TEST 8 Algebraic Equations

Marks: /60

Time: 1 hour 30 minutes

Name: ..... Date: .....

INSTRUCTIONS TO CANDIDATES

Section A (30 marks) **Time: 45 minutes**

1. Answer **all** the questions in this section.
2. Calculators may **not** be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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1 Solve the following equations.

(a) $0.2(3x - 4) = 4.7 - 0.5x$

(b) $13 - 2(5 - x) = 3(2x - 5)$

(c) $2x - 5[3 - 2(x + 4)] = 13$

Answer *(a)* $x =$ ..... [1]

(b) $x =$ ..... [1]

(c) $x =$ ..... [2]

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2 Solve the following equations.

(a) $\frac{1}{3x} - \frac{1}{4x} = \frac{1}{24}$

(b) $8 + \frac{2x - 1}{5} = x$

Answer *(a)* $x =$ ..... [1]

(b) $x =$ ..... [1]

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3 Given that $px = 12 - 9x$, find

(a) the value of x when $p = 3$,

(b) the value of p when $x = 3$.

Answer *(a)* $x =$ ..... [1]

(b) $p =$ ..... [1]

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4 Given that $(2a - b)(n - 4) = n(a - 3)$ and that $a = 5$ when $b = -2$, find the value of n. **6**

Answer $n =$ ..... [2]

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5 **(a)** If $r = \frac{p}{p - q} + 1$, find the value of p if $r = 6$ and $q = 2$.

(b) Given that $6x - 9y = 5y + 3x$, calculate the numerical value of $\frac{27x}{28y}$. **7**

Answer *(a)* $p =$ ..... [2]

(b) ..... [2]

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6 Given that $x^2 + 3x = 18$, find the values of

(a) $2x^2 + 6x + 9$,

(b) $x(x^2 + 3x) + 6x^2$.

Answer (a) ..... [1]

(b) ..... [2]

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7 The sum of three consecutive odd numbers is 129. Find the largest odd number.

Answer ..... [3]

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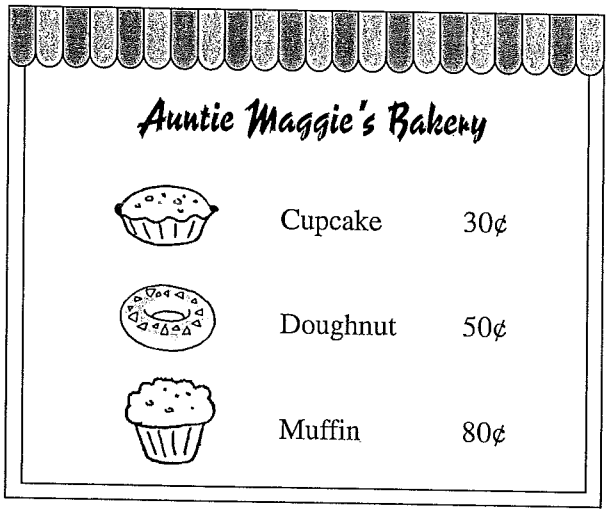
8 Mrs Lee bought x cupcakes and twice as many doughnuts. She also bought 2 fewer muffins than cupcakes. If she spent \$11 altogether, find

(a) the value of x,

(b) the amount she spent on muffins.



Answer (a) $x =$ ..... [2]

(b) \$ ..... [1]

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9 Andy has 20 marbles and Steven has 172 marbles. After each of them bought x marbles, Steven has five times as many marbles as Andy. Find the value of x.

Answer $x =$ ..... [2]

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10 Isabelle counted 25 coins in her coin box. Some of the coins were 10-cent coins while the rest were 20-cent coins.

(a) If the number of 10-cent coins Isabelle had is x, write down an expression for the number of 20-cent coins she had.

(b) Write down an expression, in terms of x, for the total value, in cents, of the 25 coins.

(c) Isabelle's brother Tom also had 25 coins in his coin box. The number of 20-cent coins he had is x and the rest of his coins were 10-cent coins. If Isabelle has 70 cents more than her brother, form an equation in x and find the value of x.

Answer *(a)* ..... [1]

(b) ..... cents [1]

(c) $x =$ ..... [3]

INSTRUCTIONS TO CANDIDATES

Section B (30 marks) **Time: 45 minutes**

1. Answer **all** the questions in this section.
2. Calculators may be used in this section.
3. All working must be clearly shown. Omission of essential working will result in loss of marks.
4. The marks for each question is shown in brackets [ ] at the end of each question.

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11 Solve the following equations.

(a) $x^2 + 2x(x + 1) = 3(x^2 - x) + 10$

(b) $\frac{x-1}{3} - \frac{x-2}{6} = \frac{x-4}{9}$

(c) $\frac{3x}{4} - \frac{2(x-1)}{3} = 1 - \frac{2x+3}{6}$

Answer *(a)* $x =$ ..... [2]

(b) $x =$ ..... [2]

(c) $x =$ ..... [3]

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12 **(a)** Given that $x = 2f + 1$ and $y = \frac{f}{2} - 1$,

(i) express $3x + 2y - 7$ in terms of f,

(ii) find the values of f, x and y when $3x + 2y - 7 = 8$.

(b) **(i)** Simplify $(-2x^2 + 5x - 11) - (-8x^2 - 9x + 7)$.

(ii) Find the values of a, b and c if
$(-2x^2 + 5x - 11) - (-8x^2 - 9x + 7) = [(ax^2 + bx + 3) - 2(x^2 - 2x) - x - c]$.

Answer (a) (i) ..... [1]

(ii) f = ...

x = ...

y = ... [2]

(b) (i) ..... [2]

(ii) a = ...

b = ...

c = ... [2]

13 Marc bought 5 cloth badges and 8 metal badges. The total cost of the badges was \$9.40. If each metal badge costs 20 cents more than each cloth badge, find the cost of a cloth badge and the cost of a metal badge.

Answer Cloth badge = \$.....

Metal badge = \$ ..... [4]

14 **(a)** The sum of two numbers is 49 and the smaller number is x.

(i) Express the larger number in terms of x.

(ii) $\frac{1}{3}$ of the smaller number is greater than $\frac{1}{5}$ of the larger number by 3. Find the value of x.

(b) Peter bought y CDs for \$148 at a music shop. He also bought 5 VCDs. Each VCD costs half as much as a CD.

(i) Write down in terms of y the price of each CD, assuming that all the CDs cost the same.

(ii) If he spent \$46.25 on the VCDs, find the value of y.

Answer (a) (i) ..... [1]

(ii) x = ... [2]

(b) (i) \$ ..... [1]

(ii) y = ... [2]

15 A toy salesman is paid a weekly salary of $\$A$, which is made up of a basic wage of \$120 plus 15 cents for each of the n toys that he sells. The formula connecting A and n in this case is

$$A = 120 + \frac{15n}{100}$$

(a) Calculate the amount of money the salesman earned in a week when he sold 265 toys.

(b) At the end of another week, the salesman earned \$228.90. How many toys did he sell?

(c) At the end of the year, his weekly salary was revised to a basic wage of \$105 plus 18 cents for each of the toys sold. Write down a formula connecting A and n.

(d) Find the number of toys the salesman needs to sell in a week for him to receive the same amount of money from using either formula.

Answer *(a)* \$ ..... [1]

(b) ..... toys [2]

(c) ..... [1]

(d) ..... toys [2]

Test 8: Algebraic Equations

Section A

1. (a) $0.2(3x - 4) = 4.7 - 0.5x$

$0.6x - 0.8 = 4.7 - 0.5x$

$0.6x - 0.8 + \mathbf{0.8} = 4.7 - 0.5x + \mathbf{0.8}$ Add 0.8 to both sides.

$0.6x = 5.5 - 0.5x$

$0.6x + \mathbf{0.5x} = 5.5 - 0.5x + \mathbf{0.5x}$ Add 0.5x to both sides.

$1.1x = 5.5$

$x = \frac{5.5}{1.1}$ Divide both sides by 1.1

$x = \mathbf{5}$

(b) $13 - 2(5 - x) = 3(2x - 5)$

$13 - 10 + 2x = 6x - 15$

$3 + 2x = 6x - 15$

$3 + 2x - \mathbf{2x} = 6x - 15 - \mathbf{2x}$ Subtract $2x$ from both sides.

$3 = 4x - 15$

$3 + \mathbf{15} = 4x - 15 + \mathbf{15}$ Add 15 to both sides.

$18 = 4x$

$x = \frac{18}{4}$ Divide both sides by 4.

$= \mathbf{4\frac{1}{2}}$

(c) $2x - 5[3 - 2(x + 4)] = 13$ Simplify expression within innermost brackets first.

$2x - 5[3 - 2x - 8] = 13$

$2x - 5[-5 - 2x] = 13$

$2x + 25 + 10x = 13$

$12x + 25 = 13$

$12x + 25 - \mathbf{25} = 13 - \mathbf{25}$ Subtract 25 from both sides.

$12x = -12$

$x = \frac{-12}{12}$

$= \mathbf{-1}$

2. (a) $\frac{1}{3x} - \frac{1}{4x} = \frac{1}{24}$ Multiply both sides by $24x$, the LCM of $3x$, $4x$ and 24.

$8 - 6 = x$

$x = \mathbf{2}$

(b) $8 + \frac{2x - 1}{5} = x$

$40 + 2x - 1 = 5x$ Multiply both sides by 5.

$2x + 39 = 5x$

$2x + 39 - \mathbf{2x} = 5x - \mathbf{2x}$ Subtract $2x$ from both sides.

$39 = 3x$

$x = \frac{39}{3}$ Divide both sides by 3.

$= \mathbf{13}$

Teacher's Tip

If an equation contains fractions, eliminate them by multiplying both sides of the equation by the LCM of the denominators. Then solve the resulting equation.

3. (a) $px = 12 - 9x$

When $p = 3$,

$3x = 12 - 9x$

$12x = 12$

$x = \frac{12}{12} = \mathbf{1}$

(b) $px = 12 - 9x$

When $x = 3$,

$3p = 12 - 27$

$= -15$

$p = \frac{-15}{3} = \mathbf{-5}$

4. $(2a - b)(n - 4) = n(a - 3)$

$[2(5) - (-2)](n - 4) = n(5 - 3)$

$(10 + 2)(n - 4) = 2n$

$12(n - 4) = 2n$

$12n - 48 = 2n$

$10n = 48$

$n = \frac{48}{10} = \mathbf{4.8}$

Teacher's Tip

Substitute the given values of a and b into the equation and simplify before expanding.

5. (a) $r = \frac{p}{p - q} + 1$

$6 = \frac{p}{p - 2} + 1$

$\frac{p}{p - 2} = 5$

$p = 5(p - 2)$ Multiply both sides by $(p - 2)$.

$p = 5p - 10$

$4p = 10$

$p = \frac{10}{4} = \mathbf{2\frac{1}{2}}$

(b) $6x - 9y = 5y + 3x$

$6x - 3x = 5y + 9y$ Collect like terms together.

$3x = 14y$

$\frac{x}{y} = \frac{14}{3}$ Divide both sides by $3y$.

$\therefore \frac{27}{28}\frac{x}{y} = \frac{\cancel{27}^{9}}{\cancel{28}_{2}}\left(\frac{\cancel{14}^{1}}{\cancel{3}_{1}}\right)$

$= \frac{9}{2}$

$= \mathbf{4\frac{1}{2}}$

6. $x^2 + 3x = 18$ (Given)

(a) $2x^2 + 6x + 9$

$= 2(x^2 + 3x) + 9$

$= 2(18) + 9$

$= 36 + 9$

$= \mathbf{45}$

(b) $x(x^2 + 3x) + 6x^2$
$= x(18) + 6x^2$
$= 6x^2 + 18x$
$= 6(x^2 + 3x)$
$= 6(18)$
$= \mathbf{108}$

7. Let the smallest of the 3 consecutive odd numbers by x.
$\therefore$ the other odd numbers are $x + 2$ and $x + 4$.
$x + (x + 2) + (x + 4) = 129$
$3x + 6 = 129$
$3x = 123$
$x = \frac{123}{3}$
$= 41$
$\therefore$ the largest odd number $= 41 + 4 = \mathbf{45}$

Teacher's Tip
Make it a habit to always check your answer.
Sum = 41 + 43 + 45 = 129

8. (a) No. of cupcakes bought = x
No. of doughnuts bought = $2x$
No. of muffins bought = $x - 2$
$30x + 50(2x) + 80(x - 2) = 1100$
$30x + 100x + 80x - 160 = 1100$
$210x = 1260$
$x = \frac{1260}{210}$
$= \mathbf{6}$

Express all amount in cents.

(b) No. of muffins bought $= 6 - 2$
$= 4$
Amount spent on muffins $= 4 \times 80¢$
$= 320¢$
$= \mathbf{\$3.20}$

9. No. of marbles Andy has now $= 20 + x$
No. of marbles Steven has now $= 172 + x$
$172 + x = 5(20 + x)$
$172 + x = 100 + 5x$
$72 = 4x$
$x = \frac{72}{4} = \mathbf{18}$

Check answer: $\frac{20 + 18}{172 + 18} = \frac{38}{190} = \frac{1}{5}$

10. (a) No. of 20-cent coins Isabelle had
$= \mathbf{25 - x}$

(b) Total value of Isabelle's coins
$= 10x + (25 - x)20$
$= 10x + 500 - 20x$
$= \mathbf{[500 - 10x]}$ cents

(c) Total value of Tom's coins
$= (25 - x)10 + 20x$
$= 250 - 10x + 20x$
$= [250 + 10x]$ cents

$500 - 10x = (250 + 10x) + 70$
$500 - 10x = 320 + 10x$
$180 = 20x$
$x = \frac{180}{20} = \mathbf{9}$

Given that Isabelle has 70 cents more than Tom.

Check answer:
Total value of Isabelle's coins = 500 − 10(9) = 410¢
Total value of Tom's coins = 250 + 10(9) = 340¢
Difference = 410¢ − 340¢ = 70¢

Section B

11. (a) $x^2 + 2x(x + 1) = 3(x^2 - x) + 10$
$x^2 + 2x^2 + 2x = 3x^2 - 3x + 10$
$3x^2 + 2x = 3x^2 - 3x + 10$
$3x^2 + 2x - \mathbf{3x^2} = 3x^2 - 3x + 10 - \mathbf{3x^2}$
$2x = -3x + 10$
$5x = 10$
$x = \frac{10}{5} = \mathbf{2}$

Subtract $3x^2$ from both sides.

(b) $\frac{x - 1}{3} - \frac{x - 2}{6} = \frac{x - 4}{9}$
$6(x - 1) - 3(x - 2) = 2(x - 4)$
$6x - 6 - 3x + 6 = 2x - 8$
$3x = 2x - 8$
$x = \mathbf{-8}$

Multiply both sides by 18, the LCM of 3, 6 and 9.

(c) $\frac{3x}{4} - \frac{2(x - 1)}{3} = 1 - \frac{2x + 3}{6}$
$3(3x) - 8(x - 1) = 12 - 2(2x + 3)$
$9x - 8x + 8 = 12 - 4x - 6$
$x + 8 = 6 - 4x$
$5x = -2$
$x = \mathbf{-\frac{2}{5}}$

Multiply both sides by 12, the LCM of 4, 3 and 6.

12. (a) $x = 2f + 1$, $y = \frac{f}{2} - 1$ (Given)

(i) $3x + 2y - 7$
$= 3(2f + 1) + 2\left(\frac{f}{2} - 1\right) - 7$
$= 6f + 3 + f - 2 - 7$
$= \mathbf{7f - 6}$

(ii) $3x + 2y - 7 = 8$

$7f - 6 = 8$ Express $3x + 2y - 7$ as $7f - 6$.

$7f = 14$

$f = \frac{14}{7} = \mathbf{2}$

$x = 2(2) + 1 = \mathbf{5}$

$y = \frac{2}{2} - 1 = \mathbf{0}$

(b) (i) $(-2x^2 + 5x - 11) - (-8x^2 - 9x + 7)$

$= -2x^2 + 5x - 11 + 8x^2 + 9x - 7$

$= \mathbf{6x^2 + 14x - 18}$

(ii) $[(ax^2 + bx + 3) - 2(x^2 - 2x) - x - c]$

$= ax^2 + bx + 3 - 2x^2 + 4x - x - c$ Simplify and collect like terms together.

$= ax^2 - 2x^2 + bx + 3x + 3 - c$

$= (a - 2)x^2 + (b + 3)x + (3 - c)$

$(-2x^2 + 5x - 11) - (-8x^2 - 9x + 7) = [(ax^2 + bx + 3) - 2(x^2 - 2x) - x - c)]$

from (b) (i) — simplify from above

$6x^2 + 14x - 18 = (a - 2)x^2 + (b + 3)x + (3 - c)$

Comparing the coefficient of x^2 term:

$6 = a - 2$

$\therefore a = \mathbf{8}$

Comparing the coefficient of x term:

$14 = b + 3$

$\therefore b = \mathbf{11}$

Comparing the constant term:

$-18 = 3 - c$

$c = \mathbf{21}$

Teacher's Tip

1. Algebraic terms having the same variable are called **like terms**.
   E.g. $2x$ and $3x$ are like terms.
   $8x^2$ and $9x^2$ are like terms.
2. The algebraic term that does not have a variable associated with it is called the **constant term**.
   E.g. In the expression $5x + 8$, 8 is the constant term.
3. In the term $5x$, 5 is called the **coefficient** of the term.
   E.g. The coefficient of $2x^2$ is 2 and the coefficient of $-3x$ is -3.

13. Let the cost of each cloth badge be x cents.

$\therefore$ the cost of each metal badge $= (x + 20)$ cents.

Cost of 5 cloth badges $= 5x$ cents

Cost of 8 metal badges $= 8(x + 20)$ cents

Total cost = \$9.40 = 940 cents

$5x + 8(x + 20) = 940$

$5x + 8x + 160 = 940$

$13x = 780$

$x = \frac{780}{13} = 60$

$\therefore$ the cost of each cloth badge is **60 cents** and the cost of each metal badge is $60 + 20 = \mathbf{80}$ **cents**.

Teacher's Tip

It is a good habit to check whether your answer is correct.

Total cost $= 5(60) + 8(80)$

$= 300 + 640$

$= 940$¢

14. (a) (i) Larger no. $= \mathbf{49 - x}$

(ii) $\frac{1}{3}x = \frac{1}{5}(49 - x) + 3$ Given that $\frac{1}{3}$ of the smaller no. (x) is greater than $\frac{1}{5}$ of the larger no. $(49 - x)$ by 3.

$5x = 3(49 - x) + 45$ Multiply both sides by 15, the LCM of 3 and 5.

$5x = 147 - 3x + 45$

$8x = 192$

$x = \frac{192}{8} = \mathbf{24}$

Check answer:
Smaller no. = 24, Larger no = 49 − 24 = 25
Sum of both nos. = 24 + 25 = 49

(b) (i) Cost of each CD $= \$\left(\frac{\mathbf{148}}{y}\right)$

(ii) Cost of each VCD $= \frac{1}{2}\left(\frac{\$148}{y}\right)$

$= \$\left(\frac{74}{y}\right)$

Cost of 5 VCDs = \$46.25 (Given)

Cost of each VCD $= \frac{\$46.25}{5} = \$9.25$

$\therefore \$\left(\frac{74}{y}\right) = \$9.25$

$74 = 9.25y$ Multiply both sides by y.

$y = \frac{74}{9.25}$

$= \mathbf{8}$

Check answer: Cost of each CD $= \frac{\$148}{8}$

$= \$18.50$

Cost of each VCD $= \frac{\$18.50}{2}$

$= \$9.25$

15. (a) $A = 120 + \frac{15n}{100}$

When $n = 265$,

$A = 120 + \frac{15(265)}{100}$

$= 120 + 39.75$

$= 159.75$

$\therefore$ amount earned is **\$159.75**.

(b) When $A = 228.90$

$$228.90 = 120 + \frac{15n}{100}$$

$$\frac{15n}{100} = 108.90$$

$$n = \frac{108.90 \times 100}{15}$$

$$= 726$$

$\therefore$ he sold **726 toys**.

(c) $A = \mathbf{105 + \frac{18n}{100}}$

Teacher's Tip

The original equation $A = 120 + \frac{15n}{100}$

$$= \text{basic wage in \$} + \frac{\left(\begin{array}{c}\text{pay per}\\ \text{toy sold in ¢}\end{array}\right) n}{100}$$

$\therefore$ substitute basic wage with \$105 and pay per toy sold with 18 cents to get the new formula.

(d) For him to receive the same amount,

$$\therefore 105 + \frac{18n}{100} = 120 + \frac{15n}{100}$$

$$\frac{18n}{100} - \frac{15n}{100} = 120 - 105$$

$$\frac{3n}{100} = 15$$

$$n = \frac{15 \times 100}{3}$$

$$= 500$$

$\therefore$ he needs to sell **500 toys** in order to receive the same amount of money from using either formula.

Check answer:

$105 + \frac{18(500)}{100} = 195$

$120 + \frac{15(500)}{100} = 195$

Mid-Year Examination
Specimen Paper A: Part 1

1. (a) $26.62 - 8.857 = \mathbf{17.763}$

(b) $15\frac{1}{3} - 8\frac{3}{5}$

$= 15\frac{5}{15} - 8\frac{9}{15}$ The LCM of 3 and 5 is 15.

$= 14\frac{20}{15} - 8\frac{9}{15}$

$= \mathbf{6\frac{11}{15}}$

2. (a) $0.005845 \approx \mathbf{0.00585}$ (correct to 3 sig. fig.)

Teacher's Tip

Zeros preceding the first non-zero digit are not significant.

(b) $0.005845 \approx \mathbf{0.01}$ (correct to 2 d.p.)

3. (a) 1.56×0.85

$= \frac{15.6}{10} \times \frac{8.5}{10}$

$= \frac{(15.6 \times 8.5)}{100}$ $15.6 \times 8.5 = 132.6$ (Given)

$= \frac{132.6}{100}$

$= \mathbf{1.326}$

(b) $15.6 \times 8.5 = 132.6$ (Given)

$\frac{132.6}{15.6} = 8.5$

$1.326 \div 156$

$= \frac{132.6}{100} \div 15.6 \times 10$

$= \frac{132.6}{100} \times \frac{1}{15.6 \times 10}$

$= \left(\frac{132.6}{15.6}\right) \times \frac{1}{1000}$ $\frac{132.6}{15.6} = 8.5$ (Given)

$= \frac{8.5}{1000}$

$= \mathbf{0.0085}$

4. (a) $\frac{\sqrt[3]{216} + 3^2}{6 \times \sqrt{81}}$ $\sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6$

$= \frac{6 + 9}{6 \times 9}$

$= \frac{\cancel{15}^{5}}{\cancel{6}_{2} \times 9}$

$= \mathbf{\frac{5}{18}}$

(b) $\{[(25 - 31) \times (-24) \div 8] - (-5 - 12)\} \div 7$

$= \{[(-6) \times (-24) \div 8] - (-17)\} \div 7$

$= (18 + 17) \div 7$

$= 35 \div 7$

$= \mathbf{5}$

5. (a) Required fraction $= \frac{1}{2}\left(\frac{1}{9} + \frac{4}{9}\right)$

$= \frac{1}{2}\left(\frac{5}{9}\right)$

$= \mathbf{\frac{5}{18}}$