SYDNEY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT
HALF-YEARLY EXAMINATIONS 2005

FORM III

MATHEMATICS

Examination date

Wednesday 18th May 2005

Time allowed

1 hour 30 minutes

Instructions

All eight questions may be attempted.
All eight questions are of equal value.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Calculators are not to be used.

Collection

Write your name, class and master clearly on the front.
Hand in all the writing paper in a single well-stapled bundle.
Keep the printed examination paper and bring it to your next Mathematics lesson.

3A: MLS	3B: GJ	3C: FMW/DS
3D: KWM/JNC	3E: JCM/BJC	3F: JMR
3G: PKH	3H: DNW	3I: WMP

Checklist

Writing paper required.
Candidature: 185 boys.

Examiner

DNW

QUESTION ONE Start a new page.

(a) Evaluate $10\frac{3}{4} - 2\frac{1}{4}$.

(b) Round 2·718 28 correct to two decimal places.

(c) What is the perimeter of a square if the length of one side is 5 cm?

(d) Find the area of a triangle with base 7 mm and perpendicular height 4 mm.

(e) Simplify $8x - y - 5x$.

(f) Evaluate $6 - 2c$ when $c = -3$.

(g) Use the index laws to simplify:

(i) $a^6 \div a^4$

(ii) $(b^3)^5$

(h) Simplify:

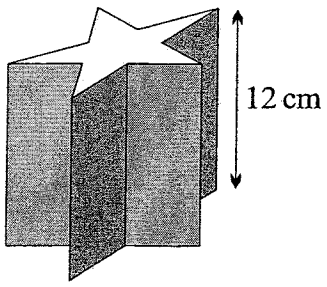
(i) $5\sqrt{3} - 2\sqrt{3}$

(ii) $\sqrt{20}$

(i) The diameter of the moon Titan is 202,752,000 inches. Write this number in scientific notation, correct to four significant figures.

QUESTION TWO Start a new page.

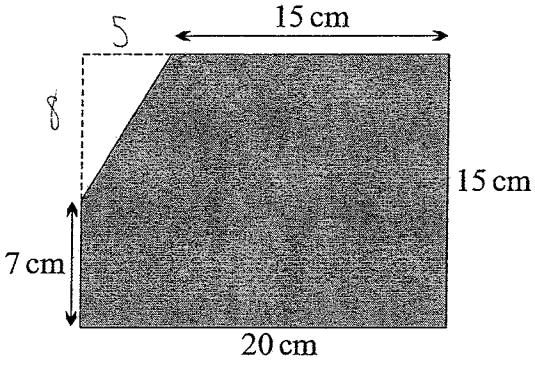
(a)



The prism above has a star-shaped base of area 5 cm^2. What is the volume of the prism?

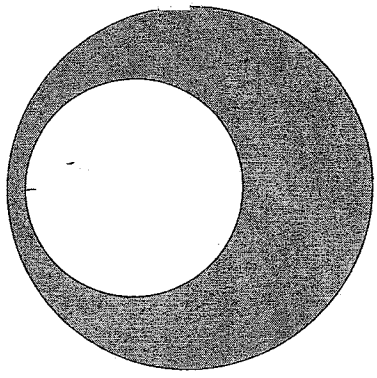
Exam continues next page ...

(b)



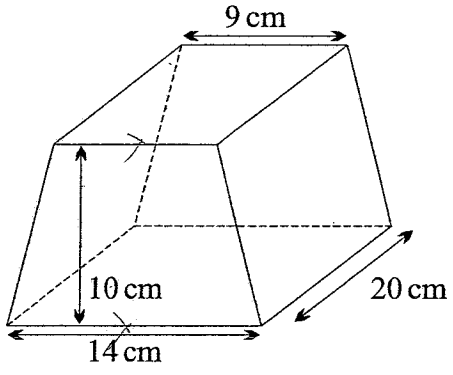
The shaded figure above is the result of cutting a corner off a rectangle. Find the shaded area.

(c)



In the diagram above, the inner circle has radius 3 cm and the outer circle has radius 5 cm. Find the area of the shaded region between the two circles, leaving your answer in terms of π.

(d)



A special container has dimensions as shown above. The shape of the front end is a trapezium.

(i) What is the area of the trapezium?

(ii) Find the volume of the container in millilitres.
(Remember that $1\,\text{cm}^3 = 1\,\text{mL}$.)

(iii) How much will it cost to fill the container with an expensive chemical, which costs 45 cents per mL?

Exam continues overleaf ...

QUESTION THREE Start a new page.

(a) Evaluate:

(i) $27^{\frac{1}{3}}$

(ii) $16^{\frac{3}{2}}$

(b) Simplify:

(i) $\sqrt{50} - \sqrt{8}$

(ii) $\left(3\sqrt{5}\right)^2$

(c) Rationalise the denominator and simplify $\dfrac{4}{\sqrt{6}}$.

(d) Simplify:

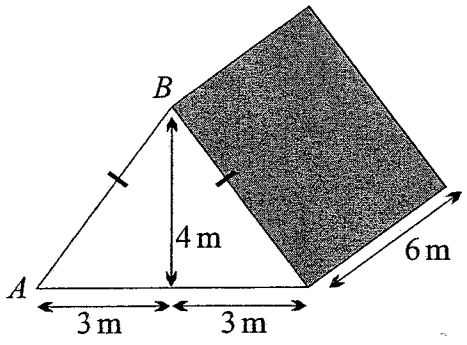
(i) $\dfrac{3}{2q^2} \times \dfrac{q^3}{6}$

(ii) $\dfrac{15a}{4b} \div \dfrac{3ab}{8}$

(iii) $\dfrac{5}{2x} + \dfrac{4}{3x}$

QUESTION FOUR Start a new page.

(a)



The diagram above shows an isosceles triangular prism.

(i) Find the length of the side marked AB.

(ii) Hence find the surface area of the prism.

(b) Solve the following equations:

(i) $3x + 2 = -7$

(ii) $4(x - 3) = 3x - 4$

(iii) $\dfrac{2x}{7} - \dfrac{x}{2} = 1$

Exam continues next page ...

(c) (i) Solve the inequation $2 - 3x \leq 17$.

(ii) Graph the solution on the number line.

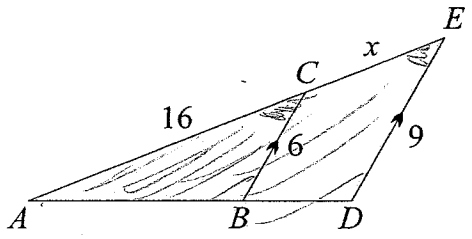
QUESTION FIVE Start a new page.

(a) Consider the formula $s = ut - 5t^2$.

(i) Find s when $u = 22$ and $t = 3$.

(ii) Make u the subject of the equation.

(b)



In the diagram above, $BC \parallel DE$, $BC = 6$, $DE = 9$ and $AC = 16$.

Let the length of CE be x.

(i) Prove that $\triangle ABC \,|||\, \triangle ADE$, giving all reasons.

(ii) Form an equation and solve it to find the value of x.

(c) Simplify:

(i) $\left(4x^6y^2\right)^{\frac{5}{2}}$

(ii) $\left(\dfrac{a^2b^{-3}}{a^{-1}b^4}\right)^{-1}$

QUESTION SIX Start a new page.

(a) Expand and simplify $(x+3)^2 - (x+5)(x-5)$.

(b) Expand and simplify $\left(\sqrt{2}+2\sqrt{3}\right)\left(3\sqrt{2}+4\sqrt{3}\right)$.

(c) Express $\dfrac{2}{\sqrt{11}-1}$ with a rational denominator and in simplest form.

(d) Solve $(x+1)(x-1) = x^2 + 2x + 3$.

Exam continues overleaf ...

QUESTION SEVEN Start a new page.

(a) Solve $\dfrac{1}{x}+\dfrac{1}{x-1}=0$.

(b) Solve the inequation $\dfrac{3x-4}{3}-\dfrac{x+2}{2}<4-x$.

(c) (i) Simplify $\dfrac{p^9q^{-3}}{p(p^2q)^3}$.

(ii) Hence express $\sqrt{\dfrac{p^9q^{-3}}{p(p^2q)^3}}$ in simplest form, and without fractions.

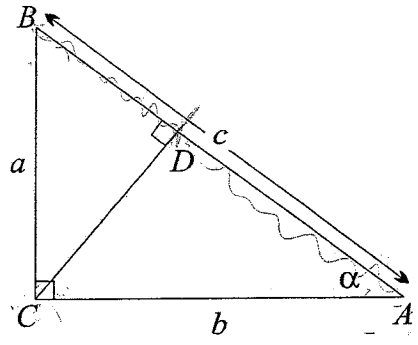
(d) In a certain cylinder, the height h is equal to the diameter. Let the radius of the cylinder be r.

(i) Draw a diagram of the situation and find h in terms of r.

(ii) Show that the ratio of the volume to the surface area is $\dfrac{r}{3}$.

QUESTION EIGHT Start a new page.

(a)



In the diagram above, let $BC=a$, $CA=b$ and $AB=c$.
Let $\angle CAB=\alpha$.

(i) Show that $\triangle ABC \;|||\; \triangle ACD$.

(ii) Hence show that $AD=\dfrac{b^2}{c}$.

(iii) Show that $BD=\dfrac{a^2}{c}$.

(iv) Hence complete a proof of Pythagoras' theorem, that is

$$c^2=a^2+b^2.$$

Exam continues next page ...

(b) The value of $\sqrt{5}$ is approximately 2·236.

Consider the problem of finding the approximate value of $\dfrac{1}{\sqrt{5}}$.

(i) You are not allowed to use calculators in this exam. Why will it be easier to rationalise the denominator first?

(ii) Hence approximate $\dfrac{1}{\sqrt{5}}$ correct to three decimal places.

(c) Let the number x be the difference between $\sqrt{5}$ and the approximation 2·236. That is,

$$x = \sqrt{5} - 2{\cdot}236\,.$$

Let

$$J = \frac{1}{\sqrt{5}} - \frac{\sqrt{5}-x}{5}$$

and let

$$M = \frac{1}{\sqrt{5}-x} - \frac{1}{\sqrt{5}}\,.$$

(i) Show that

$$M - J = \frac{x^2(\sqrt{5}+x)}{5(5-x^2)}\,.$$

(ii) Which of $\dfrac{\sqrt{5}-x}{5}$ and $\dfrac{1}{\sqrt{5}-x}$ is the better approximation to $\dfrac{1}{\sqrt{5}}$?

Explain your reasons carefully.

END OF EXAMINATION

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Candidature: 185 boys.

Examiner

DNW

V.K. Chu

QUESTION ONE Start a new page.

(a) $10\frac{3}{4} - 2\frac{1}{4} = 8\frac{1}{2}$ ☑

(b) $2{\cdot}718\,28 \doteqdot 2{\cdot}72$ (correct to two decimal places) ☑

(c) perimeter $= 4 \times 5$
$= 20\,\text{cm}$ ☑

(d) Area $= \frac{1}{2} \times 7 \times 4$
$= 14\,\text{mm}^2$ ☑

(e) $8x - y - 5x = 3x - y$ ☑

(f) $6 - 2c = 6 - 2 \times (-3)$
$= 12$ ☑

(g) (i) $a^6 \div a^4 = a^2$ ☑

(ii) $(b^3)^5 = b^{15}$ ☑

(h) (i) $5\sqrt{3} - 2\sqrt{3} = 3\sqrt{3}$ ☑

(ii) $\sqrt{20} = 2\sqrt{5}$ ☑

(i) $202\,752\,000 = 2{\cdot}028 \times 10^8$ *scientific notation* ☑
rounding ☑

Total for Question 1: 12 Marks

QUESTION TWO Start a new page.

(a) Volume $= 5 \times 12$
$= 60\,\text{cm}^3$ *answer* ☑
units ☑

(b) $\triangle = \frac{1}{2} \times 5 \times 8$ ☑
$= 20\,\text{cm}^2$ ☑
rect $= 20 \times 15$
$= 300\,\text{cm}^2$ ☑
Total Area $= 300 - 20$
$= 280\,\text{cm}^2$ ☑

Exam continues next page ...

(c) Area $= \pi \times 5^2 - \pi \times 3^2$ ☑
$= 16\pi\,\text{cm}^2$ ☑

(d) (i) Trap $= \frac{1}{2}(9+14) \times 10$
$= 115\,\text{cm}^2$ ☑

(ii) Capacity $= 115 \times 20$
$= 2300\,\text{cm}^3$
$= 2300\,\text{mL}$ ☑

(iii) Cost $= 2300 \times 45\,\text{cents}$ ☑
$= \$1035$ ☑

Total for Question 2: 12 Marks

QUESTION THREE Start a new page.

(a) (i) $27^{\frac{1}{3}} = 3$ ☑

(ii) $16^{\frac{3}{2}} = 4^3$
$= 64$ ☑

(b) (i) $\sqrt{50} - \sqrt{8} = 5\sqrt{2} - 2\sqrt{2}$ ☑
$= 3\sqrt{2}$ ☑

(ii) $\left(3\sqrt{5}\right)^2 = 9 \times 5$
$= 45$ ☑

(c) $\frac{4}{\sqrt{6}} = \frac{4}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$ ☑
$= \frac{4\sqrt{6}}{6}$
$= \frac{2\sqrt{6}}{3}$ ☑

(d) (i) $\frac{3}{2q^2} \times \frac{q^3}{6} = \frac{q}{4}$ ☑

(ii) $\frac{15a}{4b} \div \frac{3ab}{8} = \frac{15a}{4b} \times \frac{8}{3ab}$ ☑
$= \frac{10}{b^2}$ ☑

Exam continues overleaf ...

(iii) $\frac{5}{2x} + \frac{4}{3x} = \frac{15}{6x} + \frac{8}{6x}$ ☑
$= \frac{23}{6x}$ ☑

Total for Question 3: 12 Marks

QUESTION FOUR Start a new page.

(a) (i) $(AB)^2 = 3^2 + 4^2$ (by Pythagoras)
$= 25$
so $AB = 5$ ☑

(ii) Surface Area $= 2 \times \text{front} + 2 \times \text{side} + \text{bottom}$ ☑
$= 2 \times \frac{1}{2} \times 6 \times 4 + 2 \times 5 \times 6 + 6 \times 6$
$= 120\text{m}^2$ ☑

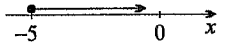
(b) (i) $3x + 2 = -7$
-2 $3x = -9$ ☑
$\div 3$ $x = -3$ ☑

(ii) $4(x-3) = 3x - 4$
expand the brackets to get
$4x - 12 = 3x - 4$ ☑
$+12$ $4x = 3x + 8$
$-3x$ $x = 8$ ☑

(iii) $\frac{2x}{7} - \frac{x}{2} = 1$
$\times 14$ $4x - 7x = 14$ ☑
so $-3x = 14$
$\div(-3)$ $x = -\frac{14}{3}$ ☑

(c) (i) $2 - 3x \leq 17$
-2 $-3x \leq 15$ ☑
$\div(-3)$ $x \geq -5$ ☑

(ii)



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Total for Question 4: 12 Marks

Exam continues next page ...

QUESTION FIVE Start a new page.

(a) (i) $S = 22 \times 3 - 5 \times 3^2$
$= 21$ ☑

(ii) $ut - 5t^2S$

$\boxed{+5t^2}$ $ut = S + 5t^2$ ☑

$\boxed{\div t}$ $u = \dfrac{S+5t^2}{t}$ (or $\frac{S}{t} + 5t$) ☑

(b) (i) In $\triangle ABC$ and $\triangle ADE$

$\angle BAC \angle DAE$ (common) ☑

$\angle ABC \angle ADE$ (corresponding angles on $BC \parallel DE$) ☑

hence $\triangle ABC \,|||\, \triangle ADE$ (AA) ☑

(ii) $\dfrac{AE}{AC} = \dfrac{DE}{BC}$ (ratio of matching sides of similar triangles)

thus $\dfrac{16+x}{16} = \dfrac{3}{2}$ ☑

$\boxed{\times 16}$ $16 + x = 24$

$\boxed{-16}$ $x = 8$ ☑

(c) (i) $(4x^6y^2)^{\frac{5}{2}} = (2x^3y)^5$ ☑

$= 32x^{15}y^5$ ☑

(ii) $\left(\dfrac{a^2b^{-3}}{a^{-1}b^4}\right)^{-1} = \left(\dfrac{a^3}{b^7}\right)^{-1}$ ☑

$= \dfrac{b^7}{a^3}$ (or b^7a^{-3}) ☑

Total for Question 5: 12 Marks

QUESTION SIX Start a new page.

(a) $(x+3)^2 - (x+5)(x-5) = x^2 + 6x + 9 - (x^2 - 25)$ ☑☑

$= 6x + 34$ ☑

(b) $\left(\sqrt{2} + 2\sqrt{3}\right)\left(3\sqrt{2} + 4\sqrt{3}\right) = 6 + 4\sqrt{6} + 6\sqrt{6} + 24$ ☑☑

$= 30 + 10\sqrt{6}$ ☑

Exam continues overleaf ...

(c) $\dfrac{2}{\sqrt{11}-1} = \dfrac{2}{\sqrt{11}-1} \times \dfrac{\sqrt{11}+1}{\sqrt{11}+1}$ ☑

$= \dfrac{2(\sqrt{11}+1)}{10}$ ☑

$= \dfrac{\sqrt{11}+1}{5}$ ☑

(d) $(x+1)(x-1) = x^2 + 2x + 3$

expand the brackets to get

$x^2 - 1 = x^2 + 2x + 3$ ☑

the x^2 terms cancel to leave

$2x + 3 = -1$ ☑

$\boxed{-3}$ $2x = -4$

$\boxed{\div 2}$ $x = -2$ ☑

Total for Question 6: 12 Marks

QUESTION SEVEN Start a new page.

(a) $\dfrac{1}{x} + \dfrac{1}{x-1} = 0$

$\boxed{\times x(x-1)}$ $x - 1 + x = 0$ ☑

$\boxed{+1}$ $2x = 1$

$\boxed{\div 2}$ $x = \frac{1}{2}$ ☑

(b) $\dfrac{3x-4}{3} - \dfrac{x+2}{2} < 4 - x$

$\boxed{\times 6}$ $2(3x-4) - 3(x+2) < 24 - 6x$ ☑

expand to get

$3x - 14 < 24 - 6x$ ☑

$\boxed{+6x - 14}$ $9x < 38$

$\boxed{\div 9}$ $x < \frac{38}{9}$ ☑

(c) (i) $\dfrac{p^9q^{-3}}{p(p^2q)^3} = \dfrac{p^9q^{-3}}{p^7q^3}$ ☑

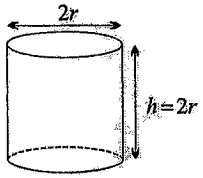
$= \dfrac{p^2}{q^6}$ (or p^2q^{-6}) ☑

(ii) $\sqrt{\dfrac{p^9q^{-3}}{p(p^2q)^3}} = \sqrt{p^2q^{-6}}$ (from part (i))

$= pq^{-3}$ ☑

Exam continues next page ...

(d) (i)



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(ii) $\text{Volume} = \pi r^2 \times 2r$

$= 2\pi r^3$ ☑

$\text{SurfaceArea} = 2\pi r^2 + 2\pi r \times 2r$

$= 6\pi r^2$ ☑

hence $\dfrac{\text{Vol}}{\text{SA}} = \dfrac{2\pi r^3}{6\pi r^2}$ ☑

$= \dfrac{r}{3}$

Total for Question 7: 12 Marks

QUESTION EIGHT Start a new page.

(a) (i) In $\triangle ABC$ and $\triangle ACD$

$\angle BAC = \angle CAD$ (common)

$\angle ACB = \angle ADC$ (given)

hence $\triangle ABC \,|||\, \triangle ACD$ (AA) ☑

(ii) In $\triangle ABC$ and $\triangle ACD$

$\dfrac{AD}{b} = \dfrac{b}{c}$ (matching sides of similar triangles) ☑

hence $AD = \dfrac{b^2}{c}$

(iii) In $\triangle ABC$ and $\triangle CBD$

$\angle ABC = \angle CBD$ (common) ☑

$\angle ACB = \angle CDB$ (given)

hence $\triangle ABC \,|||\, \triangle CBD$ (AA) ☑

so $\dfrac{BD}{a} = \dfrac{a}{c}$ (matching sides of similar triangles) ☑

thus $BD = \dfrac{a^2}{c}$

(iv) $AB = AD + DB$

so $c = \dfrac{b^2}{c} + \dfrac{a^2}{c}$ (from parts (ii) and (iii)) ☑

thus $c^2 = a^2 + b^2$

Exam continues overleaf ...

(b) (i) It is easier to evaluate $\dfrac{2{\cdot}236}{5}$ than $\dfrac{1}{2{\cdot}236}$. ☑

(ii) $\dfrac{1}{\sqrt{5}} \doteqdot \dfrac{2{\cdot}236}{5}$

$\doteqdot 0{\cdot}447$ ☑

(c) (i) $M - J = \dfrac{1}{\sqrt{5}-x} - \dfrac{1}{\sqrt{5}} - \dfrac{1}{\sqrt{5}} + \dfrac{\sqrt{5}-x}{5}$

$= \dfrac{1}{\sqrt{5}-x} + \dfrac{\sqrt{5}-x}{5} - \dfrac{2}{\sqrt{5}}$

$= \dfrac{\sqrt{5}+x}{5-x^2} + \dfrac{\sqrt{5}-x}{5} - \dfrac{2\sqrt{5}}{5}$ ☑

$= \dfrac{\sqrt{5}+x}{5-x^2} - \dfrac{\sqrt{5}+x}{5}$

$= \dfrac{5\left(\sqrt{5}+x\right) - \left(5-x^2\right)\left(\sqrt{5}+x\right)}{5(5-x^2)}$

$= \dfrac{5\sqrt{5}+5x-5\sqrt{5}+x^2\sqrt{5}-5x+x^3}{5(5-x^2)}$ ☑

$= \dfrac{x^2\sqrt{5}+x^3}{5(5-x^2)}$

$= \dfrac{x^2(\sqrt{5}+x)}{5(5-x^2)}$

(ii) J represents the error in the approximation of $\dfrac{1}{\sqrt{5}}$ by $\dfrac{\sqrt{5}-x}{5}$.

M represents the error in the approximation of $\dfrac{1}{\sqrt{5}}$ by $\dfrac{1}{\sqrt{5}-x}$.

☑

Clearly $0 < x^2 < 1$ so $5 - x^2 > 0$. Thus $M - J$ is positive.

That is, the error M is bigger than the error J.

Hence $\dfrac{\sqrt{5}-x}{5}$ is the better approximation to $\dfrac{1}{\sqrt{5}}$.

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Total for Question 8: 12 Marks

END OF EXAMINATION