-sin2x \quad \text{and} \quad y\geq 1 \qquad \text{for } 0°\leq x\leq 360°.$$

Question 3 (14 marks) ***START A NEW PAGE*** **Marks**

(a) If $sin\beta = \frac{2}{5}$, find the value of $tan\beta$ and $sec\beta$. **2**

(b) Solve $2cos^2\vartheta - 1 = 0$ for $0° \le \vartheta \le 360°$. **3**

(c)

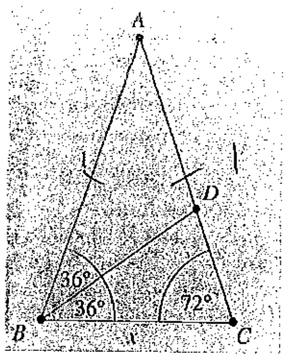


In the diagram, *ABCD* is a square. The points *P*, *Q* and *R* lie on *AB*, *BC* and *CD* respectively, such that *AP* = *BQ* = *CR*.

i. Prove that triangles PBQ and QCR are congruent. **2**

ii. Prove that $\angle PQR$ is a right angle. **2**

(d)



In the diagram *ABC* is an Isosceles triangle where $\angle ABC = \angle BCA = 72°$ and $AB = AC = 1$. $\angle ABC$ is bisected by *BD*, and *BC*$= x$.

i. Show that triangles ABC and BCD are similar. **2**

ii. By using your results in Part i, find the exact value of x. **3**

END OF EXAMINATION

Marcellin College Randwick (1)

Yr 11 Accel. Mathematics

Preliminary task 1 (2018)

SAMPLE SOLUTIONS.

1.

a) $\frac{15.7}{\sqrt{1.6+29}} = 7.4$ (1dp.)

b) $1.2424\ldots$

let this be x

then $100x = 124.2424\ldots$

$100x - x = 123$

$99x = 123$

$x = \frac{123}{99}$

c) $\frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \frac{24+8\sqrt{5}}{9-5}$

$= \frac{24+8\sqrt{5}}{4} = 6+2\sqrt{5}$

d) i) $2x^2+3x-2$

$= (2x-1)(x+2)$

ii) $2-16x^3$

$2(1-8x^3)$

$= -2(2x-1)(4x^2+2x+1)$

e) $\frac{x}{x^2-4} + \frac{2}{x-2}$

$= \frac{x}{(x+2)(x-2)} + \frac{2(x+2)}{(x-2)(x+2)}$

$= \frac{x+2x+4}{(x+2)(x-2)}$

$= \frac{3x+4}{(x+2)(x-2)}$

2. a). $|2x+1| \le 5$.

because both are positive numbers, we can square both sides

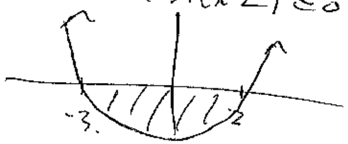
$(2x+1)^2 \le 5^2$

$4x^2+1+4x \le 25$

$4x^2+4x-24 \le 0$

$\cancel{4}(x^2+x-6) \le 0$

$(x+3)(x-2) \le 0$



$-3 \le x \le 2$

b. $4^{2x+1} = 64$

by inspection
$4^3 = 64$

$2x+1 = 3$
$x = 1$

or
$\ln 4^{2x+1} = \ln 64$

$2x+1 = \frac{\ln 64}{\ln 4}$

$2x+1 = 3$
$x = 1$

c) Sub $(1-\sqrt{3})$ for x.

$(1-\sqrt{3})^2 - 2(1-\sqrt{3}) - 2 = 0$

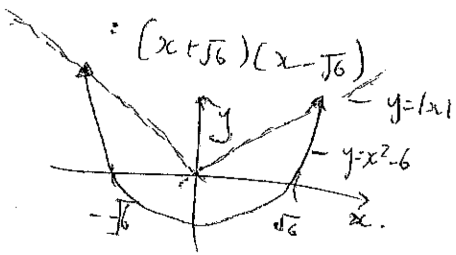
$1+3-2\sqrt{3}-2+2\sqrt{3}-2$

$= 0$

$\therefore 1-\sqrt{3}$ is a solution.

d). $y = x^2 - 6$.

$= (x+\sqrt{6})(x-\sqrt{6})$



$|x| = x^2 - 6$.

Since both are symmetrical about the y axis, we only need to find one intersection.
take $y = x$.

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

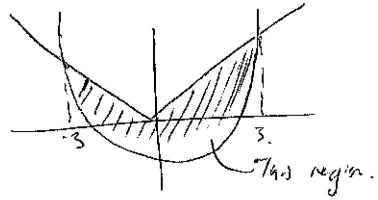
$x = 3, -2$.

we take positive case

$\therefore x = \pm 3$.

So intersect at $(-3,3)$ and $(3,3)$

iV $x^2 - 6 \le |x|$



$-3 \le x \le 3$.

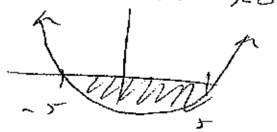
e. $y = 2\sqrt{25-x^2}$

domain $\rightarrow$ $25 - x^2 \ge 0$.

$x^2 \le 25$

$x^2 - 25 \le 0$

$(x+5)(x-5) \le 0$

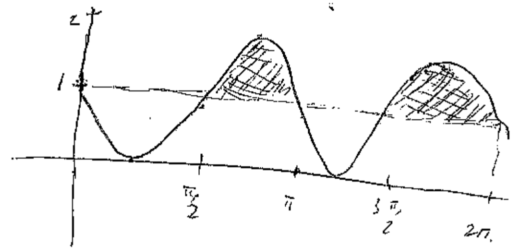


$-5 \le x \le 5$ } DOMAIN

RANGE: $0 \le y \le 10$.

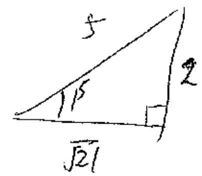
f. $y \le 1 - \sin 2x$.

$y \ge 1$



where $\pi = 180°$

5. $\sin\beta = \frac{2}{5}$.



$\tan\beta = \frac{2}{\sqrt{21}}$

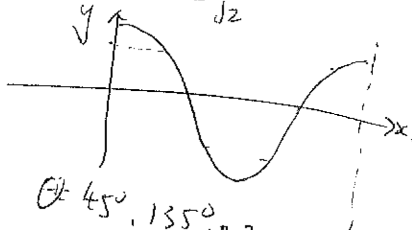
$\cos\beta = \frac{\sqrt{21}}{5}$

$\therefore \sec\beta = \frac{5}{\sqrt{21}}$

6. $2\cos^2\theta - 1 = 0$.

$\cos^2\theta = \frac{1}{2}$.

$\cos\theta = \pm\frac{1}{\sqrt{2}}$



θ: $45°, 135°, 225°, 315°$

c). lets call $AP = BQ = CR = x$.

then $BP = AB - x$.

$CQ = AB - x$.

$\therefore BP = CQ$.

$\angle QCR = \angle PBQ = 90°$ (given)

$CR = QB$ (given)

$\therefore \Delta PBQ \equiv \Delta QCR$ (SAS)

ii) let $\angle BQP$ be $\angle a$

let $\angle BPQ = \angle b$.

$180 = \angle a + \angle b + 90$

$\angle a + \angle b = 90$.

$\angle CQR = \angle BPQ = \angle b$

(corresponding angles of congruent Δ)

$\therefore \angle PQR = 90°$ (sum of straight line)

d) $\angle BDC = 180° - 72° - 36° = 72°$.

$\therefore \Delta ABC \parallel \Delta BCD$ (equiangular)

ii). corresponding sides of similar triangles are in proportion.

ie $\frac{1}{x} = \frac{x}{DC}$.

To find DC consider ΔABD

which is isosceles $\therefore AD = x$.

$\therefore DC = 1 - x$.

$$\frac{1}{x} = \frac{x}{1-x}$$

$1 - x = x^2$

$x^2 + x - 1 = 0$

quadratic formula.

$a = 1,\ b = 1,\ c = -1$

$$\frac{-1 \pm \sqrt{1 - 4(-1)}}{2}$$

$$\frac{-1 \pm \sqrt{5}}{2} = \frac{-1 + \sqrt{5}}{2}$$

since length is +ve.